MAT 121-G81-G82 | Test 3 – Spring, 2015 | (5 pts) Name KEY

100 Points   Covers Chapter 3   NO GRAPHING CALCULATORS!!!

Instructor: Harry S. Mills
EDBH 134K
x6238

Date, Time and Location:

Do you work on separate paper, provided by the testing center. Only write on one side. This should be easy, since most of the scratch paper, there, has junk on the back side, anyway. The only thing I expect to see written on this page and the next page is your *name*!

Before you turn in your work, make darn sure that your problems are in proper order. Any scratch work needs to be included in the same space as your work for the problem. Failure to organize your work, and leave margins in the upper left corner will not make my job harder. It will just cost you points.

1. (10 pts) Form a polynomial of *minimial degree* in *factored form* that has real coefficients (after expanding) and will have the given zeros. Do *not* expand your polynomial. Leave it factored! If you run out of room, you're doing it wrong!

$x = 1$, multiplicity 3;
$x = 3 - 7i$, multiplicity 1;
$x = -3$, multiplicity 2.

2. (10 pts) Use synthetic division to find $P(-2)$ if $P(x) = 3x^5 - 2x^4 - x^2 - 2x - 5$.

3. (10 pts) Represent the work you just did on the previous problem by writing $P(x)$ in the form
   Dividend = *Divisor* • *Quotient* + *Remainder* .

4. Suppose $f(x) = (x+2)^2(x-1)(x-(3+i))(x-(3-i)) = x^5 - 3x^4 - 8x^3 + 26x^2 + 24x - 40$. I'm showing you both factored and expanded form to help you answer the following:

   a. (10 pts) Provide a rough sketch of *f*, using its zeros, their respective multiplicities and its end behavior. Include *x*- and *y*-intercepts. Your graph should be smooth. Un-exaggerate the vertical for a better quality graph.

5. Solve the inequalities. Give thanks for someone factoring everything for you!

   a. (10 pts) $(x+2)^2(x-1)(x-3)(x+3) \geq 0$

   b. (5 pts) $f(x)=\dfrac{(x+2)^2(x-1)}{(x-3)(x+3)}<0$

6. Let $f(x)=9x^5+15x^4-31x^3-21x^2+48x-20$.

   a. (10 pts) Find the *real* zeros of f. Then factor f in the set of **real numbers**. This should involve an irreducible quadratic factor.

   b. (10 pts) Find the remaining (nonreal) zeros of f and factor f over the set of **complex numbers**. This step requires breaking down the irreducible – irreducible in the real number system, that is - quadratic factor. The Fundamental Theorem tells us that *nothing* is irreducible over the complex numbers.

7. (10 pts) You don't need to graph $R(x)=\dfrac{x^3-8x^2+x+42}{x^2+2x-15}=\dfrac{(x+2)(x-7)(x-3)}{(x+5)(x-3)}$, here, but I do want to see you graph its asymptotes, and locate the hole.

8. (10 pts) Sketch the graph of $R(x)=\dfrac{x^2-5x-14}{x^2+2x-15}$. Show all asymptotes and intercepts.

ANSWER ANY TWO (2) OF THE FOLLOWING for up to 10 Bonus points.

Bonus: (5 pts) Re-write $f(x)=2x^2-6x+13$ in the form $f(x)=a(x-h)^2+k$ by completing the square.

Bonus (5 pts) Solve the following absolute value inequalities:

a. $|2x-3|>11$ b. $|2x-3| \leq -11$

Bonus: (5 pts) Sketch the graph of $3\sqrt[3]{2x-4}-5$

Bonus: (5 pts) What is the domain of $f(x)=\sqrt{\dfrac{(x+2)^2(x-1)}{(x-3)(x+3)}}$? Hint: See previous work.

Bonus: (5 pts) Sketch the graph of $R(x)=\dfrac{x^3-8x^2+x+42}{x^2+2x-15}$. Hint: See previous work.

(1) 10 pts $x=1,\ m=3$; $x=3-7i,\ m=1$; $x=-3,\ m=2$

Want it to have real coefficients when expanded, but don't want to expand it!

$$\boxed{(x-1)^3(x-(3-7i))(x-(3+7i))(x+3)^2}$$

(2) 10 pts $P(x) = 3x^5-2x^4-x^2-2x-5$

$\Rightarrow$ $P(-2)$ is found by Remainder Theorem!

