121 ONLING TAKE-HOME TEST 3 SPRING 2015

$f(x) = 4x^4 + 12x^3 - 34x^2 + 28x - 10$

① End Behavior : $+x^4$:

② Descartes' Rule

Positive zeros : 3 sign changes $\Rightarrow$ 3 or 1 positive zeros

Negative Zeros :

$f(-x) = 4x^4 - 12x^3 - 34x^2 - 28x - 10$

One sign change . <u>EXACTLY</u> one negative zero

③ Rational Zeros : $a_n = a_4 = 4$

$a_0 = -10$

$\frac{p}{q}$: $\frac{a_0 \text{ factors}}{a_n \text{ factors}}$

$\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 2, \cancel{\pm \frac{2}{2}}, \cancel{\pm \frac{2}{4}},$

$\pm 5, \pm \frac{5}{2}, \pm \frac{5}{4}, \pm 10, \cancel{\pm \frac{10}{2}}, \cancel{\pm \frac{10}{4}}, \cancel{\pm \frac{5}{2}}$

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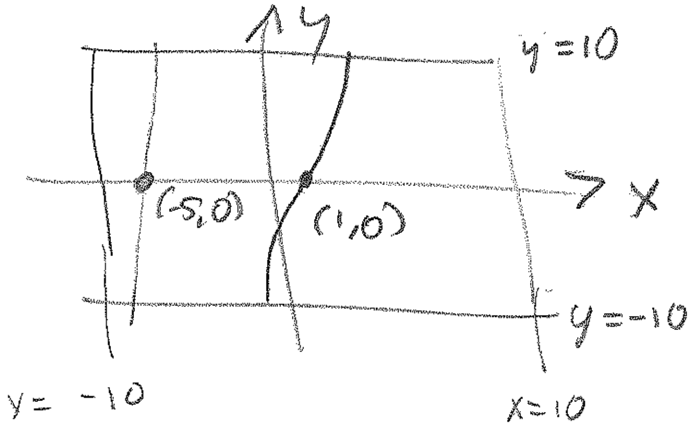
E3 TAKE-HOME

4 Graphes Gives me this ?



So I'm guessing $x=-5$ & $x=1$!

$$(x-1)(4x^3+16x^2-8x+10)$$

$$\begin{array}{r|rrrrr}
1 & 4 & 12 & -34 & 28 & -10 \\
 & & 4 & 16 & -18 & 10 \\ \hline
-5 & 4 & 16 & -18 & 10 & 0 \\
 & & -20 & 20 & -10 & \\ \hline
 & 4 & -4 & 2 & 0 &
\end{array}$$

5

$$\boxed{(x-1)(x+5)(4x^2-4x+2)}$$

Factored over (or "in") the reals, so that's #5

$4x^2-4x+2=0$

$a=4, b=-4, c=2$

$b^2-4ac = (-4)^2-4(4)(2)$

$= 16-32$

$= -16$

No more real zeros, so

It remains to solve $4x^2-4x+2=0$

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6 We have $(x-1)(x+5)(4x^2-4x+2)$

We have $a=4$, $b=-4$, $c=2$

$b^2-4ac=-16$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2(4)}$$

$$=\frac{4\pm\sqrt{-16}}{2(4)}=$$

$$=\frac{4\pm i\sqrt{16}}{8}$$

$$=\frac{4\pm 4i}{8}=\frac{4(1\pm i)}{8}$$

$$=\frac{1\pm i}{2} \quad \text{OR} \quad \frac{1}{2}\pm\frac{1}{2}i$$

So, $f(x)=4(x-1)(x+5)\left(x-\left(\frac{1+i}{2}\right)\right)\left(x-\left(\frac{1-i}{2}\right)\right)$

