Practice Test 3

State whether the function is a polynomial function or not. If it is, give its degree. If it is not, tell why not.

1) $f(x) = 14x^5 + 4x^4 + 6$

Yes, Degree 5

2) $f(x) = \dfrac{8 - x^5}{5}$

$= \frac{8}{5} - \frac{1}{5}x^5$

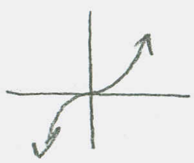
Yes. Degree 5

3) $f(x) = 1 + \dfrac{9}{x} = 1 + 9x^{-1}$

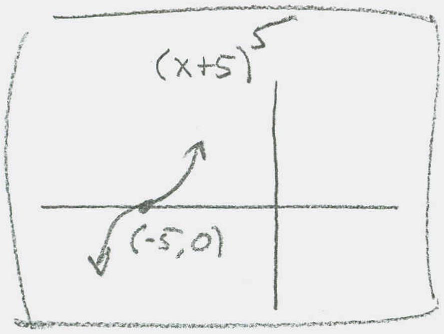
No. There's a negative power of x.

Use transformations of the graph of $y = x^4$ or $y = x^5$ to graph the function.

4) $f(x) = (x + 5)^5$



x^5



Form a polynomial whose zeros and degree are given.

5) Zeros: 3, multiplicity 2; −3, multiplicity 2; degree 4

$\boxed{(x-3)^2(x+3)^2}$ → Most of the points.

$= (x^2 - 6x + 9)(x^2 + 6x + 9)$

$$
\begin{aligned}
= \quad & x^4 + 6x^3 + 9x^2 \\
& \quad - 6x^3 - 36x^2 - 54x \\
& \qquad + 9x^2 + 54x + 81 \\
\hline
& x^4 - 36x^2 + 81
\end{aligned}
$$

$\boxed{f(x) = x^4 - 36x^2 + 81}$

↳ The 090 part (minor)

For the polynomial, list each real zero and its multiplicity. Determine whether the graph crosses or touches the x–axis at each x –intercept.

6) $f(x) = 3(x - 7)(x - 1)^3$

$x = 7$, $m = 1$, crosses

$x = 1$, $m = 3$, crosses

Looks like $y = -18(x-1)^3$ near $x = 1$.

$3(1-7)(x-1)^3 = -(3)(6)(x-1)^3$

$= -18(x-1)^3$

$3(x-7)(7-1)^3 = (3)(6)^3(x-7)$ is line with steep positive slope at $x = 7$

7) $f(x) = 3(x^2 + 4)(x^2 + 1)^2$

No real zeros!

Practice Test 3

Find the x– and y–intercepts of f.

8) $f(x) = (x + 11)^2$

$f(0) = (0+11)^2 = 11^2 = 121 \leadsto$ $\boxed{(0,121) \text{ is y-int.}}$

$f(x) = 0 \Rightarrow x = -11 \leadsto$ $\boxed{(-11,0) \text{ is x-int}}$

Find the power function that the graph of f resembles for large values of |x|.

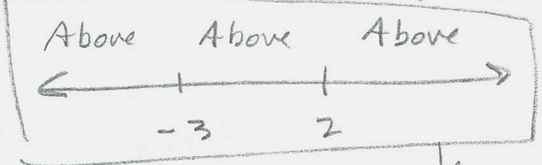
9) $f(x) = (x - 1)^6(x + 12)^2$ This is end behavior, E.B.:

For polynomials, just look at the big stuff.

$|x| \to \text{Big} \Rightarrow f(x) \to (x)^6(x)^2 = x^8$ $\nwarrow \ldots \nearrow$

Use the x–intercepts to find the intervals on which the graph of f is above and below the x–axis.

