MAT 121-G81-G82 Final Test – Spring, 2015 Name_____
200 Points Dr. Harry S. Mills NO GRAPHING CALCULATORS!!!

1. For each of the following functions, state the domain in interval notation.

a. (5 pts) $f(x)=\sqrt{2x+6}$

b. (5 pts) $g(x)=\dfrac{2x^2+x-15}{2x+6}$

c. (5 pts) $h(x)=\sqrt{\dfrac{(x+2)(x-1)}{(x-3)(x+3)}}$

d. (5 pts) $w(x)=\log_3\left(\dfrac{(x+2)(x-1)}{(x-3)(x+3)}\right)$

2. (10 pts) What is the average rate of change of the function $f(x)=x^2-3$ from $x=-1$ to $x=3$?

3. The domain of $f(x)=\sqrt{x+5}$ is $[-5,\infty)$ and the domain of $g(x)=\dfrac{2x-4}{x-1}$ is $(-\infty,1)\cup(1,\infty)$,

   a. (10 pts) Find $\dfrac{f}{g}$ and determine its domain. Do not simplify $\dfrac{f}{g}$.

   b. (10 pts) Find $f \circ g$ and determine its domain. Do not simplify $f \circ g$.

4. (10 pts) Graph $g(x) = -3\sqrt{-2x+6}+7$ by the techniques of shifting, stretching, compressing or reflecting. Start with the graph of a basic function and show all steps *as demonstrated in Videos*. I expect to see 3 points labeled in the first sketch, and to see where those points are moved to in each subsequent step. I strongly recommend using $(0,0), (1,1),$ and $(4,2)$ as the 3 points. I'm looking for 5 graphs, with the first being the basic function, $f(x) = \sqrt{x}$, and the final being $g(x)$. *None* of the graphs, between the first and the last is going to be *either* $f(x)$ *nor* $g(x)$, so, for the last time, don't call 'em all $f(x)$! *x*- and *y*-intercepts for 5 bonus points.

5. Find all real and nonreal solutions of the following equations:

   a. (10 pts) $2x^2 - 5x - 7 = 0$

   b. (10 pts) $4x^2 - 8x + 13 = 0$

   c. (10 pts) $9x^4 - 30x^3 + 38x^2 - 22x + 5 = 0$. Hint: Try $x = 1$. Heck, try it twice!

6. (10 pts) Based on your work on #5, provide a rough sketch of the graph of $f(x) = 9x^4 - 30x^3 + 38x^2 - 22x + 5$. No double jeopardy. Whatever you get for #5 is what I'll be looking for in this one.

7. Solve the following exponential and logarithmic equations. An exact answer is preferred. A decimal approximation is acceptable, if you are correct to the 5th decimal place.

   a. (10 pts) $5^x = 97$

   b. (10 pts) $\log_5(x) = 97$

   c. (5 pts) $3 \cdot 5^x = 7^x$

   d. (10 pts) $\log_7(x-4) + \log_7(x+2) = 1$

8. Solve the absolute value inequality. Give your final answer in set-builder and interval notation.

   a. (15 pts) $|2x - 7| \geq 11$

9. (15 pts) Find the sum: $\sum_{k=1}^{\infty} 3 \cdot \left(-\frac{2}{3}\right)^{k-1}$

10. (15 pts) Solve the system of linear equations:
$$\begin{aligned} 2x \quad\quad + 10z &= 48 \\ -2x + y - 11z &= -51. \\ 3x + 2y + 14z &= 71 \end{aligned}$$

11. (10 pts) Write the equation for "The half-life of the radioactive isotope, Freakazoidium-99, is 9900 years," and solve the equation for the decay constant, *k*.

12. (10 pts) Based on your work, how much radioactive Freakazoidium remains in a 512-kilogram sample, after 100,000 years?