

1. Let $f(x) = -\frac{3}{2}x + 4$

a. (5 pts) Determine the slope and y-intercept of f .

$m = -\frac{3}{2}, (0, 4) = (0, y)$

b. (5 pts) Use the slope and y-intercept to graph f , in the space on the right.

c. (5 pts) What's the average rate of change of f ?

$m = -\frac{3}{2}$

2. Suppose y varies jointly as the square root of x and the square of z and inversely as the square root of w .

a. (5 pts) Write an equation representing the relationship.

$y = k \frac{\sqrt{x} z^2}{\sqrt{w}}$

b. (5 pts) Suppose $y = 24$ when $x = 2, z = 9$ and $w = 4$. What, then, is y when $x = 2, z = 3$, and $w = 4$?

$24 = k \frac{\sqrt{2} \cdot 9^2}{\sqrt{4}} = k \frac{81\sqrt{2}}{2} = 81\sqrt{2} k \implies k = \frac{24 \cdot 2}{81 \cdot \sqrt{2}} = \frac{48}{81\sqrt{2}} = \frac{16}{27\sqrt{2}} = \frac{16\sqrt{2}}{54} = \frac{8\sqrt{2}}{27}$

3. (5 pts each) Compute the discriminant for each of the following quadratic and tell me the nature of solutions, specifically, how many distinct solutions there are and whether they're real or non-real. Do not solve the equations.

a. $9x^2 - 12x + 4 = 0$

$b^2 - 4ac = (-12)^2 - 4(9)(4) = 144 - 144 = 0$

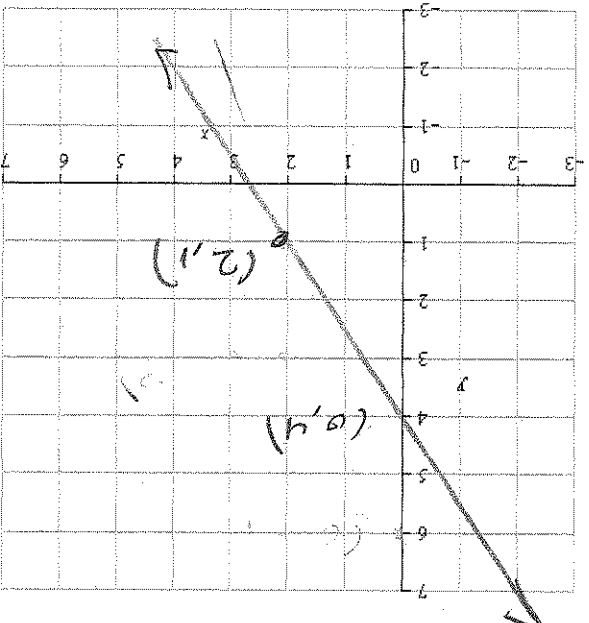
ONE Real Zero

b. $9x^2 - 18x + 8$

$b^2 - 4ac = (-18)^2 - 4(9)(8) = 324 - 288 = 36$

TWO Real (in fact, RATIONAL) solutions

3. (5 pts each) Compute the discriminant for each of the following quadratic and tell me the nature of solutions, specifically, how many distinct solutions there are and whether they're real or non-real. Do not solve the equations.



4. Solve by any method, but show all work!!! Give your final answer in fractional form (lowest terms).

a. (10 pts) $15x^2 - 92x + 140 = 0$

$$6^2 - 4ac = (-92)^2 - 4(15)(140)$$

$$= 8464 - 8400$$

$$= 64 \rightarrow \sqrt{64} = 8$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{92 \pm 8}{2(15)}$$

$$x \in \left\{ \frac{10}{3}, \frac{5}{3} \right\}$$

b. (5 pts) Solve $x^2 + 8x - 11 = 0$ by completing the square.