Divide by $(x+2)$:

$$\begin{array}{r|rrrrrr} -2 & 3 & -2 & 0 & -1 & -2 & -5 \\ & & -6 & 16 & -32 & 66 & -128 \\ \hline & 3 & -8 & 16 & -33 & 64 & \boxed{-133 = P(-2)} \end{array}$$

(3) 10 pts The above says $P(x)$

$$\boxed{= (x+2)(3x^4-8x^3+16x^2-33x+64) - 133}$$

(4) $f(x) = (x+2)^2(x-1)(x-(3+i))(x-(3-i))$

$= x^5 - 3x^4 - 8x^3 + 26x^2 + 24x - 40$

10 pts

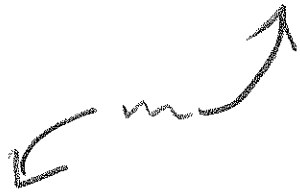
(a) We sketch $f(x)$:

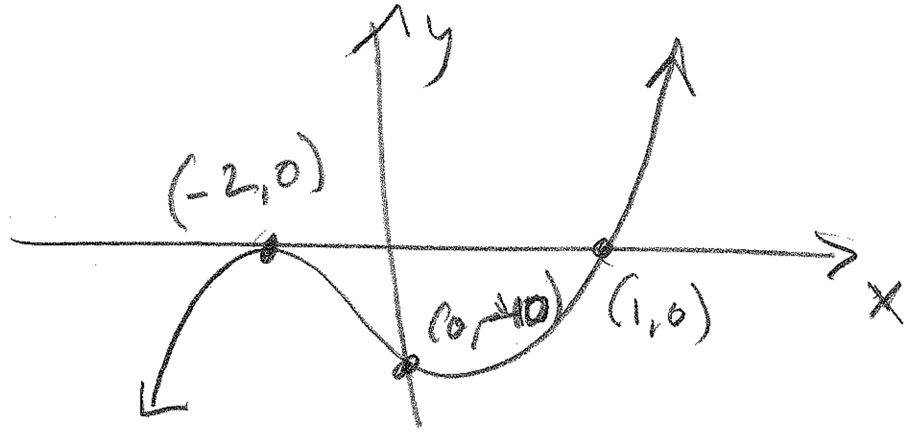
$f(0) = -40 \rightsquigarrow (0,-40)$ y-int

$(-2,0)$ $m=2$ Kiss

$(1,0)$ $m=1$ Cross

End Behavior x^5





(5) ~~10 pts~~ 10 pts (a) $(x+2)^2(x-1)(x-3)(x+3) \geq 0$

$(x^2)(x)(x)(x) = x^5$

−	−3	+	−2	+	1	−	3	+
NO	Yes	Yes	Yes	Yes	Yes	Yes	Yes	

$\Rightarrow x \in [-3,-2] \cup [-2,1] \cup [3,\infty) = \boxed{[-3,1] \cup [3,\infty)}$

5 b 5 pts

$$\frac{(x+2)^2(x-1)}{(x-3)(x+3)} < 0$$ Same Sign Pattern!

$-$	-3	$+$	-2	$+$	1	$-$	3	$+$
Yes	No	No	No	No	No	Yes	No	No

$$\boxed{x \in (-\infty, -3) \cup (1, 3)}$$

6 $f(x) = 9x^5 + 15x^4 - 31x^3 - 21x^2 + 48x - 20$

-2	9	15	-31	-21	48	-20
		-18	6	50	-58	20
-2	9	-3	-25	29	-10	0
		-18	42	-34	10	
1	9	-21	17	-5	0	
		9	-12	5		
	9	-12	5	0		

(a) 10 pts

$b^2 - 4ac = (-12)^2 - 4(9)(5)$

$= 144 - 180$

$= -36$ irreducible over $\mathbb{R}$ ∴

$$\boxed{f(x) = (x+2)^2(x-1)(9x^2 - 12x + 5)}$$

121 TEST 3

4

6 b Now break it down in the

10pts COMPLEX # system

$9x^2-12x+5=0 \Longrightarrow b^2-4ac=-36 \Longrightarrow \sqrt{b^2-4ac}=6i$

$$x=\frac{b\pm\sqrt{b^2-4ac}}{2a}=\frac{12\pm 6i}{2(9)}=\frac{6(2\pm i)}{18}$$

$$=\frac{2\pm i}{3} \Longrightarrow$$

$$f(x)=9(x+2)^2(x-1)\left(x-\left(\frac{2+i}{3}\right)\right)\left(x-\left(\frac{2-i}{3}\right)\right)$$