OR $(x-1)(x+5)(2x-(1+i))(2x-(1-i))$

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(7) Rough Sketch of

$f(x) = 4x^2 + 12x^3 - 34x^2 + 24x - 10$

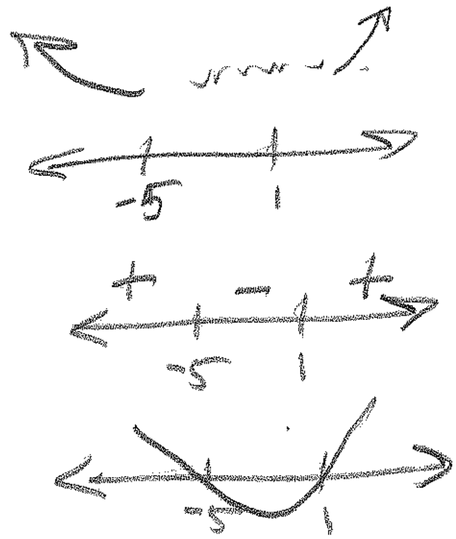
$= 4(x-1)(x+5)\left(x - \left(\frac{1+i}{2}\right)\right)\left(x - \left(\frac{1-i}{2}\right)\right)$

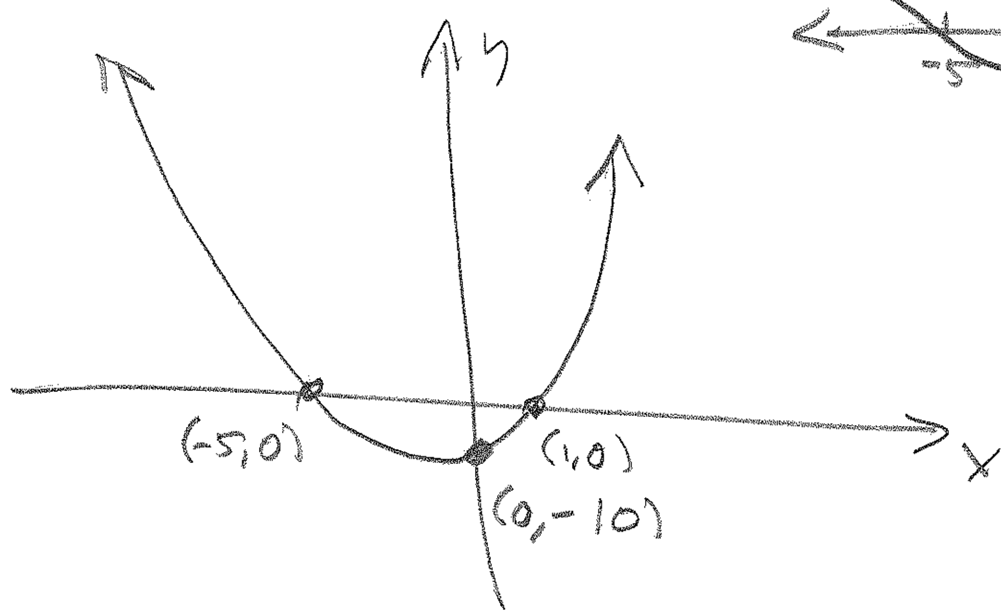
y-int: (0, −10)

x-int: (1, 0) ; (−5, 0)

$(x-1)^1$ $m=1$ odd cross

$(x+5)^1$ $m=1$ cross





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(8) $f(x) = \dfrac{x^2 - 5x - 6}{x^2 - 5x + 6} = \dfrac{(x-6)(x+1)}{(x-2)(x-3)}$

$\boxed{\mathcal{D} = \mathbb{R} \setminus \{2, 3\}}$

No holes $\Longrightarrow$

$\boxed{\text{V.A. are } x=2,\ x=3}$

$\boxed{\text{x-int: } x=6,\ x=-1}$

$\boxed{\Longrightarrow (6,0),\ (-1,0)}$

f ☆	f ☆	$f=0$	$f=0$
$x=2$	$x=3$	$x=6$	$x=-1$
$(x-2)^1$	$(x-3)^1$	$(x-6)^1$	$(x+1)^1$
$m=1$	$m=1$	$m=1$	$m=1$
ODD	ODD	ODD	ODD
change sign	change sign	change sign	change sign

$y=1$ is horizontal asymp.

