MAT 121-G81-G82 Final Test – Spring, 2015 Name KEY

200 Points Dr. Harry S. Mills NO GRAPHING CALCULATORS!!!

1. For each of the following functions, state the domain in interval notation.

a. (5 pts) $f(x)=\sqrt{2x+6}$

$2x+6 \geq 0$

$2x \geq -6 \Longrightarrow$

$\{x \mid x \geq -3\} = \mathcal{D}$

$= \boxed{[-3,\infty)}$

b. (5 pts) $g(x)=\dfrac{2x^2+x-15}{2x+6}$

$2x+6 \neq 0$

$\vdots$

$\{x \mid x \neq -3\} = \mathcal{D}$

$= \boxed{(-\infty,-3)\cup(-3,\infty)}$

c. (5 pts) $h(x)=\sqrt{\dfrac{(x+2)(x-1)}{(x-3)(x+3)}}$

$$\frac{(x+2)(x-1)}{(x-3)(x+3)} \geq 0$$

$+ \quad - \quad + \quad - \quad +$

$-3 \quad -2 \quad 1 \quad 3$

$\bigstar \quad =0 \quad =0 \quad \bigstar$

$\boxed{\mathcal{D} = (-\infty,-3)\cup[-2,1]\cup(3,\infty)}$

d. (5 pts) $w(x)=\log_3\left(\dfrac{(x+2)(x-1)}{(x-3)(x+3)}\right)$

$\boxed{\mathcal{D} = (-\infty,-3)\cup(-2,1)\cup(3,\infty)}$

2. (10 pts) What is the average rate of change of the function $f(x)=x^2-3$ from $x=-1$ to $x=3$?

$$\frac{f(3)-f(-1)}{3-(-1)} = \frac{3^2-3-((-1)^2-3)}{4} = \frac{9-3-(1-3)}{4}$$

$$= \frac{6-(-2)}{4} = \frac{6+2}{4} = \frac{8}{4} = \boxed{2 = m_{AVG}}$$

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3. The domain of $f(x)=\sqrt{x+5}$ is $[-5,\infty)$ and the domain of $g(x)=\dfrac{2x-4}{x-1}$ is $(-\infty,1)\cup(1,\infty)$,

a. (10 pts) Find $\dfrac{f}{g}$ and determine its domain. Do not simplify $\dfrac{f}{g}$.

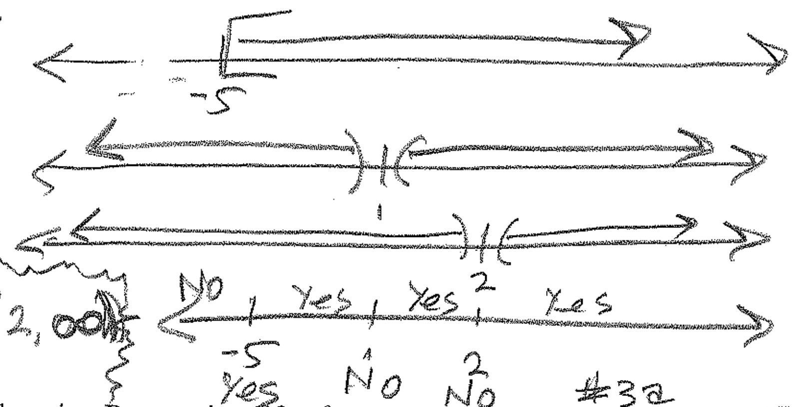
$$\left(\frac{f}{g}\right)(x)=\frac{\sqrt{x+5}}{\frac{2x-4}{x-1}} \qquad \mathcal{D}=\{x \mid x\in\mathcal{D}(f) \text{ and } x\in\mathcal{D}(g) \text{ and } g(x)\neq 0\}$$

$=$

$g(x)\neq 0$ $\quad 2x\neq 4$

$\dfrac{2x-4}{x-1}\neq 0$ $\quad x\neq 2$

$2x-4\neq 0$ $\quad (-\infty,2)\cup(2,\infty)$



$$\boxed{[-5,1)\cup(1,2)\cup(2,\infty)}$$

b. (10 pts) Find $f\circ g$ and determine its domain. Do not simplify $f\circ g$.