7 $R(x)=\dfrac{x^3-8x^2+x+42}{x^2+2x-15}=\dfrac{(x+2)(x-7)(x-3)}{(x+5)(x-3)}$

10pts $=\dfrac{(x-2)(x-7)}{x+5}$, where $x\neq 3$

$\Longrightarrow \mathcal{D}=\mathbb{R}\setminus\{-5,3\}$

$\boxed{V.A.\ x=-5}$

Hole @ $x=3$:

$$\frac{(3+2)(3-7)}{3+5}=\frac{(5)(-4)}{8}=$$

$= -\frac{5}{2} \rightsquigarrow \boxed{(3,-\frac{5}{2})}$ .3

$\boxed{\text{hole}}$

$\boxed{\text{y-int: } (0,\frac{42}{-15}) = (0,-\frac{14}{5})}$

x-int: $(2,0),(7,0)$

(5)

\#7

$\dfrac{3^{rd}\text{ degree}}{2^{nd}\text{ degree}} \Longrightarrow$ Oblique Asymptote

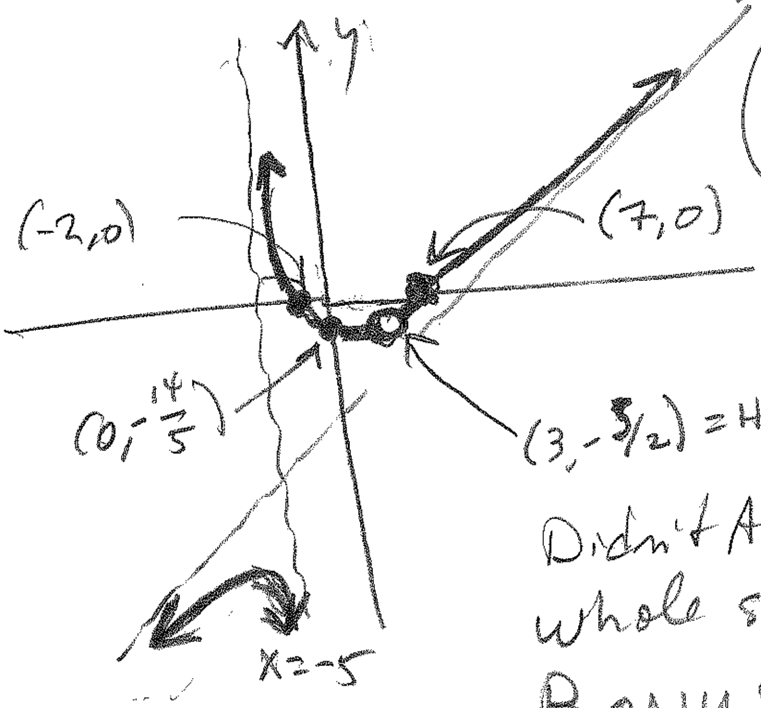
$$\begin{array}{r} x-10 \\ x^2+2x-15 \,\overline{\big)\, x^3-8x^2+x+42} \\ -(x^3+2x^2-15x) \\ \hline -10x^2+16x+42 \end{array}$$

$\boxed{y = x-10 \text{ is O.A.}}$

$y = x-10$ Asymptotes Hole

"skeleton" 10 pts

BONUS #5 5 pts Finish graph



Didn't Ask for whole sketch, until BONUS Page!

