$(x-3)^2$

§2.7 #23

$(0,1)$

$f(x) = -2x^2 + 8x + 1$

$= -2(x^2 - 4x + 2^2) + 1 + 2(2^2)$

$a(x-h)^2 + k$

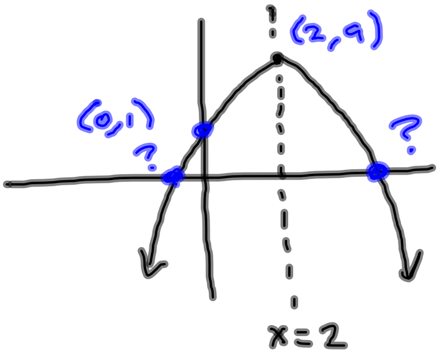
$= -2(x-2)^2 + 9$

This gives (i), (ii), (v), (vi)

(i) vertex
(ii) axis of symmetry
(iii) y-int.
(iv) x-int(s)

ON GRAPH

(v) $\mathcal{D} = \mathbb{R}$
(vi) $\mathcal{R} = (-\infty, 9]$
(vii) inc.: $(-\infty, 2)$
(viii) dec.: $(2, \infty)$



$\mathcal{R} = (-\infty, 9]$

$(x+5)^2 - 11$

$(h,k) = (-5, -11)$

x-ints: $f(x) = 0$

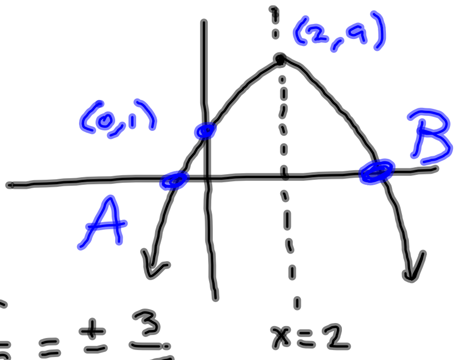
$-2(x-2)^2 + 9 = 0$

$\Longrightarrow -2(x-2)^2 = -9$

$\Longrightarrow (x-2)^2 = \frac{-9}{-2} = \frac{9}{2}$

$\Longrightarrow x - 2 = \pm\sqrt{\frac{9}{2}} = \pm\frac{\sqrt{9}}{\sqrt{2}} = \pm\frac{3}{\sqrt{2}}$

$\Longrightarrow x = 2 \pm \frac{3}{\sqrt{2}}$



Let $A = \left(2 - \frac{3}{\sqrt{2}}, 0\right)$

$B = \left(2 + \frac{3}{\sqrt{2}}, 0\right)$

$f(x) = -2x^2 + 8x + 1 \rightsquigarrow (0,1)$ is y-int

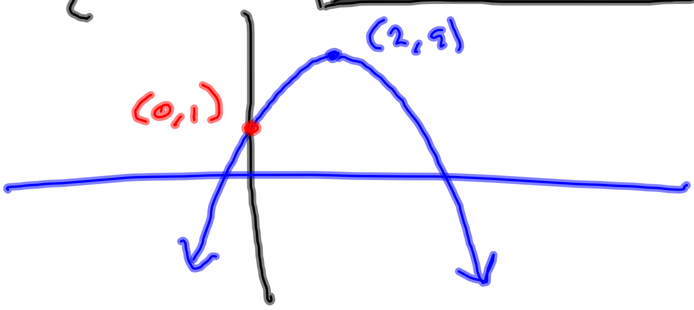
$a = -2,\ b = 8,\ c = 1$

$-\frac{b}{2a} = -\frac{8}{2(-2)} = 2 = h$

$f(-\frac{b}{2a}) = f(2) = -2(2)^2 + 8(2) + 1 = -8 + 16 + 1 = 9 = k$

$(h,k) = \text{vertex} = (2,9)$ gives you this:

$f(x) = \boxed{-2(x-2)^2 + 9}$



Find zeros:

$b^2 - 4ac =$

$8^2 - 4(-2)(1)$

$= 64 + 8 = 72$

$= 2^3 \cdot 3^2 = 2^1 \cdot 2^2 \cdot 3^2$

$= 2^2 \cdot 3^2 \cdot 2^1$

So $\sqrt{2^2 \cdot 3^2 \cdot 2} = 2 \cdot 3\sqrt{2}$

$= 6\sqrt{2} = \sqrt{b^2 - 4ac}$

$$\begin{array}{l|l} 2 & 72 \\ 2 & 36 \\ 2 & 18 \\ 3 & 9 \\ & 3 \end{array}$$

& $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-8 \pm 6\sqrt{2}}{2(-2)} = \frac{-8 \pm 6\sqrt{2}}{-4}$