$$x^2 + 8x - 11 = 0$$

$$x^2 + 8x = 11$$

$$x^2 + 8x + 4^2 = 11 + 16$$

$$(x+4)^2 = 27$$

a. (5 pts) $f(x) = x^2 + 8x - 11$

$$= (x+4)^2 - 27$$

SEARCH

$$\frac{10}{9} + \frac{7}{9} = \frac{17}{9}$$

$$\frac{196}{289} = \frac{14}{17}$$

$$(15)(140) = 2100$$

$$-92 = -80 - 12 \rightarrow 960$$

$$= -60 - 32 \rightarrow 1920$$

$$= -50 - 42 \rightarrow 2100$$

$$18x^2 - 92x - 42x + 140 = 0$$

$$5x(3x-10) - 14(3x-10) = 0$$

$$(3x-10)(5x-14) = 0$$

$$x \in \left\{ \frac{10}{3}, \frac{14}{5} \right\}$$

$$\sqrt{(x+4)^2} = \sqrt{27}$$

$$|x+4| = 3\sqrt{3}$$

$$x+4 = \pm 3\sqrt{3}$$

$$x = -4 \pm 3\sqrt{3}$$

6. Complete the square and re-write each of the following in the form $f(x) = a(x-h)^2 + k$.

b. (5 pts) $f(x) = 7x^2 - 3x - 10$

$$\textcircled{1} \frac{1}{4} f(x) = x^2 - \frac{3}{4}x - \frac{10}{4}$$

$$\textcircled{2} \frac{1}{4} f(x) + \frac{9}{16} = x^2 - \frac{3}{4}x - \frac{10}{4} + \frac{9}{16}$$

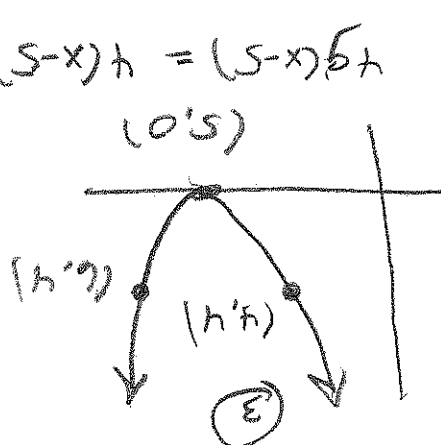
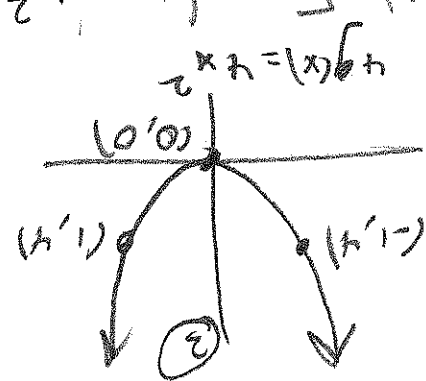
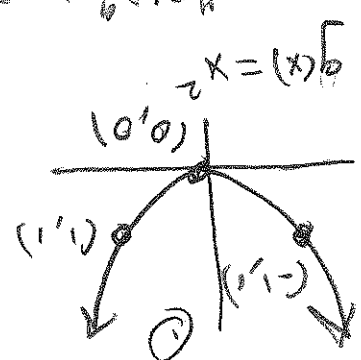
$$\textcircled{3} \frac{1}{4} f(x) + \frac{9}{16} + \left(\frac{14}{3}\right)^2 = x^2 - \frac{3}{4}x + \left(\frac{14}{3}\right)^2$$

$$\textcircled{4} \frac{1}{4} f(x) + \frac{9}{16} + \left(x - \frac{14}{3}\right)^2 = \frac{196}{81}$$

$$\textcircled{5} \frac{1}{4} f(x) = \left(x - \frac{14}{3}\right)^2 - \frac{196}{81}$$

$$\textcircled{6} f(x) = 4\left(x - \frac{14}{3}\right)^2 - \frac{196}{27}$$

7. a. (10 pts) Sketch the graph of $f(x) = 4(x-5)^2 - 3$. I want to see it step-by-step, using techniques I gave you. Your first graph of the basic function should include the points $(-1,1)$, $(0,0)$, and $(1,1)$. Track where those points end up at each stage (each graph), as demonstrated in videos.

