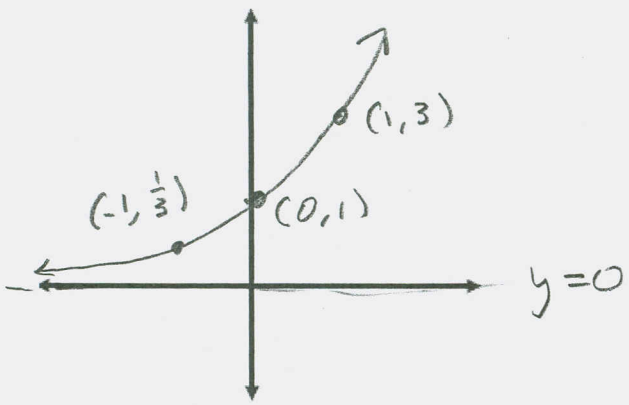
MAT 121-G81 Test 2 Name KEY

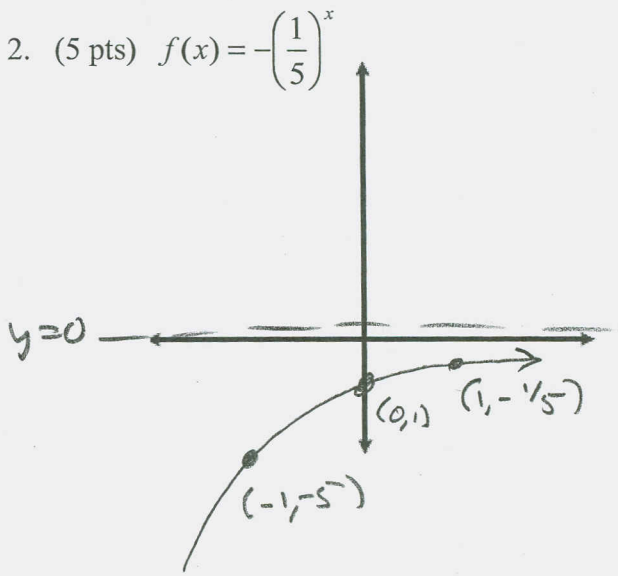
100 Points EIIIE, 12.1 – 12.3

Graph:

1. (5 pts) $f(x)=3^x$



2. (5 pts) $f(x)=-\left(\frac{1}{5}\right)^x$



3. (10 pts) Solve $8=16^{2x+1}$ =

$2^3=(2^4)^{2x+1}=2^{8x+4}$

$3=8x+4$

$-1=8x$

$-\frac{1}{8}=x$

4. (10 pts) Write an exponential function to model the situation. Tell what each variable represents. A population of red ants is initially at 100,000 ants and grows (exponentially) at 20% per week.

$P(t)=100{,}000(1+.2)^t$ was accepted, but

$P(t)=100{,}000e^{.2t}$ was preferred.

t = time (in weeks)

P = Ant Population

This second model is more of a "continuous compounding" type model.

5. (5 pts) Write the equation $\log_x 27=\frac{5}{7}$ in exponential form.

$27=x^{\frac{5}{7}}$

6. (5 pts) Evaluate $\log_3\left(\frac{1}{243}\right)$.

$= \log_3(3^{-5}) = -5$

$3 \mid 243$
$3 \mid 81$
$3 \mid 27$
$3 \mid 9$
3

7. (10 pts) Express $\log_7\left(\sqrt[5]{\frac{5x^3y^7}{z^{11}}}\right)$ in terms of logarithms of x, y, and z.

$= \frac{1}{5}\left[\log_7(5) + 3\log_7 x + 7\log_7 y - 11\log_7 z\right]$

8. (10 pts) Solve correct to four decimal places: $4^x = 3^{2x-1}$

$\ln(4^x) = \ln(3^{2x-1})$

$x \ln 4 = (2x-1)\ln 3 = (2\ln 3)x - \ln 3$

$\Rightarrow (\ln 4)x - (2\ln 3)x = -\ln 3$

$\Rightarrow (\ln 4 - 2\ln 3)x = -\ln 3$

$\Rightarrow x = \frac{-\ln 3}{\ln 4 - 2\ln 3} \approx 1.354755646$

≈ 1.3548

9. (5 pts) Solve: $\log_2(x-7)=4$

$x-7 = 2^4$

$x = 2^4 - 7 = 16+7 = 23 = x$

10. (5 pts) Find the value of the expression: $\left(\frac{1}{8}\right)^{\log_2 7}$

$= \left(2^{-3}\right)^{\log_2 7} = 2^{-3\log_2 7} = 2^{\log_2(7^{-3})} = 7^{-3} = \frac{1}{7^3}$