$$\frac{x^3-8x^2+x+42}{x^2+2x-15} = \frac{(x+2)(x-7)(x-3)}{(x+5)(x-3)}$$

121-online Test 3

(8) $R(x) = \dfrac{x^2-5x-14}{x^2+2x-15} = \dfrac{(x-7)(x+2)}{(x+5)(x-3)}$

10 pts

$D = \mathbb{R} \setminus \{-5, 3\}$

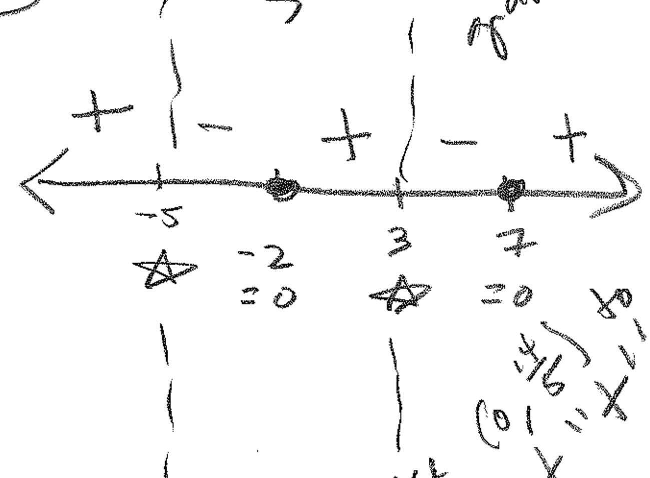
V.A.: $\boxed{x=-5,\ x=3}$

H.A.: $y = \frac{x^2}{x^2} = \boxed{1 = y}$

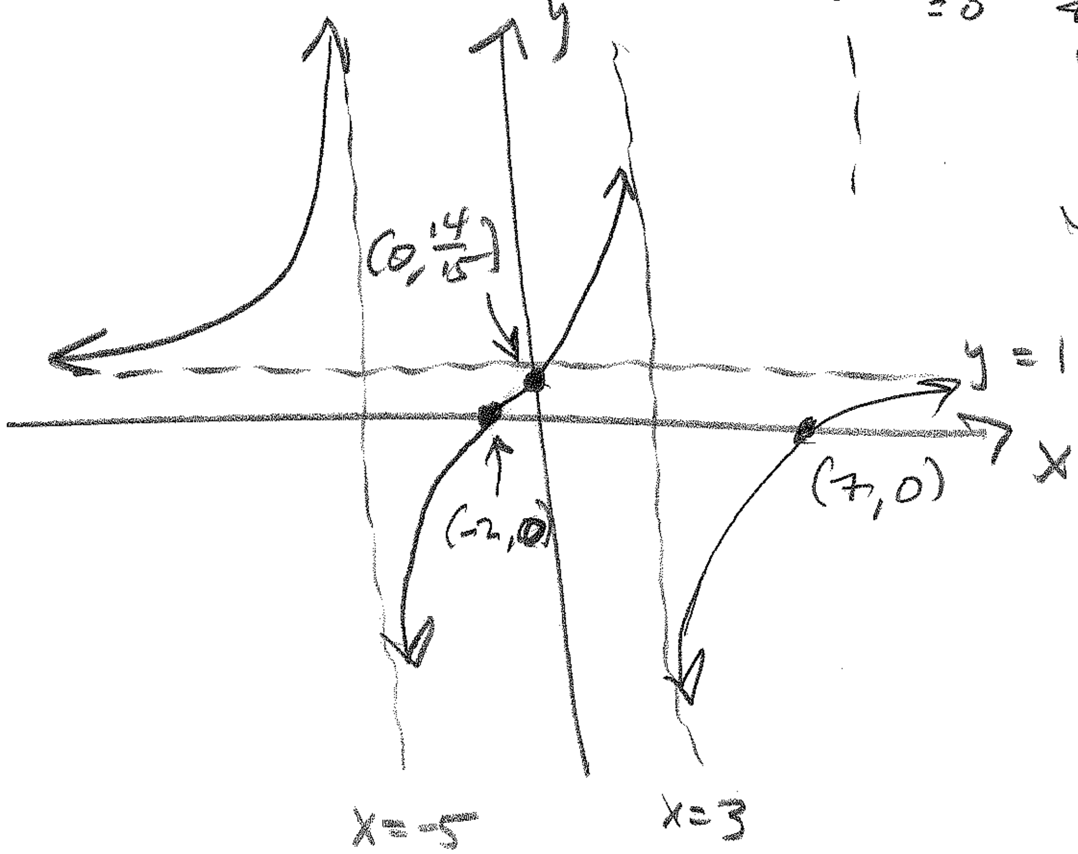
x-int: $(-2,0), (7,0)$

y-int: $(0, \frac{14}{15})$

Sign Pattern, key to graphing!