10) $f(x) = (x - 2)^2(x + 3)^2$ $\underset{\text{touch}}{x=2,\ m=2}$; $\underset{\text{touch}}{x=-3,\ m=2}$



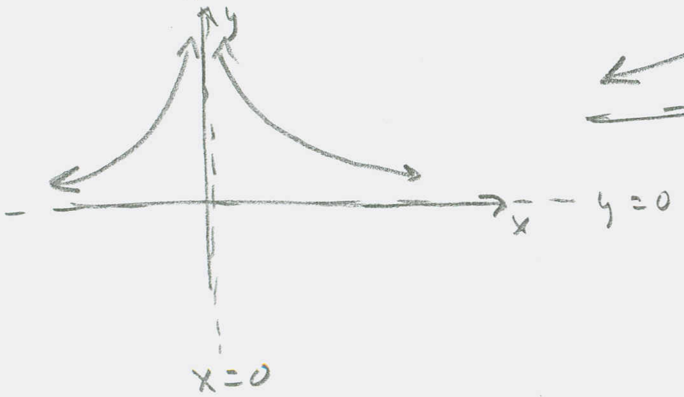
FINAL ANS

E.B.: $(x)^2(x)^2 = x^4$ $\nwarrow \ldots \nearrow$

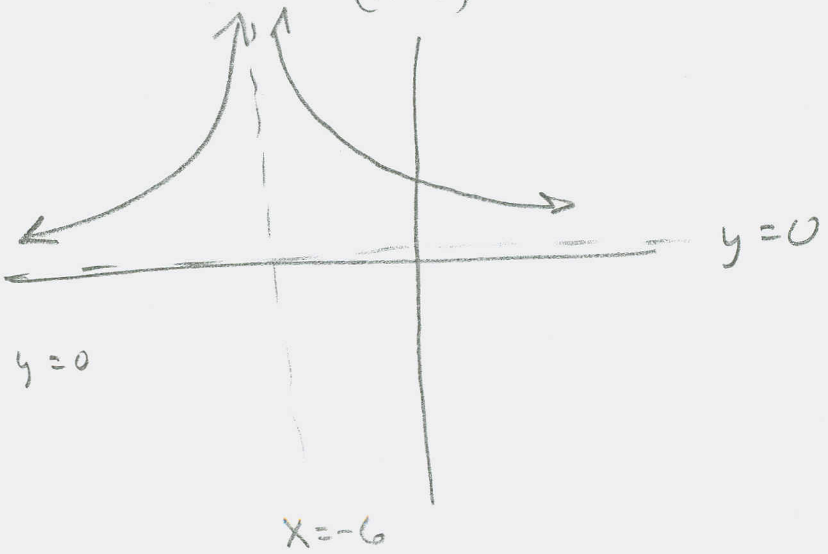
Graph the function using transformations.

13) $f(x) = \dfrac{3}{(6 + x)^2}$

$\dfrac{1}{x^2}$



$\dfrac{3}{(x+6)^2}$



Practice Test 3

Analyze the graph of the given function f as follows:

(a) Determine the end behavior: find the power function that the graph of f resembles for large values of $|x|$.

(b) Find the x– and y–intercepts of the graph.

(c) Determine whether the graph crosses or touches the x–axis at each x–intercept.

(d) Graph f using a graphing utility. You can't do (d) on test

(e) Use the graph to determine the local maxima and local minima, if any exist. Round turning points to two decimal places. Not sweating part (e), either.

(f) Use the information obtained in (a) – (e) to draw a complete graph of f by hand. Label all intercepts and turning points.

(g) Find the domain of f. Use the graph to find the range of f.

(h) Use the graph to determine where f is increasing and where f is decreasing.

