MAT 121-G81-G82 Final Test – Fall, 2014 Name KEY

200 Points Comprehensive NO GRAPHING CALCULATORS!!!

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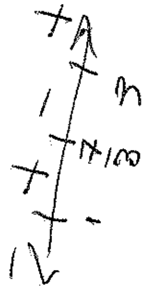


1. For each of the following functions, state the domain in interval notation.

a. (5 pts) $f(x)=\sqrt{3x-7}$

Need $3x-7\ge 0$

$3x\ge 7$

$\mathcal{D}=\{x \mid x\ge \frac{7}{3}\}$

$=\boxed{[7/3,\infty)}$

b. (5 pts) $g(x)=\dfrac{x^2-4x+3}{3x-7}$

Need $3x-7\ne 0$

$3x\ne 7$

$\mathcal{D}=\{x \mid x\ne 7/3\}$

$=\boxed{(-\infty,7/3)\cup(7/3,\infty)}$

c. (5 pts) $h(x)=\sqrt{\dfrac{(4x-16)(x-1)}{(2x-5)(x+3)}}$

Need $\dfrac{4(x-4)(x-1)}{(2x-5)(x+3)}\ge 0$

$+ \quad - \quad + \quad - \quad +$

$-3 \quad 1 \quad \frac{5}{2} \quad 4$

$\not\exists \quad =0 \quad \not\exists \quad =0$

$\mathcal{D}=\boxed{(-\infty,-3)\cup[1,5/2)\cup[4,\infty)}$

d. (5 pts) $w(x)=\log_4\left(\dfrac{(4x-16)(x-1)}{(2x-5)(x+3)}\right)$

$\mathcal{D}=$

$\boxed{(-\infty,-3)\cup(1,5/2)\cup(4,\infty)}$

2. (10 pts) What is the average rate of change of the function $f(x)=x^2+5$ from $x=1$ to $x=3$?

$\dfrac{f(3)-f(1)}{3-1}=\dfrac{3^2+5-(1^2+5)}{2}=\dfrac{9+5-(1+5)}{2}$

$=\dfrac{14-6}{2}=\dfrac{8}{2}=\boxed{4=m_{AVG}}$

3. The domain of $f(x)=\sqrt{x-1}$ is $[1,\infty)$ and the domain of $g(x)=\dfrac{2x-4}{x+2}$ is $(-\infty,-2)\cup(-2,\infty)$,

a. (5 pts) Find $\dfrac{f}{g}$ and determine its domain. Do not simplify $\dfrac{f}{g}$.

$\dfrac{f}{g}=\dfrac{\sqrt{x-1}}{\dfrac{2x-4}{x+2}}$ $\mathcal{D}=\{x \mid x\in\mathcal{D}(f) \text{ and } x\in\mathcal{D}(g) \text{ and } g(x)\neq 0\}$

$=\{x \mid x\geq 1 \text{ and } x\neq -2 \text{ and } x\neq 2\}$

$g(x)\neq 0$ $2x-4\neq 0$

$\dfrac{2x-4}{x+2}\neq 0$ $2x\neq 4$

$x\neq 2$

$=\boxed{[1,2)\cup(2,\infty)}$

b. (5 pts) Find $f\circ g$ and determine its domain. Do not simplify.

$f\circ g=\sqrt{\dfrac{2x-4}{x+2}-1}$ $\mathcal{D}=\{x \mid x\in\mathcal{D}(g) \text{ and } g(x)\in\mathcal{D}(f)\}$

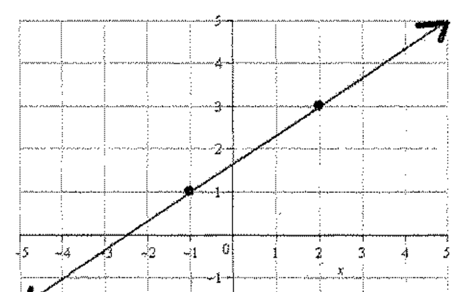
$=\{x \mid x\neq -2 \text{ and } g(x)\geq 1\}$

$\dfrac{2x-4}{x+2}-1\geq 0$ $=\{x \mid x\neq -2 \text{ and } \dfrac{2x-4}{x+2}\geq 1\}$

$\dfrac{2x-4-(x+2)}{x+2}\geq 0$ $=\{x \mid x\neq -2 \text{ and } (x\leq -2 \text{ or } x\geq 6)\}$

$\dfrac{x-6}{x+2}\geq 0$ + − + (−2, 6) $=\boxed{(-\infty,-2)\cup[6,\infty)}$

4. (10 pts) Use the points marked as dots to derive an equation of the line from its graph.