Use $(0,\frac{14}{15})$ to get "+" between $x=-2$ & $x=3$, work out from there!



121 ONLINE TEST 3

B1 5pts

$f(x) = 2x^2 - 6x + 13$

$\frac{1}{2} f(x) = x^2 - 3x + \frac{13}{2}$

$\frac{1}{2} f(x) - \frac{13}{2} = x^2 - 3x$

$\frac{1}{2} f(x) - \frac{13}{2} + \frac{9}{4} = x^2 - 3x + \left(\frac{3}{2}\right)^2$

$\frac{1}{2} f(x) - \frac{26}{4} + \frac{9}{4} = \left(x - \frac{3}{2}\right)^2$

$\frac{1}{2} f(x) - \frac{17}{4} = \left(x - \frac{3}{2}\right)^2$

$\frac{1}{2} f(x) = \left(x - \frac{3}{2}\right)^2 + \frac{17}{4}$

$$\boxed{f(x) = 2\left(x - \frac{3}{2}\right)^2 + \frac{17}{2}}$$

5pts B2

a) $|2x - 3| > 11$

$2x - 3 > 11$ OR $2x - 3 < -11$

$2x > 14$ $\quad 2x < -8$

$\{x \mid x > 7$ OR $x < -4\}$

$= (-\infty, -4) \cup (7, \infty)$

b) $|2x - 3| \leq -11$

Nevah!