$+$ -1 2 3 6 $+$

-1	2	3	6
$=0$	☆	☆	$=0$
$m=1$	$m=1$	$m=1$	$m=1$

Sign-change manage

$+$ -1 $-$ 2 $+$ 3 $-$ 6 $+$

End Behavior

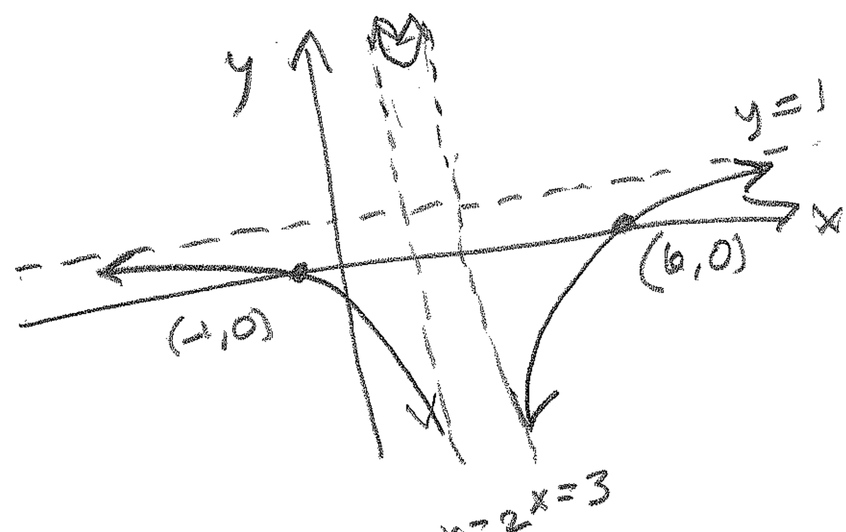
$\dfrac{x^2 - 5x - 6}{x^2 - 5x + 6}$ degree $= 2$ / degree $= 2$

Degree Tie

Divide highest-degree terms:

$\dfrac{x^2}{x^2} = 1$

$\boxed{\text{H.A.: } y = 1}$



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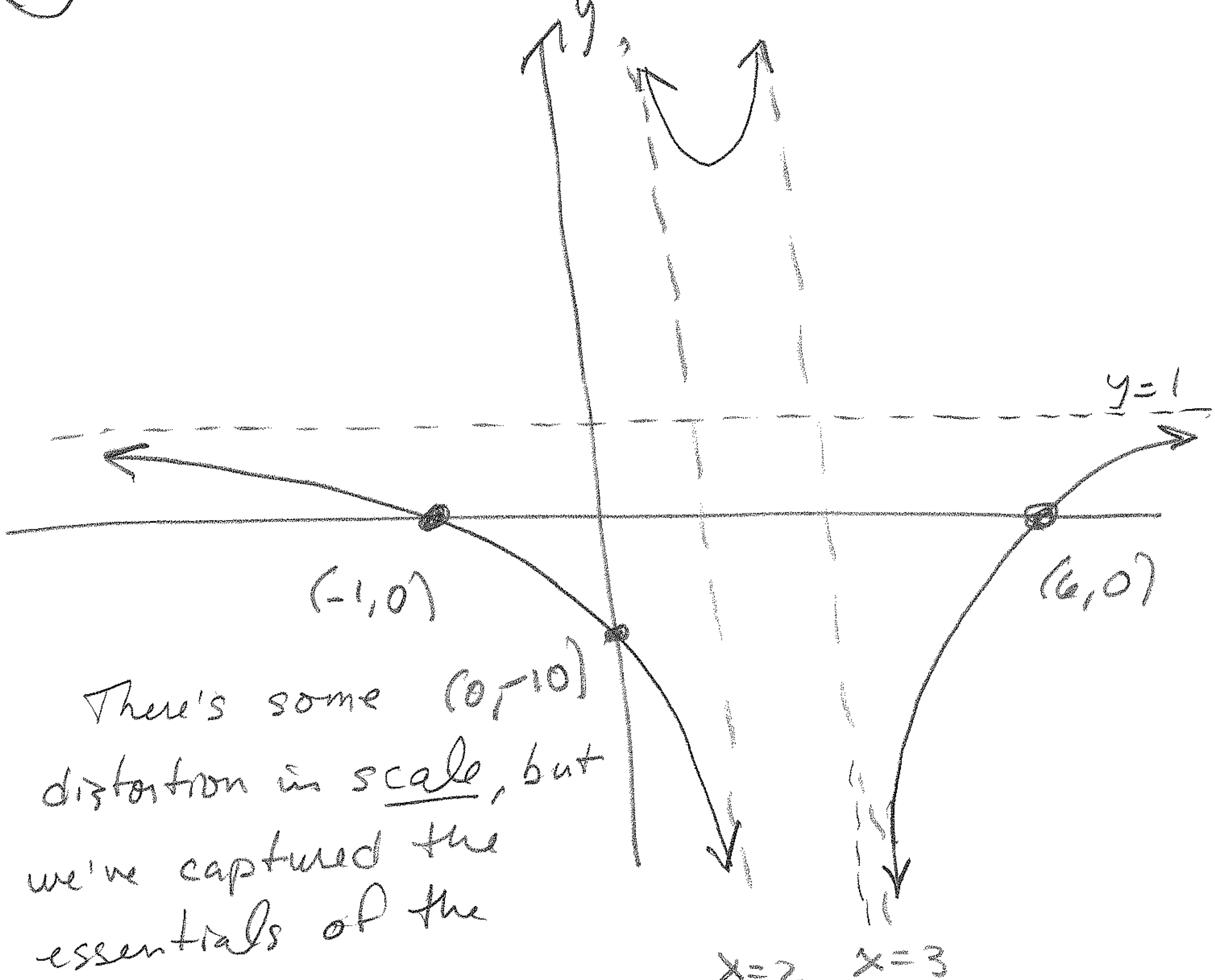
8 Re-Sketch



There's some distortion in scale, but we've captured the essentials of the graph.

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(9) $g(x) = \dfrac{x^3 - 9x^2 + 14x + 24}{x^3 - 9x^2 + 26x + 6}$

Says it's almost the same as #8.

Look for same x-intercepts:

$$\begin{array}{r|rrrr} -1 & 1 & -9 & 14 & 24 \\ & & -1 & 10 & -24 \\ \hline 6 & 1 & -10 & 24 & 0 \\ & & 6 & -24 & \\ \hline & 1 & -4 & 0 & \end{array}$$

So $x^3 - 9x^2 + 14x + 24$
$= (x+1)(x-6)(x-4)$

Look for same vertical asymptotes

$$\begin{array}{r|rrrr} 2 & 1 & -9 & 26 & -24 \\ & & 2 & -14 & 24 \\ \hline 3 & 1 & -7 & 12 & 0 \\ & & 3 & -12 & \\ \hline & 1 & -4 & 0 & \end{array}$$

So $x^3 - 9x^2 + 26x - 24$
$= (x-2)(x-3)(x-4)$

So $g(x) =$

$$\frac{(x+1)(x-6)(x-4)}{(x-2)(x-3)(x-4)}$$

$= \dfrac{(x+1)(x-6)}{(x-2)(x-3)}$ with

a HOLE @ $x = 4$!

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9 Cnt'd So, the graph of $g(x)$ is the same as the graph of $f(x)$, only $g(x)$ has a hole @ $x=4$.