$= \frac{-\cancel{2}(4 \pm 3\sqrt{2})}{-\cancel{4}_2}$

$= \frac{4 \pm 3\sqrt{2}}{2} = \frac{4}{2} \pm \frac{3\sqrt{2}}{2}$

$= 2 \pm \frac{3\sqrt{2}}{2}$

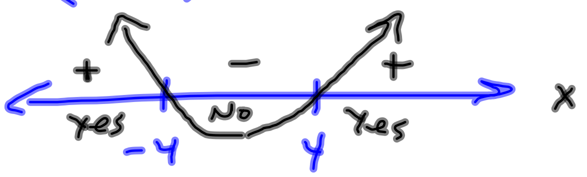
Before: $2 \pm \frac{3}{\sqrt{2}}$

$= 2 \pm \frac{3}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = 2 \pm \frac{3\sqrt{2}}{2}$

Domain of $\sqrt{x^2-16}$

Need $x^2 - 16 \geq 0$

$(x-4)(x+4) \geq 0$



want +, because we want ≥ 0

so $x \in (-\infty,-4] \cup [4,\infty)$

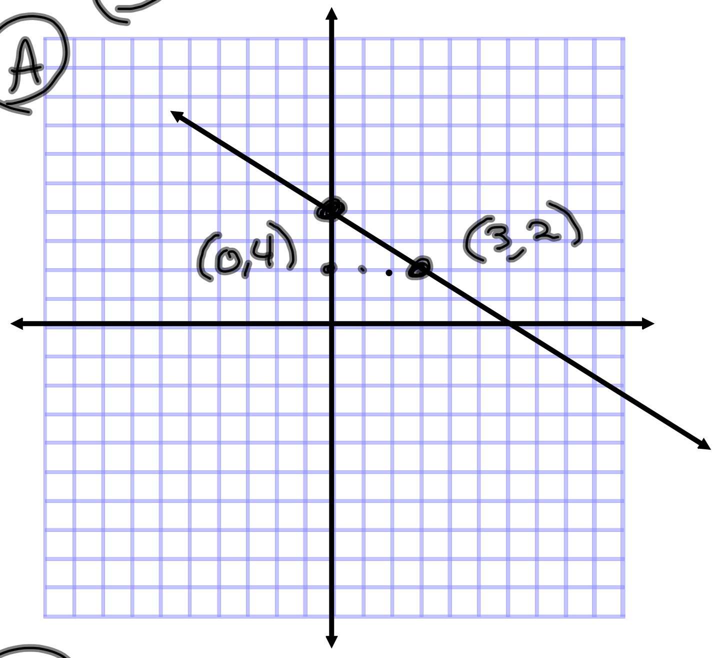
$\boxed{\sqrt{x} - 4 \geq 0 \text{ is garbage}}$

Graph lines!

(A) slope & y-int.

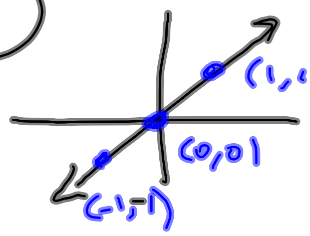
(B) Transformations on $f(x) = x$

(A)

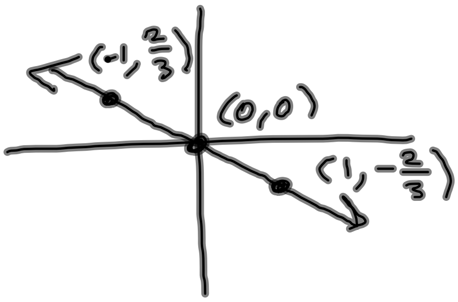


$y = -\frac{2}{3}x + 4$

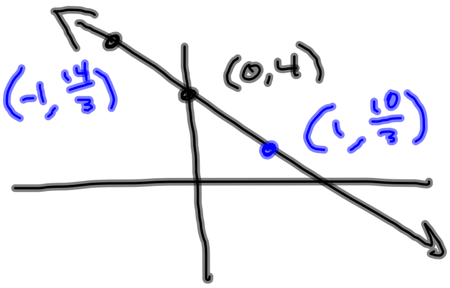
(B)



$y = x$



$-\frac{2}{3}x$



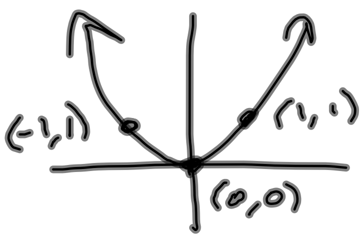
$-\frac{2}{3}x + 4$

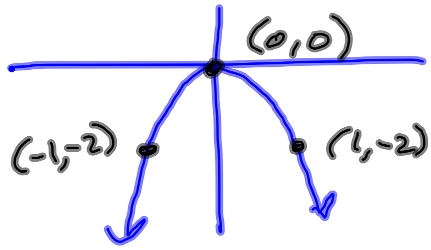
$4 + \frac{2}{3} = \frac{12}{3} + \frac{2}{3} = \frac{14}{3}$