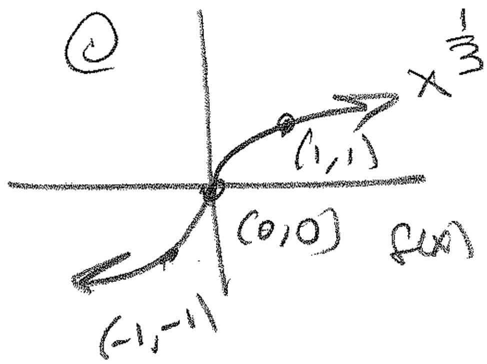
$\emptyset$

B3 5pts

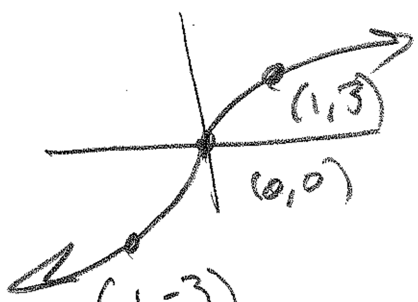
$g(x) = 3\sqrt[3]{2x-4} - 5$

$= 3\sqrt[3]{2(x-2)} - 5$

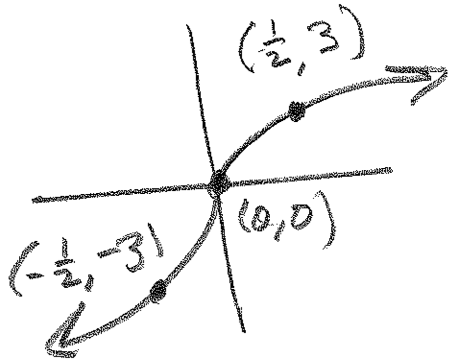
① $x^{\frac{1}{3}}$ OR $\sqrt[3]{x}$



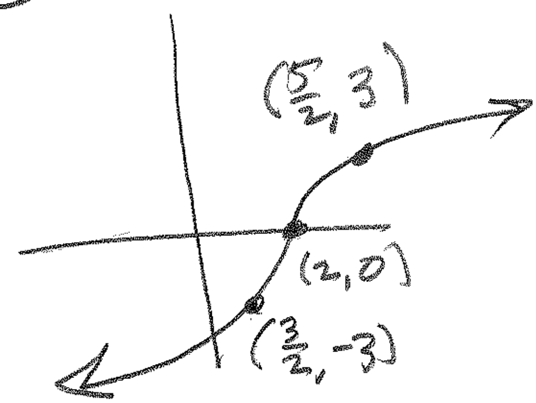
② $3\sqrt[3]{x} = 3f(x)$



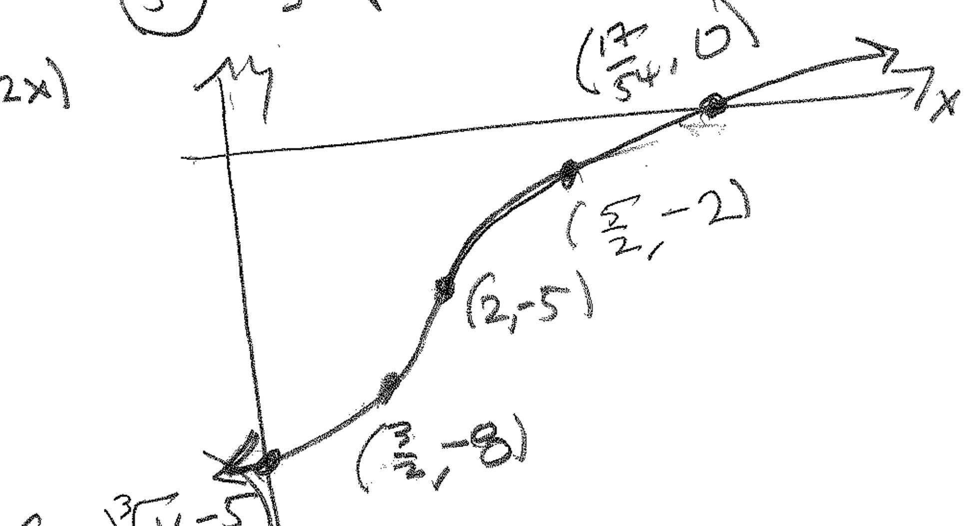
③ $3\sqrt[3]{2x} = 3f(2x)$



④ $3\sqrt[3]{2(x-2)} = 3f(2(x-2))$



⑤ $3\sqrt[3]{2(x-2)} - 5 = g(x)$



$3\sqrt[3]{2x-4} - 5 = 0$

$\sqrt[3]{2x-4} = \frac{5}{3}$

$2x-4 = \frac{125}{27}$

$2x = \frac{125-108}{27} = \frac{17}{27}$

$x = \frac{17}{54} \rightsquigarrow \left(\frac{17}{54}, 0\right)$

For B4, see #7!

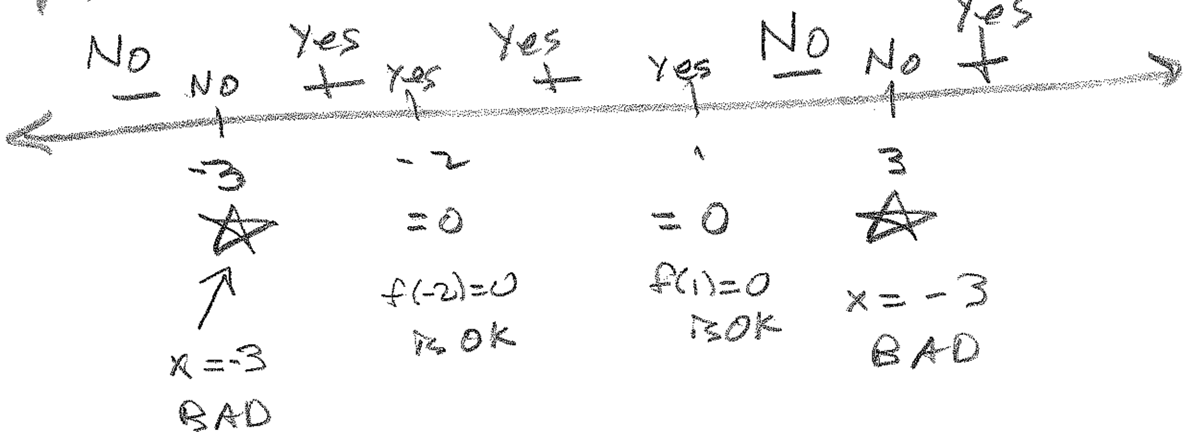
121 F3

B4 5 pts Domain of $f(x) = \sqrt{\dfrac{(x+2)^2 (x-1)}{(x-3)(x+3)}}$

is obtained by solving

$\dfrac{(x+2)^2 (x-1)}{(x-3)(x+3)} \geq 0$ Sign Pattern from

previous work



$\mathcal{D}(f) = (-3,-2] \cup [-2,1] \cup (3,\infty)$

$= \boxed{(-3,-1] \cup (3,\infty)}$