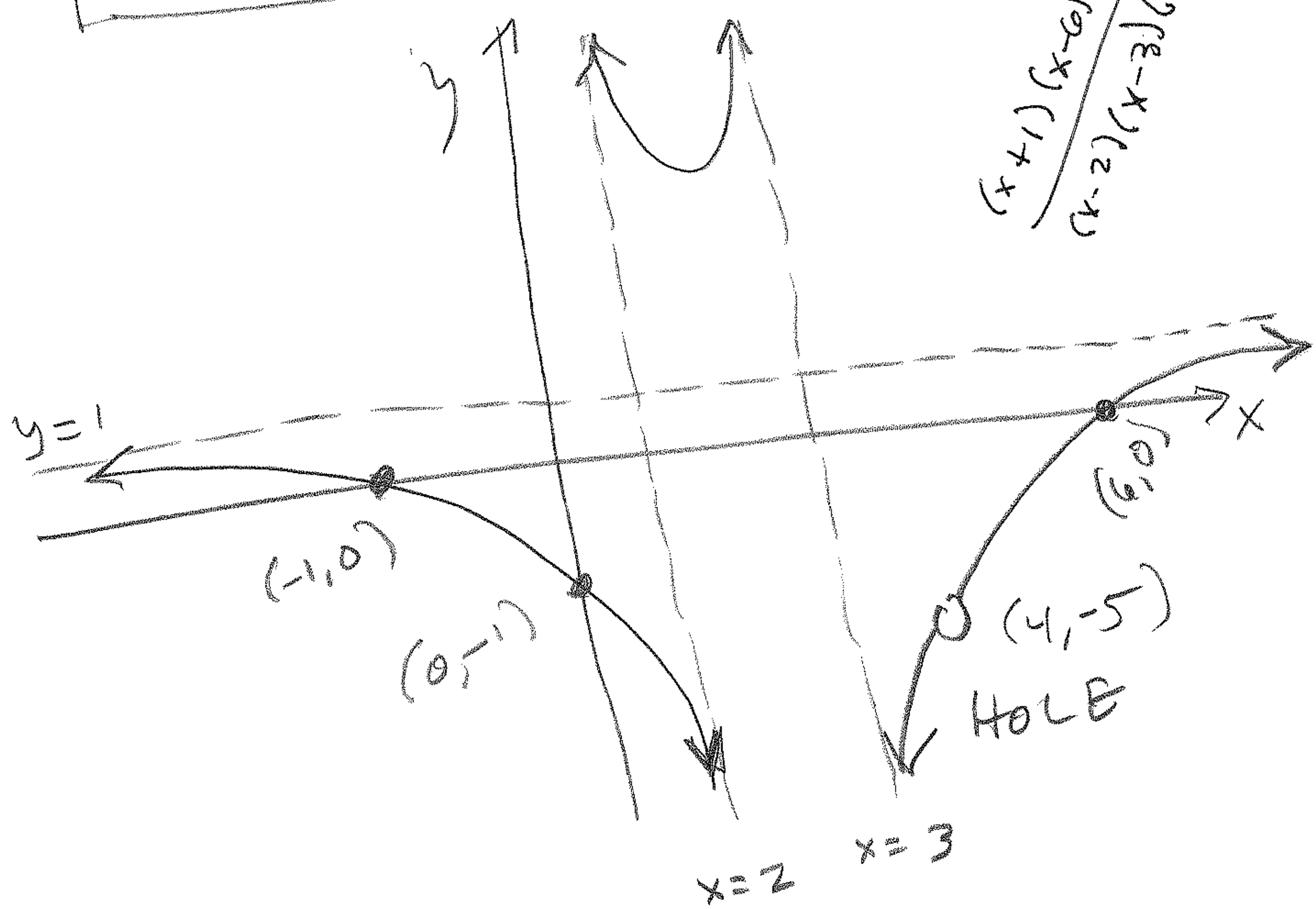
Locate y-coordinate of the hole:

$$f(4) = \frac{(4+1)(4-6)}{(4-2)(4-3)} = \frac{(5)(-2)}{(2)(1)} = -5$$

$\boxed{\text{HOLE @ } (4,-5)}$

Otherwise, same as f(x) graph

$$\frac{(x+1)(x-6)(x-4)}{(x-2)(x-3)(x-4)}$$



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(10) $h(x) = \dfrac{2x^2-5x-3}{x-4} = \dfrac{(2x+1)(x-3)}{x-4}$

