

Date, Time:

Do your own work on separate paper. Leave plenty of margin and plenty of room around your work. I'm not impressed if you squeeze more work into a smaller space. To the contrary. At the end, please make sure your problems are in order. I'm too old and ornery to want to go on a scavenger hunt to award you points.

1. (20 pts) Starting with  $f(x) = 2^x$ , sketch the graph of  $g(x) = -3 \cdot 2^{x+5} + 9$  in 4 steps (counting  $f(x) = 4^x$  as the first step). Use  $x = -1$ ,  $x = 0$ , and  $x = 1$  to find 3 points in the first graph, and show how these 3 points are moved around by each step in the transformation to  $g(x)$ . Finding the  $x$ - and  $y$ -intercepts is a separate problem, so don't worry about them, on this page. Label each sketch as some variation on  $f(x)$ , for instance,  $7 \cdot 2^{x-11} - 4$  would be  $7f(x-11) - 4$ .
2. (10 pts) Let . Find the  $x$ - and  $y$ -intercepts for this function, rounded to 4 decimal places. For 5 bonus points, label these intercepts on your final graph for #1.
3. Let  $f(x) = \sqrt{3x-6}$  and  $g(x) = \frac{x+10}{x+4}$ .
  - a. (5 pts) What is the domain of  $f$ ?
  - b. (5 pts) What is the domain of  $g$ ?
  - c. (5 pts) Write the function  $\frac{f}{g}$ . Do not simplify.
  - d. (5 pts) What is the domain of  $\frac{f}{g}$ ?
  - e. (5 pts) Write the function  $f \circ g$ . Do not simplify.
  - f. (5 pts) What is the domain of  $f \circ g$ ?
4. Find the domain:
  - a. (5 pts)  $\sqrt{\frac{(x+3)(x-9)^2}{(x-13)^3(x-5)^2}}$ . (Sign Pattern!)
  - b. (5 pts)  $\log_3\left(\frac{(x+3)(x-9)^2}{(x-13)^3(x-5)^2}\right)$  (Reinterpret previous sign pattern in the current context!)
5. (10 pts) Solve  $\log_7(2x+3) + \log_7(x-3) = \log_7(4x+6)$ .
6. (10 pts) Solve  $3^{x^2-12} \cdot 3^{-2x} = 27$
7. (10 pts) The half-life of a radioactive isotope is 950 years. Find how old a sample is, if 82% of the isotope in an ancient manuscript has decayed (i.e., if only 18% of the radioactive isotope remains.). Give this answer to the nearest year.

**Solve any two (3) Bonus problems for up to 15 points. I'll grade the first two I come to.**

**1. BONUS (5 pts)** Solve the equation  $2 \cdot (1.3)^x = 5 \cdot (1.1)^x$ . Give an exact answer and a decimal answer, rounded to 4 places.



**2. BONUS (5 pts)** Solve the absolute value inequality  $|2x - 7| \geq 8$ . Use a number line and either union or intersection ('and' or 'or') to find the solution.

**3. BONUS (5 pts)** The absolute value inequality  $|2x - 7| \geq -8$  is always true, since absolute value can never be negative. But show the steps and manage your and's and or's, with a number line graph at the end to interpret what the algebra is telling you.

**4. BONUS (5 pts)** The absolute value inequality  $|2x - 7| < -8$  is never true, since absolute value can never be negative. But show the steps and manage your and's and or's, with a number line graph at the end to interpret what the algebra is telling you.

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1} \text{ and } y = m(x - x_1) + y_1$$

$$\text{Continuous growth/decay/compounding: } A(t) = A_0 e^{kt}$$

$$\text{Periodic Compounding: } A(t) = A_0 \left(1 + \frac{r}{m}\right)^{mt} \text{ or } = P \left(1 + \frac{r}{m}\right)^{mt} \text{ or } = P(1 + i)^n.$$

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$ax^2 + bx + c = a(x - h)^2 + k, \text{ where } (h, k) = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$$

$$\text{Difference Quotient} = \frac{f(x+h) - f(x)}{h} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \text{average slope.}$$

$$a + ar + ar^2 + \dots + ar^{n-2} + ar^{n-1} = \sum_{k=1}^n ar^{k-1} = a \left(\frac{1 - r^n}{1 - r}\right) \text{ or } a \left(\frac{r^n - 1}{r - 1}\right)$$

$$\text{If } |r| < 1, \text{ then } a + ar + ar^2 + \dots + ar^{n-2} + ar^{n-1} + \dots = \sum_{k=1}^{\infty} ar^{k-1} = a \left(\frac{1}{1 - r}\right)$$