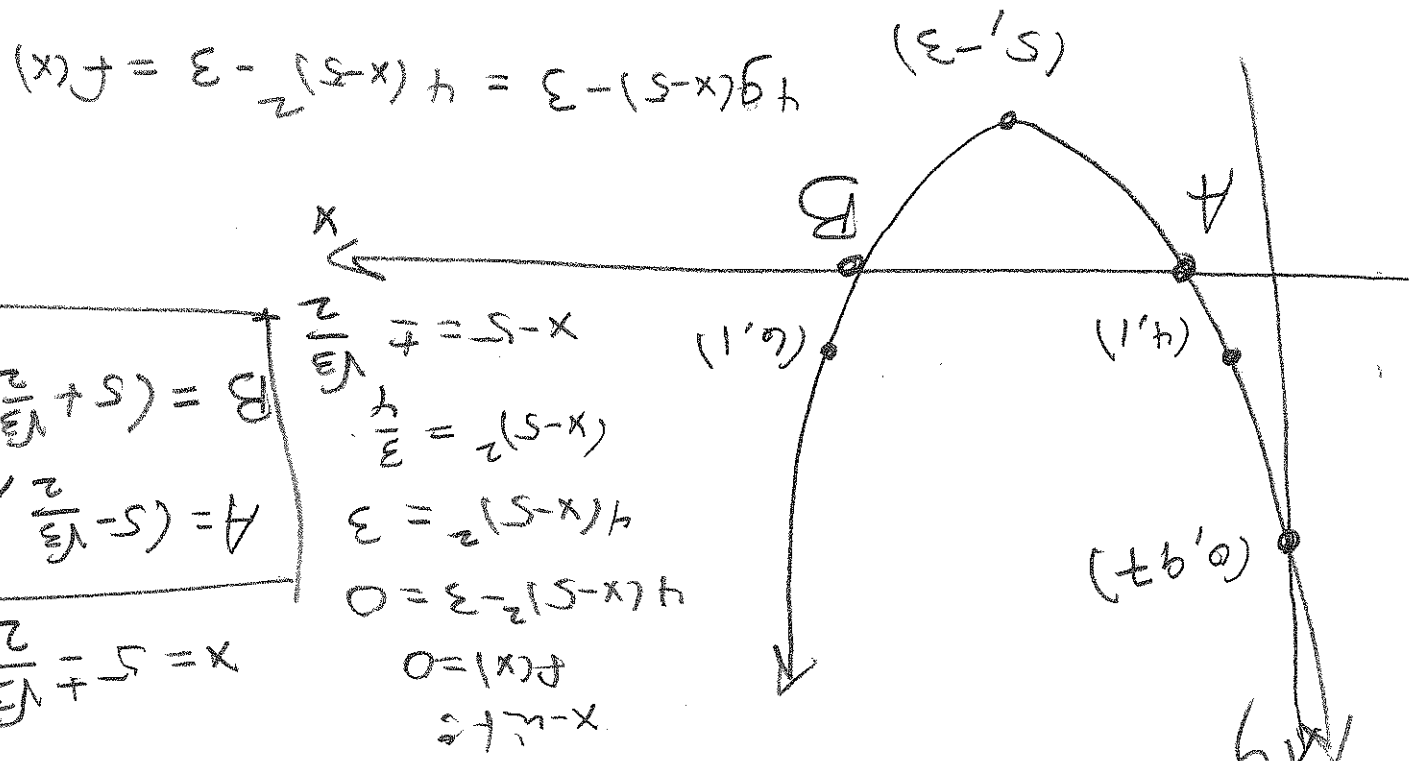


$4g(x-5) = 4(x-5)^2$

" $g(x) = x^2$ is Basic Function" x^2

$f(x) = 0$
 $4(x-5)^2 - 3 = 0$
 $4(x-5)^2 = 3$
 $(x-5)^2 = \frac{3}{4}$
 $x-5 = \pm \sqrt{\frac{3}{4}}$
 $x-5 = \pm \frac{\sqrt{3}}{2}$