11) $f(x) = -x^2(x-1)(x+3)$

(a) E.B.: $-x^2(x)(x)$

$= -x^4$

(b) x-int: $(0,0), (1,0), (-3,0)$

y-int: $(0, f(0)) = (0,0)$

(c) $x = 0,\ m = 2 \Rightarrow$ Touch

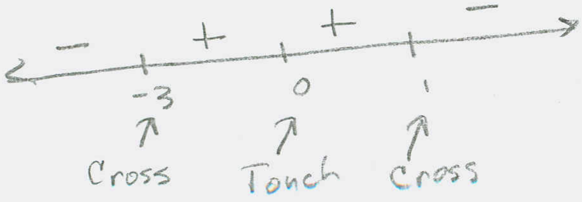
$x = 1,\ m = 1 \longrightarrow$ Cross

$x = -3,\ m = 1 \longrightarrow$ Cross

(f) Sign Pattern:

E.B. gives

Part (c) says:

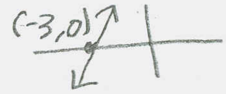


Further Analysis: Behavior near intercepts:

$x = -3$: $-(-3)^2(-3-1)(x+3)$

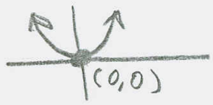
$= -9(-4)(x+3)$

$= 36(x+3)$ Steep positive slope



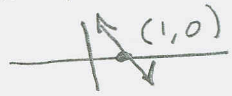
$x = 0$: $-x^2(0-1)(0+3)$

$= -x^2(-1)(3) = 3x^2$



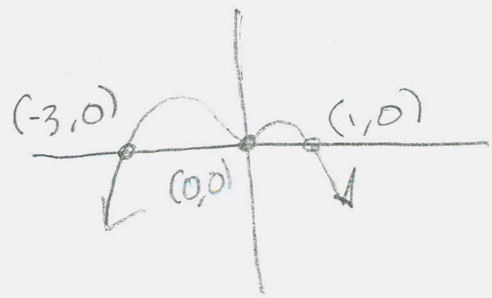
$x = 1$: $-(1)^2(x-1)(1+3)$

$= -4(x-1)$



Combine:

FINISH:



Practice Test 3

12) For the polynomial function $f(x) = 2x^4 - 7x^3 + 11x - 4$

a) Find the x- and y-intercepts of the graph of f. Round to two decimal places, if necessary.

b) Determine whether the graph crosses or touches the x-axis at each x-intercept.

c) End behavior: find the power function that the graph of f resembles for large values of $|x|$.

d) Use a graphing utility to graph the function. Approximate the local maxima rounded to two decimal places, if necessary. Approximate the local minima rounded to two decimal places, if necessary.

e) Determine the number of turning points on the graph.

f) Put all the information together, and connect the points with a smooth, continuous curve to obtain the graph of f.

(a) $f(0) = -4 \rightarrow \boxed{(0,-4) \text{ is y-int.}}$

$f(x) = 0$:

Descartes: 3 or 1 positive and 1 negative zero (Check this!)

Rational Zeros: $a_0 = -4$, $a_n = 2$

$\frac{p}{q} = \pm 1, \pm \frac{1}{2}, \pm 2, \pm \frac{2}{2}, \pm 4, \pm \frac{4}{2}$ → Redundant

YOU check: $x = \pm 1, \pm 2$ don't work.

Try $x = 4$:

$$\begin{array}{r|rrrrr} 4 & 2 & -7 & 0 & 11 & -4 \\ & & 8 & 4 & 16 & \text{Nope} \\ \hline & 2 & 1 & 4 & 27 & \end{array}$$

$$\begin{array}{r|rrrrr} -4 & 2 & -7 & 0 & 11 & -4 \\ & & -8 & 60 & -\text{HUGE} & \\ \hline & 2 & -15 & 60 & \text{No way} & \end{array}$$

This problem is a graphing calculator problem. I should've caught this earlier. My bad. Please Disregard.

Practice Test 3

Graph the function.

14) $f(x) = \dfrac{2x}{(x-3)(x-1)}$

KEY : $x = 0, 1, 3$

x-int: (0,0)

V.A.: x=1, x=3

E.B.: y = 0 is H.A.