$$(f\circ g)(x)=\sqrt{\frac{2x-4}{x-1}+5}$$

$$\mathcal{D}=\{x \mid x\in\mathcal{D}(g) \text{ and } g(x)\in\mathcal{D}(f)\}$$

$$=\{x \mid x\neq 1 \text{ and } \frac{2x-4}{x-1}+5\geq 0\}$$

$$\frac{2x-4}{x-1}+5\geq 0$$

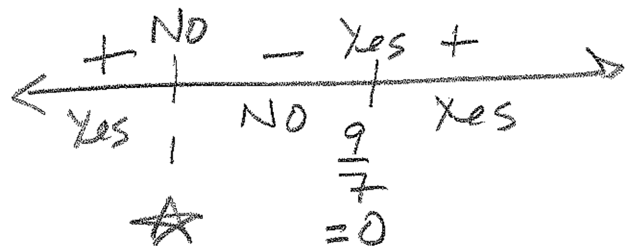
$$\frac{2x-4}{x-1}+\frac{5(x-1)}{x-1}\geq 0$$

$$\frac{2x-4+5x-5}{x-1}\geq 0$$

$$\frac{7x-9}{x-1}\geq 0 \Rightarrow$$

$7x-9=0$ & $x-1=0$ are where sign can (possibly) change

$x=\frac{9}{7}$, & $x=1$



$$\boxed{(-\infty,1)\cup[\tfrac{9}{7},\infty)}$$

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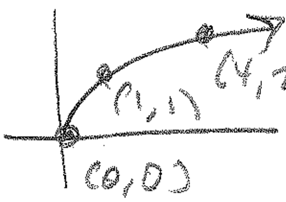
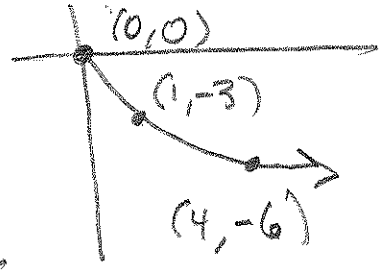
neither

4. (10 pts) Graph $g(x) = -3\sqrt{-2x+6}+7$ by the techniques of shifting, stretching, compressing or reflecting. Start with the graph of a basic function and show all steps *as demonstrated in Videos*. I expect to see 3 points labeled in the first sketch, and to see where those points are moved to in each subsequent step. I strongly recommend using $(0,0), (1,1)$, and $(4,2)$ as the 3 points. I'm looking for 5 graphs, with the first being the basic function, $f(x) = \sqrt{x}$, and the final being $g(x)$. *None* of the graphs, between the first and the last is going to be *either* $f(x)$ *nor* $g(x)$, so, for the last time, don't call 'em all $f(x)$! x- and y-intercepts for 5 bonus points.

1 $f(x) = \sqrt{x}$



2 $-3f(x) = -3\sqrt{x}$

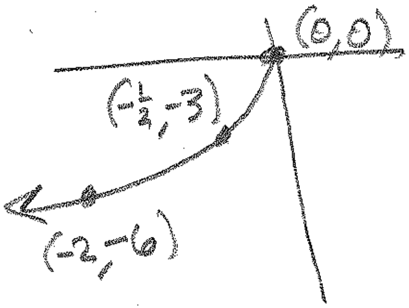


$-2x+6$

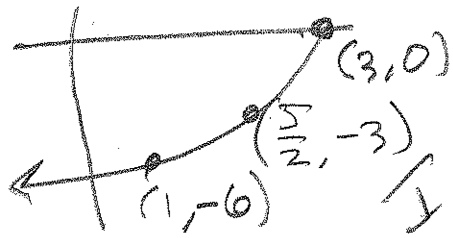
$= -2(x-3)$:

$x \longmapsto -2x \longmapsto -2(x-3)$

3 $-3f(-2x) = -3\sqrt{-2x}$

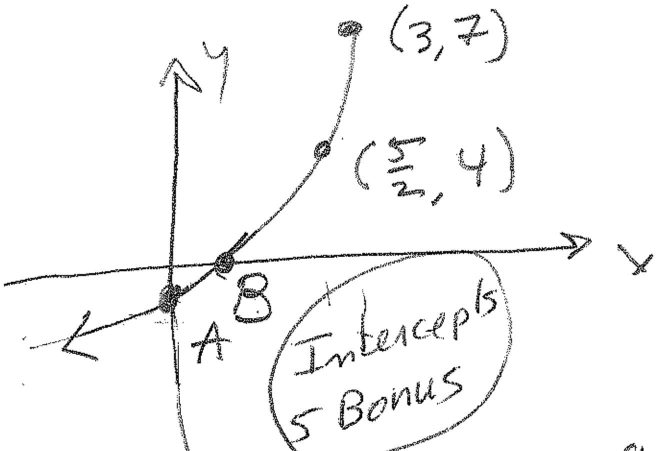


4 $-3f(-2(x-3)) = -3\sqrt{-2(x-3)}$



I made this worth 20 pts in grading

5 $-3f(-2(x-3))+7 = g(x)$!



y-int:

$g(0) = -3\sqrt{6}+7$

$\approx .348469228 3$

$\boxed{A \Rightarrow (0, -3\sqrt{6}+7) \approx (0, .348)}$

x-int: $g(x) = 0$

$-3\sqrt{-2x+6} = -7$

$\sqrt{-2x+6} = \frac{7}{3}$

$-2x+6 = \frac{49}{9}$

$-2x = \frac{49-54}{9} = \frac{-5}{9} \div -.\overline{5}$

$\Rightarrow x = \frac{5}{18} = .2\overline{77}$

$\boxed{B = (\frac{5}{18}, 0) = (.27, 0)}$

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5. Find all real and nonreal solutions of the following equations:

a. (10 pts) $2x^2 - 5x - 7 = 0$

$a=2, b=-5, c=-7$

$b^2-4ac = (-5)^2-4(2)(-7)$

$= 25+56 = 81$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \dfrac{5 \pm \sqrt{81}}{2(2)} = \dfrac{5 \pm 9}{4}$ $\frac{14}{4} = \frac{7}{2}$, $-\frac{4}{4} = -1$ $\boxed{x \in \{-1, \frac{7}{2}\}}$

b. (10 pts) $4x^2 - 8x + 13 = 0$

$a=4, b=-8, c=13$

$b^2-4ac = (-8)^2-4(4)(13)$

$= 64-208 = -144 \rightsquigarrow 12i$

$x = \dfrac{8 \pm 12i}{2(4)} = \boxed{\dfrac{2 \pm 3i}{2}}$

$x \in \left\{\frac{2 \pm 3i}{2}\right\}$

c. (10 pts) $9x^4 - 30x^3 + 38x^2 - 22x + 5 = 0$. Hint: Try $x = 1$. Heck, try it twice!

$$\begin{array}{r|rrrrr} 1 & 9 & -30 & 38 & -22 & 5 \\ & & 9 & -21 & 17 & -5 \\ \hline 1 & 9 & -21 & 17 & -5 & 0 \\ & & 9 & -12 & 5 & \\ \hline & 9 & -12 & 5 & 0 & \end{array}$$

$x = 1, m = 2 \rightarrow$ Touch

$x = \frac{2 \pm i}{3}, m = 1 \rightarrow$ Irrelevant to graph

$9x^2 - 12x + 5 = 0 \Rightarrow$

$a=9, b=-12, c=5$

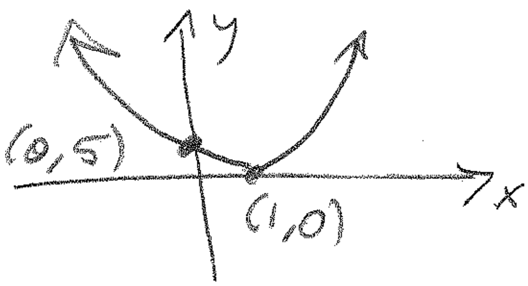
$x \in \left\{1, \frac{2 \pm i}{3}\right\}$

$b^2-4ac = (-12)^2-4(9)(5)$

$= 144-180 = -36$

$x = \dfrac{12 \pm 6i}{2(9)} = \dfrac{12 \pm 6i}{18} = \dfrac{6(2 \pm i)}{18} = \dfrac{2 \pm i}{3}$

6. (10 pts) Based on your work on #5, provide a rough sketch of the graph of $f(x) = 9x^4 - 30x^3 + 38x^2 - 22x + 5$. No double jeopardy. Whatever you get for #5 is what I'll be looking for in this one.