$A = (5 - \frac{\sqrt{3}}{2}, 0)$
 $B = (5 + \frac{\sqrt{3}}{2}, 0)$



$4g(x-5) - 3 = 4(x-5)^2 - 3 = f(x)$

$D = \mathbb{R} = (-\infty, \infty)$; $R = [-3, \infty)$

8. (5 pts) Based on your work on the previous, state the domain and range of $f(x)$.

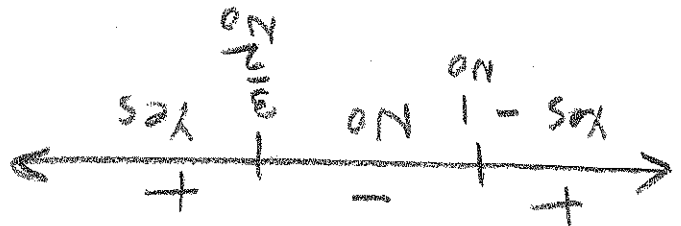
See above for $f(x) = 0$ work.

$f(0) = 4(-5)^2 - 3 = 97$
 $= 4(25) - 3 = 97$

10. (10 pts) Solve the inequality $3x^2 + 2x - 1 > x^2 + 3x + 2$. Give your final answer in set notation and interval notation.

$$2x^2 - x - 3 > 0$$

$$(2x - 3)(x + 1) > 0$$



$$x \in (-\infty, -1) \cup (\frac{3}{2}, \infty)$$

notation.

Bonus (5 pts) Solve $2x^2 - 4x + 7 \leq x^2 + 2x$. Give an exact final answer (simplified radical form), in set notation and interval notation.



$$x^2 - 6x + 7 \leq 0$$

$$x^2 - 6x + 7 = 0$$

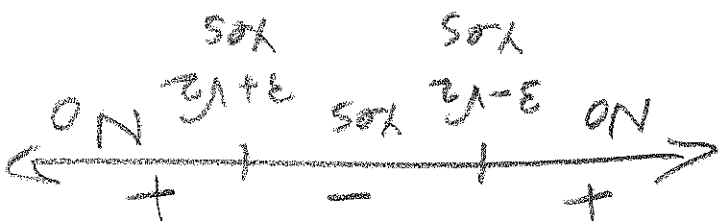
$$x^2 - 6x = -7$$

$$x^2 - 6x + 3^2 = -7 + 9$$

$$(x - 3)^2 = 2$$

$$x - 3 = \pm\sqrt{2}$$

$$x = 3 \pm \sqrt{2}$$



$$x \in [3 - \sqrt{2}, 3 + \sqrt{2}]$$

11. Solve the absolute value inequalities. Give your answers in set-builder and interval notation.

a. (5 pts) $|3x - 5| \leq 4$

$$3x - 5 \leq 4 \text{ AND } 3x - 5 \geq -4$$

$$3x \leq 9 \text{ AND } 3x \geq 1$$

$$\left\{ x \mid x \leq 3 \text{ AND } x \geq \frac{1}{3} \right\}$$

$$= \left[\frac{1}{3}, 3 \right]$$

b. (5 pts) $|3x - 5| > 4$

$$3x - 5 > 4 \text{ OR } 3x - 5 < -4$$

$$3x > 9 \text{ OR } 3x < 1$$

$$\left\{ x \mid x > 3 \text{ OR } x < \frac{1}{3} \right\}$$

$$= (-\infty, \frac{1}{3}) \cup (3, \infty)$$