$\approx .002915451 9$

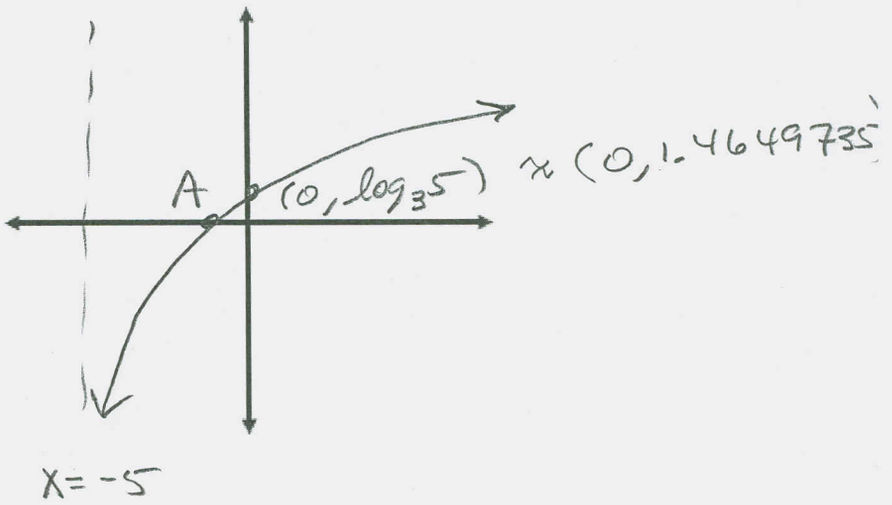
11. (5 pts) Graph: $\log_3(x+5)$

$\log_3(x+5) = 0$

$x+5 = 3^0 = 1$

$x = 1-5 = -4$

$A = (-4, 0)$



12. (5 pts) Write the following as the logarithm of a single expression. Assume that variables represent positive numbers. $3\log_5(x+2) - 4\log_5(x-7) + \log_5 9$

$= \log\left(\frac{(x+2)^3(9)}{(x-7)^4}\right)$

13. (5 pts) The population (in millions of people) of Soretoothistan t years after 2000 is given by $R(t) = 15e^{kt}$. If there are 12 million people in Soretoothistan in 2005, find k.

$R(5) = 12 = 15e^{5k}$

$e^{5k} = \frac{12}{15}$

$5k = \ln\left(\frac{12}{15}\right)$

$k = \frac{\ln\left(\frac{12}{15}\right)}{5} \approx -.044628\,7103$

14. Cobalt-60 is a radioactive substance that decays according to the model $A(t) = A_0 e^{-0.1308t}$, where $A = A(t)$ is the amount of cobalt-60 present at time t (in years).

a. (5 pts) Find the half-life of cobalt-60. You may leave your final answer in terms of $\ln\left(\frac{1}{2}\right)$.

$A_0 e^{-.1308t} = \frac{1}{2} A_0$

$e^{-.1308t} = \frac{1}{2}$

$-.1308t = \ln\left(\frac{1}{2}\right)$

$t = \frac{\ln\left(\frac{1}{2}\right)}{-.1308} \approx 5.29929037\,1$ years.

b. (5 pts) To the nearest 10^{th} of a year, what is the half-life of cobalt-60, according to this model? (Base your answer on your result from part a.)

$5.29929037\,1 \approx$ 5.3 years

15. (10 pts bonus) The half-life of carbon-14 is (approximately) 5730 years.

a. Find an exponential model $A(t) = A_0 e^{kt}$ that gives the amount of radioactive carbon-14 present in a charcoal sample after t years.

$A_0 e^{5750k} = \frac{1}{2} A_0$

$e^{5750k} = \frac{1}{2}$

$5730k = \ln(\frac{1}{2})$

$k = \frac{\ln(\frac{1}{2})}{5730} \approx -1.209680894 \times 10^{-4}$

$\approx -.00012$

$A(t) = A_0 e^{-.00012t}$

b. How old is a sample from a neolithic fire pit if it is found that 30% of naturally-occurring carbon-14 is present in the sample?

$A(t) = .3A_0$

$A_0 e^{kt} = .3A_0$

$e^{kt} = .3$

$kt = \ln(.3)$

$t = \frac{\ln(.3)}{k} \approx 9952.812857$

based on 1st Answer: $k \approx -.000120968094$

Based on $k = -.00012$:

$t \approx 10{,}033.1067$ yrs.

Both are close to 10,000 years

16. (5 pts) Solve the equation: $\log_3 x + \log_3(x-24) = 4$.

$\log_3(x(x-24)) = 4$

$x^2 - 24x = 3^4 = 81$

$x^2 - 24x - 81 = 0$

$x^2 - 27x + 3x - 81 = 0$

$x(x-27) + 3(x-27) = 0$

$(x-27)(x+3) = 0 \Rightarrow x \in \{-3, 27\}$

$x = -3 \notin$ Domain

Scratch:

$81 = 3 \cdot 3 \cdot 3 \cdot 3$

$27 - 3 = 24$

Check:

$\log_3 27 + \log_3 3 = 3 + 1 = 4$ ✓

$x = 27$ FINAL ANS