MAT 121-G81-G82 Test 1 – Spring, 2015 Name KEY

100 Points Covers Chapter 1 NO GRAPHING CALCULATORS!!!

1. For each of the following functions, state the domain in set-builder notation and in interval notation.

a. (5 pts) $f(x)=\sqrt{3x+5}$

$3x+5 \geq 0$

$3x \geq -5$

$\left\{x \mid x \geq -\frac{5}{3}\right\} = \left[-\frac{5}{3}, \infty\right)$

b. (5 pts) $g(x)=\frac{x^2+13}{3x+5}$

$3x+5 \neq 0$

$3x \neq -5$

$\left\{x \mid x \neq -\frac{5}{3}\right\} =$

$= \left(-\infty, -\frac{5}{3}\right) \cup \left(-\frac{5}{3}, \infty\right)$

c. (5 pts) $h(x)=\frac{x^2+13}{\sqrt{3x+5}}$

$3x+5 \geq 0$ and $\sqrt{3x+5} \neq 0$

$\Rightarrow 3x+5 > 0 \Rightarrow \left\{x \mid x > -\frac{5}{3}\right\} = \left(-\frac{5}{3}, \infty\right)$

2. (5 pts) If $f(x)=|3x+5|$, what is $f(-8)$?

$f(-8) = |3(-8)+5| = |-24+5| = |-19| = 19$

3. (10 pts) What is the average rate of change of the function $f(x)=x^2+1$ from $x=3$ to $x=4$?

$\frac{f(4)-f(3)}{4-3} = \frac{4^2+1-(3^2+1)}{1} = 16+1-(9+1) = 17-10 = 7$

$= m_{AVG}$

4. (5 pts) Consider the relation $\mathbf{R} = \{(2,4),(3,-2),(5,6),(6,4),(7,2)\}$ and fill in the following:

a. Domain $= \{2,3,5,6,7\}$

b. Range $= \{4,-2,6,2\}$

c. Is **R** a function? Yes

d. If **R** is a function, is it a 1-to-1 function? (Yes, No or DNA)

No. $x=2 \rightarrow y=4$ and $x=6 \rightarrow y=4$

5. Let $f(x)=\frac{x-2}{x-3}$ and $g(x)=\sqrt{5x+10}$.

a. (5 pts) What is the domain of f?

$\{x \mid x \neq 3\}$

OR $(-\infty, 3) \cup (3, \infty)$

b. (5 pts) What is the domain of g?

$5x+10 \geq 0$

$5x \geq -10$

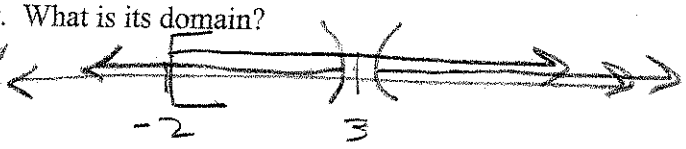
$\{x \mid x \geq -2\}$

OR $[-2, \infty)$

c. (5 pts) Write the function $(f-g)(x)$. Do not simplify. What is its domain?

$f-g = \frac{x-2}{x-3} - \sqrt{5x+10}$

"AND"



$\mathcal{D} = \{x \mid x \neq 3 \text{ and } x \geq -2\} = [-2, 3) \cup (3, \infty)$

d. (5 pts) Write the function $\left(\frac{f}{g}\right)(x)$. Do not simplify. What is its domain?

$\frac{\frac{x-2}{x-3}}{\sqrt{5x+10}}$

$\mathcal{D}$ = same, but need $\sqrt{5x+10} \neq 0$, also.

This amounts to snipping $x=-2$ out:

$\{x \mid x \neq 3 \text{ and } x > -2\} = (-2, 3) \cup (3, \infty)$

e. (5 pts) Write the function $(f \circ g)(x)$. Do not simplify. What is its domain?