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7. Solve the following exponential and logarithmic equations. An exact answer is preferred. A decimal approximation is acceptable, if you are correct to the 5th decimal place.

a. (10 pts) $5^x = 97$

$\log_5(5^x) = \log_5(97)$

$\boxed{x = \log_5(97)}$

$= \frac{\ln 97}{\ln 5}$ $\boxed{2.84243}$

$\approx \boxed{2.842427}746$

b. (10 pts) $\log_5(x) = 97$

$5^{\log_5(x)} = 5^{97}$

$\boxed{x = 5^{97}}$

$\approx 6.310887242 \times 10^{67}$

c. (5 pts) $3 \cdot 5^x = 7^x$

$\ln(3 \cdot 5^x) = \ln(7^x)$

$\ln(3) + \ln(5^x) = (\ln(7))x$

$\ln(3) + (\ln(5))x = (\ln(7))x$

$A + Bx = Cx$

$Bx - Cx = -A$

$(B-C)x = -A$

$x = \frac{-A}{B-C} = \boxed{\frac{-\ln(3)}{\ln(5)-\ln(7)}} \approx 3.265090456 \approx \boxed{3.26509}$

d. (10 pts) $\log_7(x-4) + \log_7(x+2) = 1$

$\log_7[(x-4)(x+2)] = 1$

$(x-4)(x+2) = 7^1 = 7$

$x^2 - 2x - 8 = 7$

$x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

$\boxed{x = 5}$ $(x \neq -3 \notin \mathcal{D})$

8. Solve the absolute value inequality. Give your final answer in set-builder and interval notation.

a. (15 pts) $|2x-7| \geq 11$

$2x - 7 \geq 11$ OR $2x - 7 \leq -11$

$2x \geq 18$ OR $2x \leq -4$

$\boxed{\{x \mid x \geq 9 \text{ OR } x \leq -2\} = (-\infty, -2] \cup [9, \infty)}$

9. (15 pts) Find the sum: $\sum_{k=1}^{\infty} 3 \cdot \left(-\frac{2}{3}\right)^{k-1} = a\left(\frac{1}{1-r}\right) = \frac{3}{1+\frac{2}{3}} = \frac{3}{\frac{5}{3}} = \boxed{\frac{9}{5}}$

$a = 3,\ r = -\frac{2}{3}$

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10. (15 pts) Solve the system of linear equations:

$$\begin{aligned} 2x \quad\quad +10z &= 48 \\ -2x + y - 11z &= -51. \\ 3x + 2y + 14z &= 71 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 10 & 48 \\ -2 & 1 & -11 & -51 \\ 3 & 2 & 14 & 71 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 5 & 24 \\ 0 & 1 & -1 & -3 \\ 3 & 2 & 14 & 71 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 5 & 24 \\ 0 & 1 & -1 & -3 \\ 0 & 2 & -1 & -1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 5 & 24 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 1 & 5 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$$\longrightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix}$$

11. (10 pts) Write the equation for "The half-life of the radioactive isotope, ~~Freakazoidium-99~~, is 9900 years," and solve the equation for the decay constant, k.

$$A_0 e^{9900K} = \frac{1}{2} A_0 \qquad \boxed{K = \frac{-\ln(2)}{9900}}$$

$$e^{9900K} = \frac{1}{2} \qquad \approx -7.00148667 \times 10^{-5}$$

$$9900K = \ln(1/2) = -\ln 2 \quad = -0.0000700148667$$

12. (10 pts) Based on your work, how much radioactive Freakazoidium remains in a 512-kilogram sample, after 100,000 years?

$$A(100{,}000) = A_0 e^{100000K} = 512 e^{\left(-\frac{\ln(2)}{9900}\right)(100000)}$$

$$\approx \boxed{.46618\,99792 \text{ Kg}}$$

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