$(x_1,y_1)=(2,3)$, $(x_2,y_2)=(-1,1)$

$m=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{1-3}{-1-2}=\dfrac{-2}{-3}=\dfrac{2}{3}$

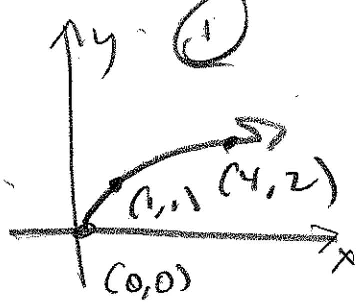
$y=m(x-x_1)+y_1$

$\boxed{y=\frac{2}{3}(x-2)+3}=\frac{2}{3}x-\frac{4}{3}+\frac{9}{3}=\boxed{\frac{2}{3}x+\frac{5}{3}=y}$

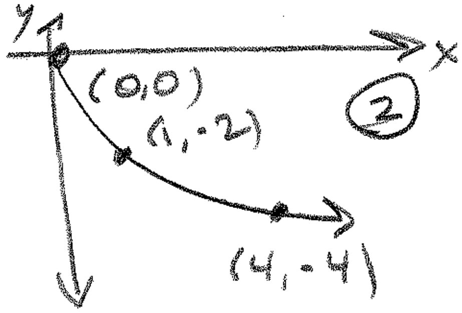
$\boxed{y=\frac{2}{3}(x+1)+1}$ also accepted

5. (10 pts) Graph $g(x)=-2\sqrt{-3x+6}+5$ by the techniques of shifting, stretching, compressing or reflecting. Start with the graph of a basic function and show all steps *as demonstrated in Videos*. I expect to see 3 points labeled in the first sketch, and to see where those points are moved to in each subsequent step. I strongly recommend using $(0,0),(1,1)$, and $(4,2)$ as the 3 points. I'm looking for 5 graphs, with the first being the basic function, $f(x)=\sqrt{x}$, and the final being $g(x)$. *None* of the graphs, between the first and the last is going to be *either* $f(x)$ *nor* $g(x)$, so, for the last time, don't call 'em all $f(x)$! x- and y-intercepts for 5 bonus points.