$4 - \frac{2}{3} = \frac{12-2}{3} = \frac{10}{3}$

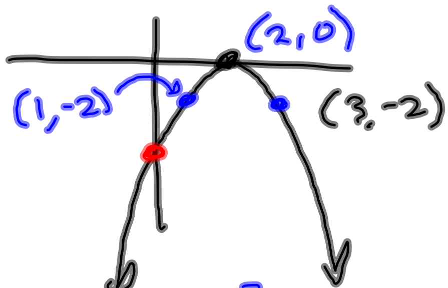
$$-2(x-2)^2+9 = -2x^2+8x+1$$

x^2

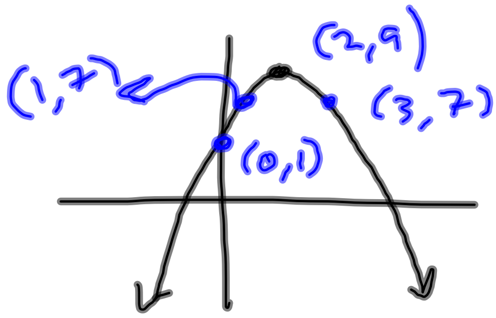




$-2x^2$



$-2(x-2)^2$



$-2(x-2)^2+9$

Line? If so, find the slope

x	f(x)
-2	1
4	3
7	4

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{4-(-2)} = \frac{2}{6} = \frac{1}{3}$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-3}{7-4} = \frac{1}{3}$

Yes! $m = \frac{1}{3}$

Equilibrium

Proportionality $y=kx$ §2.2

$|3x-5|=11 \Rightarrow$

$3x-5=11$ OR $3x-5=-11$

$\Rightarrow 3x=16$ OR $3x=-6$

$\Rightarrow x=\frac{16}{3}$ OR $x=-\frac{6}{3}=-2$

$\Rightarrow x\in\left\{-2, \frac{16}{3}\right\}$

$|3x-5|=-11$

No Sol'n

$|3x-5|\leq -11$

No Sol'n

$|3x-5|>-11$

Always True

Soln Set $=\mathbb{R}$

$$|3x-5| \le 11$$

$$-11 \le 3x-5 \le 11 \Longrightarrow$$

$$+5 \quad = \quad +5 = +5$$

$$-6 \le 3x \le 16 \Longrightarrow$$

$$\frac{-6}{3} \le \frac{3x}{3} \le \frac{16}{3} \Longrightarrow$$

$$-2 \le x \le \frac{16}{3} \Longrightarrow$$

Set-builder:

$$\{x \mid -2 \le x \le 16/3\}$$

Interval:

$$\left[-2, \frac{16}{3}\right]$$

$$|3x-5| + 7 > 18$$

$$\Longrightarrow |3x-5| > 11$$

$$\Longrightarrow 3x-5 > 11 \quad \text{OR} \quad 3x-5 < -11$$

$$\Longrightarrow 3x > 16 \quad \text{OR} \quad 3x < -6$$

$$\Longrightarrow x > \frac{16}{3} \quad \text{OR} \quad x < -2$$

$$\left\{x \mid x > \frac{16}{3} \text{ OR } x < -2\right\}$$

$$= \left(\frac{16}{3}, \infty\right) \cup (-\infty, -2)$$

Factoring Skill:

Solve by completing the square

$6x^2 - x - 12$

Want $-x$ in the middle & $ac = (6)(-12) = -72$

$-x \to -1 \cdot x$

Looking for sum of -1 & product of -72

$-2+1=-1 \qquad (-2)(1) = -2$

$-3+2=-1 \qquad (-3)(2) = -6$

$(-4)(3) = -12$

$(-5)(4) = -20$

$(-6)(5) = -30$

$(-7)(6) = -42$

$(-8)(7) = -56$

$(-9)(8) = -72$! Bingo!

$6x^2 - x - 12$

$= 6x^2 - 9x + 8x - 12$

$= 3x(2x-3) + 4(2x-3)$

$= (2x-3)(3x+4)$

$3x(2x-3) + 4(2x-3)$

$= (2x-3)\left[\dfrac{3x\cancel{(2x-3)}}{\cancel{2x-3}} + \dfrac{4\cancel{(2x-3)}}{\cancel{2x-3}}\right]$

$= (2x-3)[3x+4]$