$(f \circ g)(x) = \frac{\sqrt{5x+10}-2}{\sqrt{5x+10}-3}$

$\mathcal{D} = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

$\mathcal{D} = \{x \mid x \geq -2 \text{ and } g(x) \neq 3\}$

$= \{x \mid x \geq -2 \text{ and } x \neq -\frac{1}{5}\}$

$= [-2, -\frac{1}{5}) \cup (-\frac{1}{5}, \infty)$

Scratch:

$g(x) \neq 3$

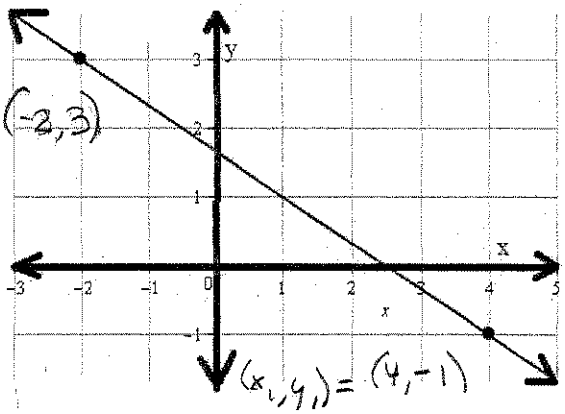
$\sqrt{5x+10} \neq 3$

$5x+10 \neq 9$

$5x \neq -1$

$x \neq -\frac{1}{5}$

6. (10 pts) Determine the equation of the line from its graph. Give the equation in…
   a. … point-slope form and
   b. … slope-intercept form.



$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-1)}{-2-4} = \frac{4}{-6} = -\frac{2}{3}$ $(x_2, y_2) = (-2, 3)$

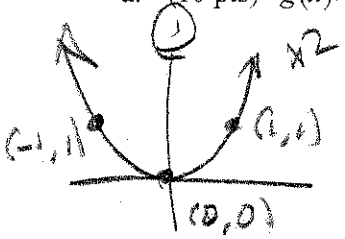
(a) $y = m(x - x_1) + y_1$

$y = -\frac{2}{3}(x-4) - 1$

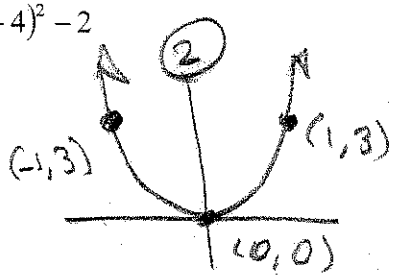
(b) $= -\frac{2}{3}x + \frac{8}{3} - \frac{3}{3} = -\frac{2}{3}x + \frac{5}{3} = y$

7. Graph each of the following by the techniques of shifting, stretching, compressing or reflecting. Start with the graph of a basic function and show all steps *as demonstrated in Videos*. I expect to see 3 points labeled in the first sketch, and to see where those points are moved to in each subsequent step.

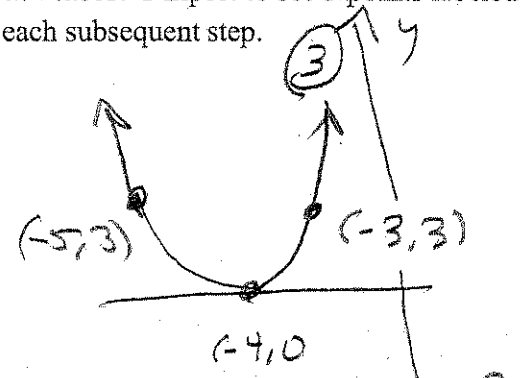
   a. (10 pts) $g(x) = 3(x+4)^2 - 2$



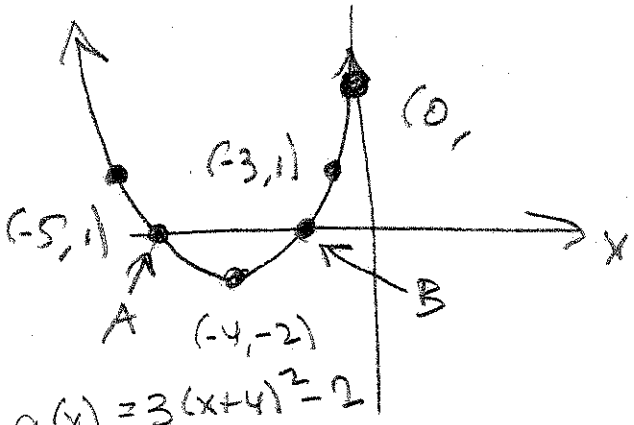
$f(x) = x^2$



$3f(x) = 3x^2$



$3f(x+4) = 3(x+4)^2$



$g(x) = 3(x+4)^2 - 2$

y-int: $3(0+4)^2 - 2$

$= 3(16) - 2$

$= 48 - 2 = 46 \rightsquigarrow (0, 46)$

x-int:

$3(x+4)^2 = 2$

$(x+4)^2 = \frac{2}{3}$

$x + 4 = \pm\sqrt{\frac{2}{3}} = \pm\frac{\sqrt{6}}{3}$

$x = \frac{12 \pm \sqrt{6}}{3}$

$A = \left(\frac{12-\sqrt{6}}{3}, 0\right)$

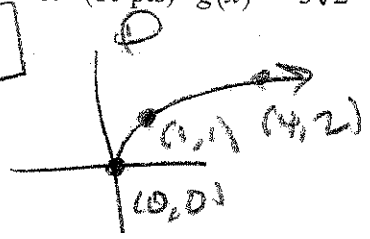
$B = \left(\frac{12+\sqrt{6}}{3}, 0\right)$

$x \xrightarrow{\text{flip}} -x \xrightarrow{\text{shift}} -(x-2)$

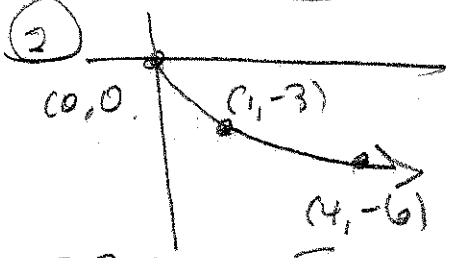
$2-x = -(x-2)$

b. (10 pts) $g(x) = -3\sqrt{2-x}+4$

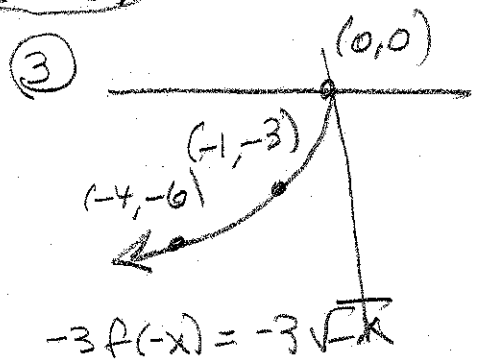
M1



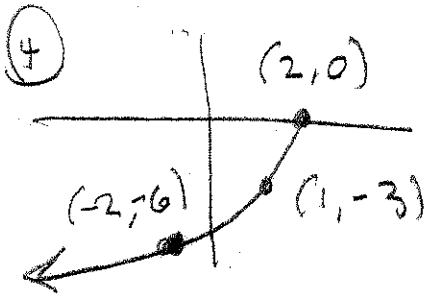
$f(x) = x^2$



$-3f(x) = -3\sqrt{x}$

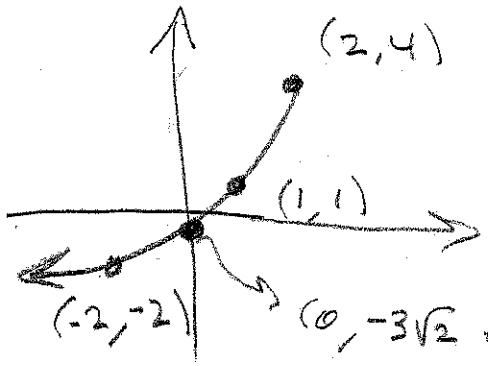


$-3f(-x) = -3\sqrt{-x}$



$-3f(-(x-2))$

$= -3\sqrt{-(x-2)}$



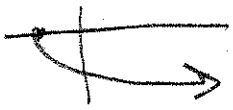
$\approx (0, -.2426)$

$g(x) = -3f(-(x-2)) + 4$

$= -3\sqrt{-(x-2)} + 4$

M2 Swap ③ & ④ sort of

③ $-3\sqrt{x+2}$



④ $-3\sqrt{-x+2}$

Students who do M2 generally don't go to get it right.

8. (10 pts) Sketch the graph of the piecewise-defined function $f(x) = \begin{cases} -x+3 & \text{if } -3 \le x \le 2 \\ (x-2)^2 - 1 & \text{if } x \ge 2 \end{cases}$. You don't have to show all steps with that 2nd piece, like I wanted for #7a, above. You can safely read the vertex from the definition. But I do want to see special care taken at the endpoints of the pieces. I also expect to see the x- and y- intercepts for f.

$(2-2)^2 - 1 = -1 \rightsquigarrow (2,-1)$ ●

$-2+3 = 1 \rightsquigarrow (2,1)$ ○

x-int! $(x-2)^2 - 1 = 0$

$(x-2)^2 = 1$

$x-2 = \pm\sqrt{1} = \pm 1$

$x = 2 \pm 1$ → $3 \rightsquigarrow (3,0)$, $1 \rightsquigarrow (1,0)$