MAT 121 Test 2

100 Points Name (5 pts) ______________________

1. (5 pts) Determine whether the given function is linear or nonlinear. If it is linear, determine the slope.

x	$y=f(x)$
-2	1
-1	3
0	5
1	7
2	9

$\frac{3-1}{-1+2} = \frac{2}{1}$

$\frac{5-3}{0+1} = \frac{2}{1}$

$\frac{7-5}{1-0} = 2$

$\frac{9-7}{2-1} = 2$

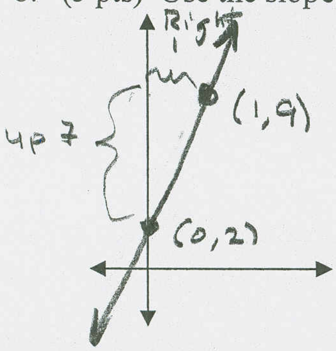
$\boxed{\text{Yes, } m = 2}$

2. Let $f(x) = 7x + 2$ in the following:

a. (5 pts) Determine the slope and y-intercept of f.

$\boxed{m = 7 \ \& \ (0,b) = (0,2)}$

b. (5 pts) Use the slope and y-intercept to graph f here:



c. (5 pts) Determine the average rate of change of f.

$m = 7$

d. (5 pts) Is f increasing, decreasing or constant?

Increasing

3. (5 pts) The velocity v of a falling object on the moon is directly proportional to the time t of the fall. If, after 2 seconds, the velocity of the object is 8 feet per second, what will its velocity be after 3 seconds? .

$v = kt$

$8 = k \cdot 2$

$4 = k$

$v = 4t \Rightarrow$

$\boxed{v = 4(3) = 12 \text{ ft per second after 3 seconds.}}$

4. Let $f(x) = 6x^2 + 5x - 6$.

a. (5 pts) Find the zeros of f by factoring.

$(6)(-6) = -(3)(2)(3)(2)$. Want $+5$ sum:

$+9 - 4 = 5$. cool.

$6x^2 + 9x - 4x - 6$

$= 3x(2x+3) - 2(2x+3)$

$= (2x+3)(3x-2) \overset{\text{SET}}{=} 0$

$\to 2x+3=0$ OR $3x-2=0$

$2x = -3$ $\quad$ $3x = 2$

$x = -\frac{3}{2}$ $\quad$ $x = \frac{2}{3}$

$\Longrightarrow \boxed{x \in \{-\frac{3}{2}, \frac{2}{3}\}}$

b. (5 pts) Find the zeros of f by completing the square.

$6x^2 + 5x - 6 = 0$

$x^2 + \frac{5}{6}x - 1 = 0$

$x^2 + \frac{5}{6}x + \left(\frac{5}{12}\right)^2 = 1 + \left(\frac{5}{12}\right)^2$

$\left(x + \frac{5}{12}\right)^2 = \frac{144}{144} + \frac{25}{144} = \frac{169}{144}$

$x + \frac{5}{12} = \pm\sqrt{\frac{169}{144}} = \pm\frac{13}{12}$

$\Longrightarrow x = \frac{-5 \pm 13}{12}$

$\frac{-5+13}{12} = \frac{8}{12} = \frac{2}{3}$ $\quad$ $\frac{-5-13}{12} = \frac{-18}{12} = -\frac{3}{2}$

$\boxed{x \in \{-\frac{3}{2}, \frac{2}{3}\}}$

c. (5 pts) Find the zeros of f by using the quadratic formula.

$a = 6, b = 5, c = -6$

$\to b^2 - 4ac = 5^2 - 4(6)(-6)$

$= 25 + 144$

$= 169$

& $\sqrt{169} = 13$, so $\to$ $x = \frac{-5 \pm 13}{2(6)}$ by quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

See (b) for simplified version

5. $f(x) = (x-3)^2 - 7$

a. (5 pts) Find the zeros of $f(x)$ using the Square Root Method.

$(x-3)^2 - 7 = 0 \Longrightarrow (x-3)^2 = 7 \Longrightarrow x - 3 = \pm\sqrt{7}$

$\Longrightarrow x = 3 \pm \sqrt{7} \Longrightarrow x \in \{3 \pm \sqrt{7}\}$

b. (5 pts) What are the x-intercepts of the graph of $f(x)$?

$(3-\sqrt{7}, 0)$ and $(3+\sqrt{7}, 0)$

6. (10 pts) Graph $f(x) = x^2 - 4x - 2$. I expect to see all of the following information on (or next to) your graph. You may use completing the square or the $-\dfrac{b}{2a}$ method.:

i.	vertex	v. domain
ii.	axis of symmetry	vi. range
iii.	y-intercept	vii. interval(s) of increase
iv.	x-intercep(s), if any	viii. interval(s) of decrease