$\mathcal{D} = \mathbb{R} \setminus \{4\}$

No Holes $\Longrightarrow$

$\boxed{\text{V.A.}: \; x = 4}$

x-int:

$(2x+1)(x-3) = 0$

$\Longrightarrow x = -\frac{1}{2}, 3$

$\Longrightarrow \boxed{\text{x-int}: (-\tfrac{1}{2},0), (3,0)}$

y-int:

$h(0) = \dfrac{-3}{-4} = +\dfrac{3}{4}$

$\boxed{\text{y-int}: (0, \tfrac{3}{4})}$

End behavior

$\dfrac{2x^2-5x-3}{x-4} = \dfrac{\text{degree} = 2}{\text{degree} = 1}$

Numerator of greater degree $\Longrightarrow$ Oblique asymptote

$\Longrightarrow$ Polynomial Division

Long Division:

$$
\begin{array}{r}
2x + 3 \;\; r\, 9 \\
x-4 \,\big)\, \overline{2x^2 - 5x - 3} \\
-\underline{(2x^2 - 8x)} \\
3x - 3 \\
-\underline{(3x - 12)} \\
9
\end{array}
$$

$\boxed{\begin{array}{c} y = 2x+3 \\ \text{O.A.} \end{array}}$

Synthetic Division:

$$
\begin{array}{r|rrr}
4 & 2 & -5 & -3 \\
 & & 8 & 12 \\
\hline
 & 2 & 3 & -9 \\
 & x & c & r
\end{array}
$$

Interpret:

$2x^2-5x-3 = (x-4)(2x+3) - 9$

$\dfrac{2x^2-5x-3}{x-4} = 2x+3 - \dfrac{9}{x-4}$

OR

Oblique Asymptote

$y = 2x+3$

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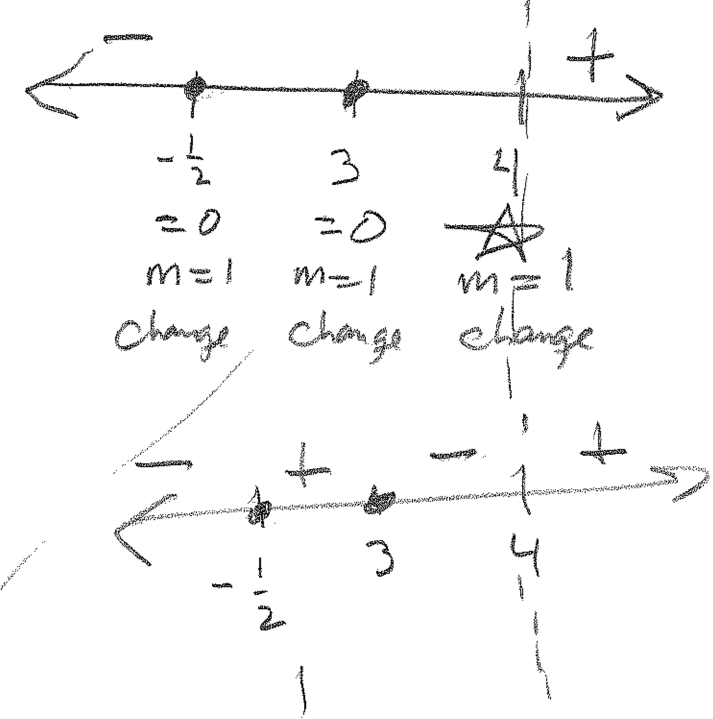
10 cnt'd