Also, x ⟶ large positive

⟹ $\dfrac{2x}{(x-3)(x-1)}$ is postive

15) $f(x) = \dfrac{x^2 - 7x + 10}{(x-4)^2} = \dfrac{(x-5)(x-2)}{(x-4)^2}$

$\boxed{\begin{array}{l}\text{V.A.: } x=4 \\ \text{H.A.: } y=1\end{array}}$ ⟶ End Behavior!

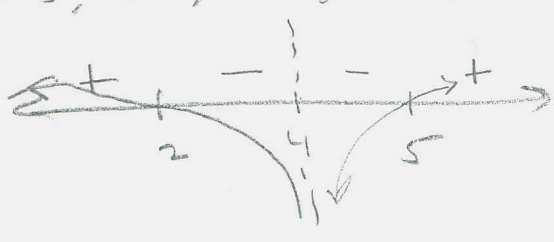
x-int: $\boxed{(5,0), (2,0)}$

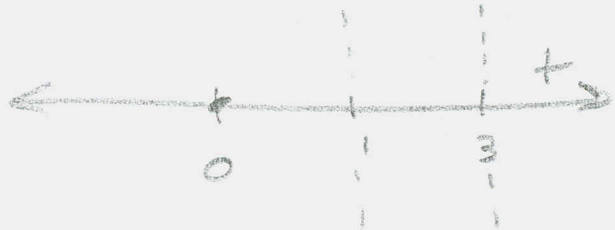
y-int: $\dfrac{+10}{(-4)^2} = \dfrac{10}{16} = \dfrac{5}{8} \to \boxed{(0, \frac{5}{8})}$

x = 2, m=1, changes sign

x = 4, m=2, No sign change!!!

x=5, m=1, changes sign



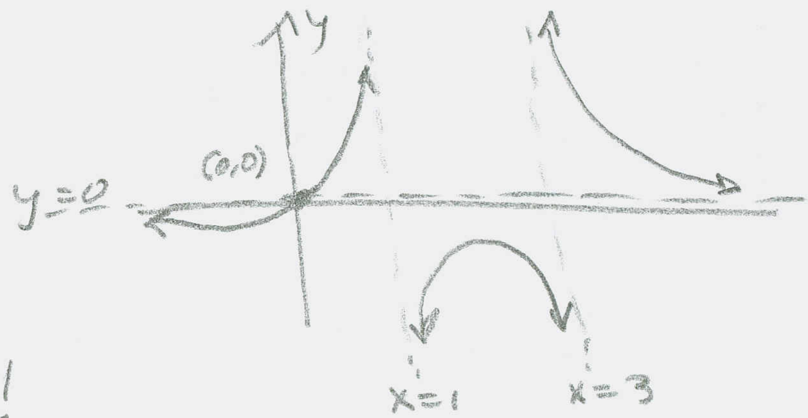


x = 0, m = 1 changes sign

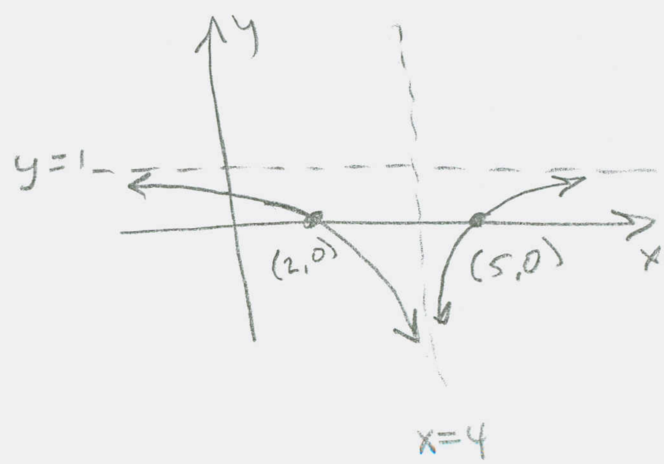
x = 1, m = 1 changes sign

x = 3, m = 1 changes sign

Graph must look like:



Graph must look like:



Practice Test 3

Solve the inequality.