$f(x)=\sqrt{x}$



$-2f(x)=-2\sqrt{x}$



$-3x+6=-3(x-2)$

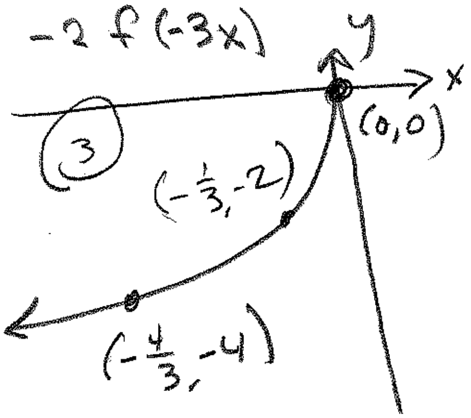
$g(0)=-2\sqrt{6}+5$

$\approx .1010205144$

$\boxed{(0,-2\sqrt{6}+5)\ A \approx (0, 0.1010205144)}$

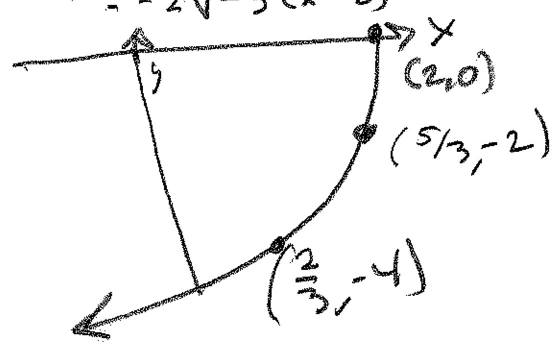
$-2\sqrt{-3x}$

$-2f(-3x)$



$-2f(-3(x-2))=$

$=-2\sqrt{-3(x-2)}$



$g(x)=0$

$-2\sqrt{-3x+6}+5=0$

$-2\sqrt{-3x+6}=-5$

$\sqrt{-3x+6}=\frac{5}{2}$

$-3x+6=\frac{25}{4}$

$-3x=\frac{25-24}{4}$

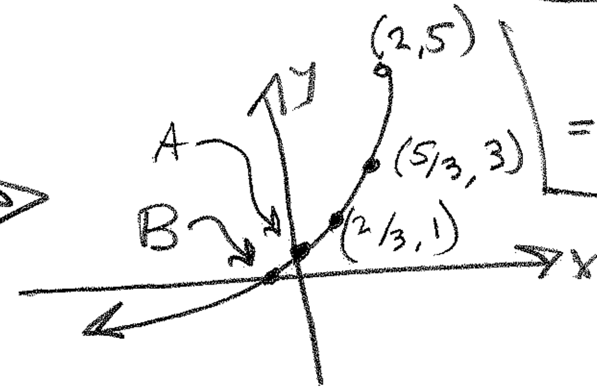
$x=\frac{-1}{12}$

$\boxed{(-\frac{1}{12},0)\ B = (-.08\overline{3},0)}$

$-2f(-3(x-2))+5$

$=-2\sqrt{-3x+6}+5$

$=g(x)$



6. Find all real and nonreal solutions of the following equations:

a. (10 pts) $2x^2 - x - 10 = 0$

$(2x - 5)(x + 2) = 0$

$\boxed{x \in \{-2, \frac{5}{2}\}}$

$a = 2, b = -1, c = -10$

$b^2 - 4ac = (-1)^2 - 4(2)(-10)$

$= 1 + 80 = 81$

$x = \frac{1 \pm \sqrt{81}}{2(2)} = \frac{1 \pm 9}{4}$ $\frac{10}{4} = \frac{5}{2}$, $-\frac{8}{4} = -2$

b. (10 pts) $4x^2 - 12x + 10 = 0$

$\overset{a}{2}x^2 - \overset{b}{6}x + \overset{c}{5} = 0$

$b^2 - 4ac = (-6)^2 - 4(2)(5)$

$= 36 - 40 = -4$

$x = \frac{6 \pm \sqrt{-4}}{2(2)} = \frac{6 \pm 2i}{4}$

$= \frac{3 \pm i}{2}$

$\boxed{x \in \{\frac{3 \pm i}{2}\}}$

c. (10 pts) $2x^4 - 2x^3 - 13x^2 + 28x - 15 = 0$

$\pm 1, \pm \frac{1}{2}, \pm 3, \pm \frac{3}{2}, \pm 5, \pm \frac{5}{2}, \pm 15, \pm \frac{15}{2}$

-3	2	-2	-13	28	-15
		-6	24	-33	15
1	2	-8	11	-5	0
		2	-6	5	
	2	-6	5	0	

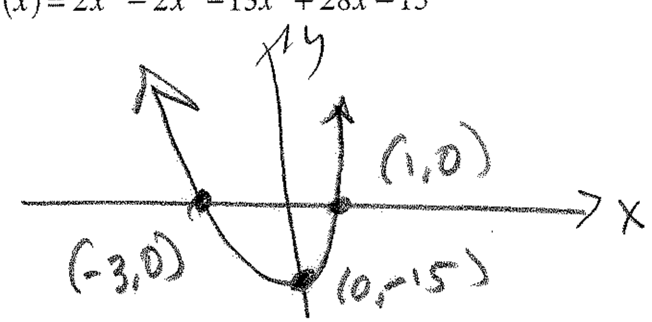
1	2	-2	-13	28	-15
		2	0	-13	15
	2	0	-13	15	0

$2x^2 - 6x + 5 = 0$

$\Rightarrow x \in \{\frac{3 \pm i}{2}\}$ by previous work, so

$\boxed{x \in \{-3, 1, \frac{3 \pm i}{2}\}}$

7. (10 pts) Based on your work on #6, provide a rough sketch of the graph of $f(x) = 2x^4 - 2x^3 - 13x^2 + 28x - 15$



$$\log_7(3\cdot 5^x) = \log_7(7^x)$$

$$\log_7(3) + \log_7(5^x) = \log_7(7^x)$$

$$\log_7(3) + (\log_7(5))x = \log_7(7)x = x$$

$$(\log_7(5))x - x = \log_7(3)$$

$$Ax - x = B$$

$$(A-1)x = B$$

$$x = \frac{B}{A-1} = \frac{\log_7(3)}{\log_7(5)-1}$$

8. Solve the following exponential and logarithmic equations. An exact answer is preferred. A decimal approximation is acceptable, if you are correct to the 5th decimal place.