$x^2 - 4x + 2^2 - 2^2 - 2$

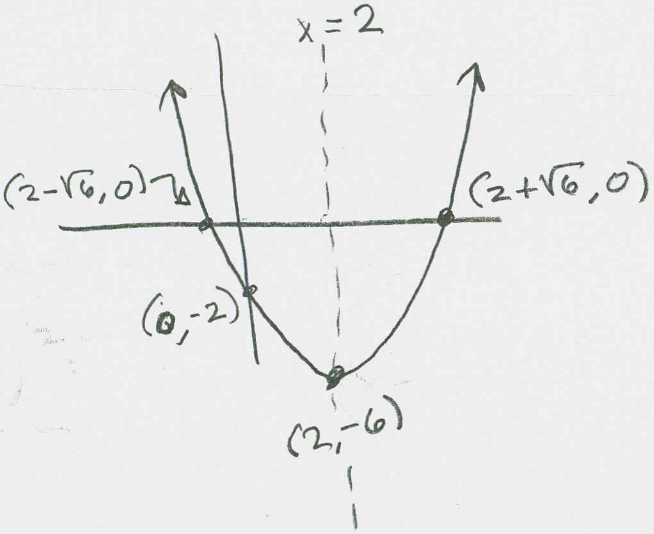
$= (x-2)^2 - 6$

x-int: $(x-2)^2 - 6 = 0$

$(x-2)^2 = 6$

$x - 2 = \pm\sqrt{6}$

$x = 2 \pm \sqrt{6}$



$\mathcal{D} = \mathbb{R}$

$\mathcal{R} = [-6, \infty)$

Increasing: $(2, \infty)$

Decreasing: $(-\infty, 2)$

7. Consider the quadratic function $h(x) = 6x^2 - 5x + 3$.

a. (5 pts) Compute the discriminant for h.

$b^2 - 4ac = (-5)^2 - 4(6)(3)$

$= 25 - 72 = -47$

b. (5 pts) Based on your answer to part a., describe the nature of the zeros of h. In other words, state how many zeros h has, and whether they're real or nonreal.

No real zeros,

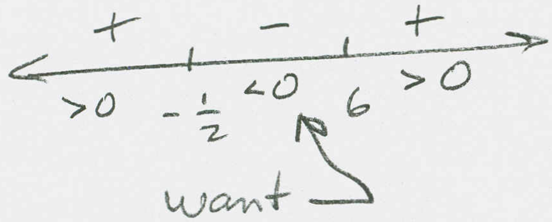
Two, nonreal, nonequal zeros.

8. (5 pts) Solve $2x^2 < 5x + 3$. Express your answer in both set-builder and interval notation.

$2x^2 - 5x - 3 < 0$

$(2x+1)(x-6) < 0$

Key values: $x = -\frac{1}{2}, 6$



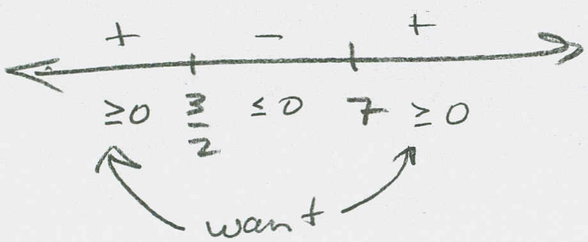
$$\boxed{\begin{aligned} &x \in \left(-\tfrac{1}{2}, 6\right) \\ &= \left\{x \mid -\tfrac{1}{2} < x < 6\right\} \end{aligned}}$$

9. (5 pts) Solve $2x^2 - 17x \geq -21$. Express your answer in both set-builder and interval notation.

$2x^2 - 17x + 21 \geq 0$

$(2x-3)(x-7) \geq 0$

Key values: $x = \frac{3}{2}, 7$



$$\boxed{\begin{aligned} &x \in \left(-\infty, \tfrac{3}{2}\right] \cup [7, \infty) \\ &= \left\{x \mid x \leq \tfrac{3}{2} \text{ OR } x \geq 7\right\} \end{aligned}}$$

10. Find the complex zeros of $f(x) = x^2 - 6x + 10$

$x^2 - 6x + 10 = 0$

$x^2 - 6x = -10$

$x^2 - 6x + 3^2 = -10 + 9$

$(x-3)^2 = -1$

$x - 3 = \pm\sqrt{-1} = \pm i$

$x = 3 \pm i$

$$\boxed{x \in \{3 \pm i\}}$$

11. Without solving, determine the character of the solutions of each equation in the complex number system.

a. (2 pts) $x^2+2x+6=0$

$b^2-4ac = 2^2-4(1)(6) = 4-24=-20$

$\boxed{\text{Two distinct nonreal solutions}}$

b. (2 pts) $4x^2-12x+9=0$

$b^2-4ac = (-12)^2-4(4)(9) = 144-144=0$

$\boxed{\text{ONE, real zero of multiplicity 2}}$

c. (2 pts) $2x^2-4x+1=0$

$b^2-4ac=(-4)^2-4(2)(1) = 16-8=8$

$\boxed{\text{Two distinct, real solns}}$

13. Solve each of the following absolute value equations:

a. (2 pts) $|2x-1|=3 \implies 2x-1=\pm 3$

$2x = 1 \pm 3$

$x = \dfrac{1\pm 3}{2}$

$\dfrac{1+3}{2} = \dfrac{4}{2} = 2$

$\dfrac{1-3}{2} = \dfrac{-2}{2} = -1$

$\boxed{x \in \{-1, 2\}}$

b. (2 pts) $|2x-1|=-3$ NEVER! $\varnothing$.

14. Solve each of the following absolute value inequalities. Give your answer in set-builder *and* interval notation.

a. (3 pts) $|3x-5|>-2$ Always. $\boxed{x \in \mathbb{R}}$

b. (3 pts) $|3x-5|\le -2$ Never $\boxed{\varnothing}$

c. (3 pts) $|x-3|<2$

$-2 < x-3 < 2$

$1 < x < 5$

$\boxed{x \in \{x \mid 1<x<5\} = (1,5)}$

d. (3 pts) $|2x+1|\ge 3$

$2x+1 \ge 3$ OR $2x+1 \le -3$

$2x \ge 2$ $\quad 2x \le -4$

$x \ge 1$ $\quad x \le -2$

$\boxed{x \in \{x \mid x \le -2 \text{ OR } x \ge 1\} = (-\infty, -2] \cup [1, \infty)}$