16) $x^3 - 3x^2 - 40x > 0$

$x(x^2-3x-40)>0$

$x(x-8)(x+5)>0$

$x=0$, $m=1$, changes

$x=-5$, $m=1$, changes

$x=8$, $m=1$, changes

E.B.: x^3

From E.B., we get this much:

$-$ ... $+$

-5 0 8

Now use change/doesn't change or "touch/cross" criteria:

$-$ $+$ $-$ $+$

-5 0 8

Now analyze original question:

want ">0" means

want "$+$" means

$-$ $+$ $-$ $+$

No -5 Yes 0 No 8 Yes

So, $\boxed{x \in (-5,0)\cup(8,\infty)}$

17) $\dfrac{(x + 10)(x - 5)}{x - 1} \geq 0$

Same idea, but keep in mind $x \neq 1$:

$-$ $+$ $-$ $+$

N -10 Y 1 N 5 Y

$\boxed{x \in [-10,1)\cup[5,\infty)}$

Form a polynomial f(x) with real coefficients having the given degree and zeros.

21) Degree: 3; zeros: -2 and $3 + i$.

$(x+2)(x-(3+i))(x-(3-i))$ 8 of 10pts

$= (x+2)(x^2-(3-i)x-(3+i)x+(3+i)(3-i))$

$= (x+2)(x^2-3x+\underline{ix}-3x-\underline{ix}+3^2+1^2)$

$= (x+2)(x^2-6x+10)$

$= x^3-6x^2+10x$
$\quad +2x^2-12x+20$

$\boxed{x^3-4x^2-2x+20 = f(x)}$

The other 2 pts.

Practice Test 3

Use Descartes' Rule of Signs and the Rational Zeros Theorem to find all the real zeros of the polynomial function. Use the zeros to factor f over the real numbers.

18) $f(x) = \overbrace{4x^4 - 7x^3}^{1} \overbrace{+11x^2}^{2} \overbrace{-14x}^{3} \overbrace{+6}^{4}$ This one is REALLY hard if you can't factor by grouping!

Descartes: 4, 2, or 0 positive zeros

$f(-x) = 4x^4 + 7x^3 + 11x^2 + 14x + 6$

No negative zeros.

Rational zeros: $a_0 = 6$, $a_n = 4$

$\frac{p}{q} = \pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 2, \pm 3, \pm \frac{3}{2}, \pm \frac{3}{4}, \pm 6$

$\boxed{x = 1}$

$$\begin{array}{r|rrrrr} 1 & 4 & -7 & 11 & -14 & 6 \\ & & 4 & -3 & 8 & -6 \\ \hline & 4 & -3 & 8 & -6 & 0 \end{array}$$

Yes!

So, $f(x) = (x-1)(4x^3 - 3x^2 + 8x - 6)$

So we now look at breaking down

$4x^3 - 3x^2 + 8x - 6$ It factors by grouping!

$4x^3 - 3x^2 + 8x - 6$

$= x^2(4x-3) + 2(4x-3)$

$= (4x-3)(x^2+2)$

Doesn't break down

Solve $4x - 3 = 0$

$\Rightarrow 4x = 3$

$\Rightarrow \boxed{x = \frac{3}{4}}$

$f(x) = (x-1)(4x-3)(x^2+2)$

& real zeros are

$x = 1, \frac{3}{4}$

19) $f(x) = x^4 - 4x^3 - x^2 + 10x + 6$ I cooked this one up fairly carefully to come out "nice," but have irrationals.

Descartes: 2 or 0 positive zeros

$f(-x) = x^4 + 4x^3 - x^2 - 10x + 6$

2 or 0 negative zeros

Rational zeros: $a_0 = 6$, $a_n = 1$

$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 6$

$$\begin{array}{r|rrrrr} 1 & 1 & 4 & -1 & -10 & 6 \\ & & 1 & 5 & 4 & -6 \\ \hline & 1 & 5 & 4 & -6 & 0 \end{array}$$

Yes!