a. (10 pts) $3^x = 97$

$\boxed{x = \log_3(97)}$

$= \dfrac{\ln(97)}{\ln(3)}$

≈ 4.164081383

≈ 4.16408

b. (10 pts) $\log_4(x) = 97$

$\boxed{x = 4^{97}}$

$\approx 2.5108406094 \times 10^{58}$

c. (5 pts) $3 \cdot 5^x = 7^x$

$\ln(3 \cdot 5^x) = \ln(7^x)$

$\ln 3 + (\ln 5)x = (\ln 7)x$

$A + Bx = Cx$

$Bx - Cx = -A$

$(B-C)x = -A$

$x = \dfrac{A}{C-B} = \boxed{\dfrac{\ln 3}{\ln 7 - \ln 5}} \approx 3.265090456$

Other variations: $\dfrac{\log_7(3)}{\log_7(5) - 1}$

d. (10 pts) $\log_2(x^2 + 2x + 1) = 4$

$x^2 + 2x + 1 = 2^4 = 16$

$x^2 + 2x - 15 = 0$

$(x+5)(x-3) = 0$

$\boxed{x \in \{-5, 3\}}$

9. Solve the absolute value inequality. Give you final answer in set-builder and interval notation.

a. (15 pts) $|2x - 7| > 11$

$2x - 7 > 11$ OR $2x - 7 < -11$

$2x > 18$ $2x < -4$

$\boxed{\{x \mid x > 9 \text{ OR } x < -2\}} = \boxed{(-\infty, -2) \cup (9, \infty)}$

10. (15 pts) Find the sum: $3 - 6 + 12 - 24 + \cdots\cdots + 768$

$\boxed{a = 3, r = -2}$

$768 = 3 \cdot 2^8 = 3 \cdot 2^{n-1} \Longrightarrow n = 9$

$S_n = 3\left(\dfrac{1-(-2)^9}{1-(-2)}\right) = 3\left(\dfrac{1+512}{3}\right) = 513$

$2 \mid 768$
$2 \mid 384$
$2 \mid 192$
$2 \mid 96$
$2 \mid 48$
$2 \mid 24$
$2 \mid 12$
$2 \mid 6$
3

11. (15 pts) Solve the system of linear equations:
$$\begin{aligned} x-5y+13z&=-35 \\ 3x-14y+37z&=-98 \\ 4x-18y+49z&=-127 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & -5 & 13 & -35 \\ 3 & -14 & 37 & -98 \\ 4 & -18 & 49 & -127 \end{array}\right] \begin{array}{l} R1 \\ -3R1+R2 \\ -4R1+R3 \end{array} \left[\begin{array}{ccc|c} 1 & -5 & 13 & -35 \\ 0 & 1 & -2 & 7 \\ 0 & 2 & -3 & 13 \end{array}\right]$$

$$\begin{array}{l} R1 \\ R2 \\ -2R2+R3 \end{array} \left[\begin{array}{ccc|c} 1 & -5 & 13 & -35 \\ 0 & 1 & -2 & 7 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$$\boxed{(x,y,z) \in \{(3,5,-1)\}}$$

$z=-1$ $y-2(-1)=7$ $x-5(5)+13(-1)=-35$

$y+2=7$ $x-25-13=-35$

$y=5$ $x-38=-35$

$x=3$

12. (10 pts) Write the equation for "The half-life of the radioactive isotope, Freakazoidium-99, is 450 years," and solve the equation for the decay constant, *k*.

$A(450)=A_0e^{450k}=\frac{1}{2}A_0$

$e^{450k}=\frac{1}{2}$

$$\boxed{k=-\frac{\ln(2)}{450}\approx .0015403271}$$

$450k=\ln(\frac{1}{2})=-\ln(2)$

13. (10 pts) Based on your work, how much radioactive Freakazoidium remains in a 512-kilogram sample, after 3600 years? (You can logic this one out, but I'm *looking* for something based on your previous work.)

$A_0=512$

$A(3600)=512e^{3600k}=512e^{(3600)(-\ln(2)/450)}=2\text{ kg}!$

0 → 512
450 → 256
900 → 128
1350 → 64
1800 → 32
2250 → 16
2700 → 8
3150 → 4
3600 → 2 ✓

$\approx$

Slope $= m = \dfrac{y_2 - y_1}{x_2 - x_1}$ and $y = m(x - x_1) + y_1$

Continuous growth/decay/compounding: $A(t) = A_0 e^{kt}$

Periodic Compounding: $A(t) = A_0\left(1 + \dfrac{r}{m}\right)^{mt}$ or $= P\left(1 + \dfrac{r}{m}\right)^{mt}$, depending on how much you like the letter 'P.'

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$ax^2 + bx + c = a(x-h)^2 + k$, where $(h,k) = \left(-\dfrac{b}{2a}, f\left(-\dfrac{b}{2a}\right)\right)$

Difference Quotient $= \dfrac{f(x+h) - f(x)}{h} = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1} =$ average slope.

$$a + ar + ar^2 + \cdots + ar^{n-2} + ar^{n-1} = \sum_{k=1}^{n} ar^{k-1} = a\left(\frac{1-r^n}{1-r}\right) \text{ or } a\left(\frac{r^n - 1}{r - 1}\right)$$

If $|r| < 1$, then $a + ar + ar^2 + \cdots\cdots + ar^{n-2} + ar^{n-1} + \cdots\cdots = \sum_{k=1}^{\infty} ar^{k-1} = a\left(\dfrac{1}{1-r}\right)$