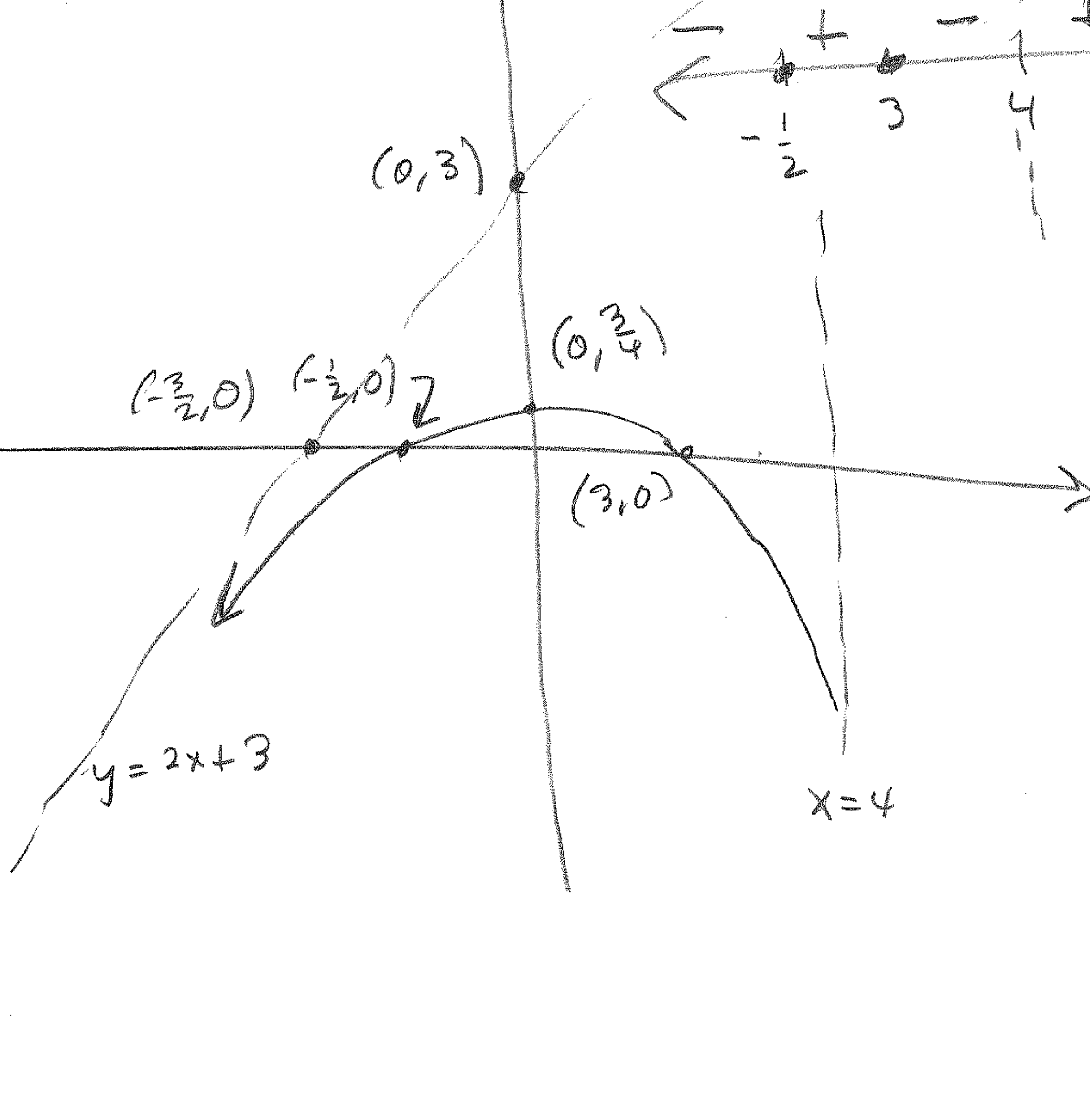
$y=2x+3$

End behavior

$2x$

O.A. $y=2x+3$





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(10) cnt'd

$y = 2x + 3 \overset{\text{SET}}{=} 0$

$\Rightarrow 2x = -3$

$\Rightarrow x = -\frac{3}{2}$

$\rightsquigarrow \left(-\frac{3}{2}, 0\right)$

$x = 0 \Rightarrow$

$y = 2(0) + 3 = 3$

$\rightsquigarrow (0, 3)$