$(x-1)(x^3 + 5x^2 + 4x - 6)$

Now break down $x^3 + 5x^2 + 4x - 6$

$\pm 1, \pm 2, \pm 3, \pm 6$, again

And we Know there's another positive root!

$$\begin{array}{r|rrrr} 1 & 1 & 5 & 4 & -6 \\ & & 1 & 6 & \\ \hline & 1 & 6 & 10 & \end{array}$$

No

$$\begin{array}{r|rrrr} 2 & 1 & 5 & 4 & -6 \\ & & 2 & 14 & \\ \hline & 1 & 7 & & \end{array}$$

No

$$\begin{array}{r|rrrr} 3 & 1 & 5 & 4 & -6 \\ & & 3 & 24 & \\ \hline & 1 & 8 & & \end{array}$$

No

$$\begin{array}{r|rrrr} 6 & 1 & 5 & 4 & -6 \\ & & 6 & & \\ \hline & 1 & 11 & & \end{array}$$

Nope.

#19. By previous work, we know that the 2nd positive real zero must be irrational since none of our positive rational candidates worked. Continuing with x^3+5x^2+4x-6:

-1	1	5	4	-6
		-1	-4	0
	1	4	0	NO

-2	1	5	4	-6
		-2	-6	4
	1	3	-2	NO

-3	1	5	4	-6
		-3	-6	6
	1	2	-2	0 Yes!

Now, we have

$f(x) = (x-1)(x+3)(x^2+2x-2)$

& we have x^2+2x-2 to break down.

$a=1,\ b=2,\ c=-2$;

$b^2-4ac = 2^2-4(1)(-2)$

$= 4+8 = 12$

So, $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \dfrac{-2 \pm \sqrt{12}}{2(1)} = \dfrac{-2 \pm 2\sqrt{3}}{2}$

$= \dfrac{-1 \pm \sqrt{3}}{1} \begin{array}{l} \nearrow 1+\sqrt{3} \\ \searrow 1-\sqrt{3} \end{array}$

This gives

$$\boxed{f(x) = (x-1)(x+3)(x-(1-\sqrt{3}))(x-(1+\sqrt{3}))}$$

and the real zeros are $\boxed{x \in \{-3, 1, 1 \pm \sqrt{3}\}}$

Practice Test 3

Solve the equation in the real number system.

20) $3x^3 - x^2 - 15x + 5 = 0$ Factors by grouping!

$x^2(3x-1) - 5(3x-1)$

$= (3x-1)(x^2-5)$

$= (3x-1)(x-\sqrt{5})(x+\sqrt{5}) = 0$

$\Rightarrow x \in \{\frac{1}{3}, \pm\sqrt{5}\}$!

Find all zeros of the function and write the polynomial as a product of linear factors.

This one has non-real roots.

22) $f(x) = x^3 + 5x^2 + 11x + 7$

$\pm 1, \pm 7$, No pos. real roots

$$\begin{array}{r|rrrr} -1 & 1 & 5 & 11 & 7 \\ & & -1 & -4 & -7 \\ \hline & 1 & 4 & 7 & \end{array}$$ Yes!

Now solve $x^2 + 4x + 7 = 0$:

$x^2 + 4x = -7$

$x^2 + 4x + 2^2 = -7 + 4$

$(x+2)^2 = -3$

$x + 2 = \pm i\sqrt{3}$

$x = -2 \pm i\sqrt{3}$

So,

$\boxed{f(x) = (x+1)(x-(-2+i\sqrt{3}))(x-(-2-i\sqrt{3}))}$ & $x \in \{-1, -2 \pm i\sqrt{3}\}$ is the set of zeros of $f(x)$.