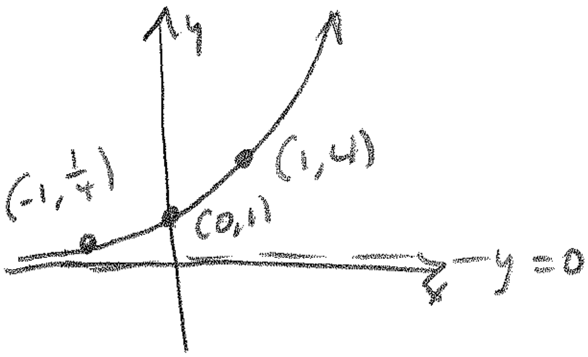
MAT 121-G81
100 Points

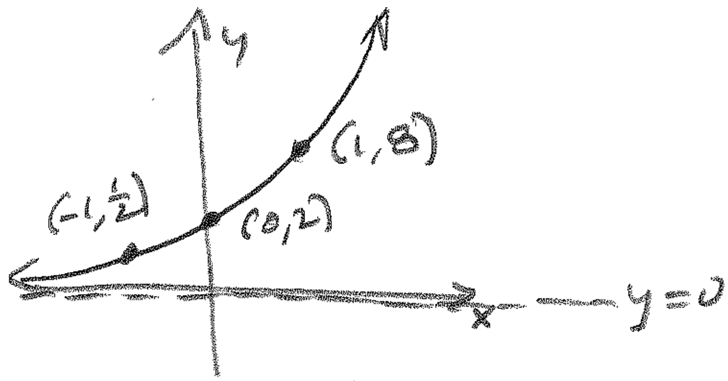
Test 4 - Fall, 2014
Covers Chapter 4

Name KEY

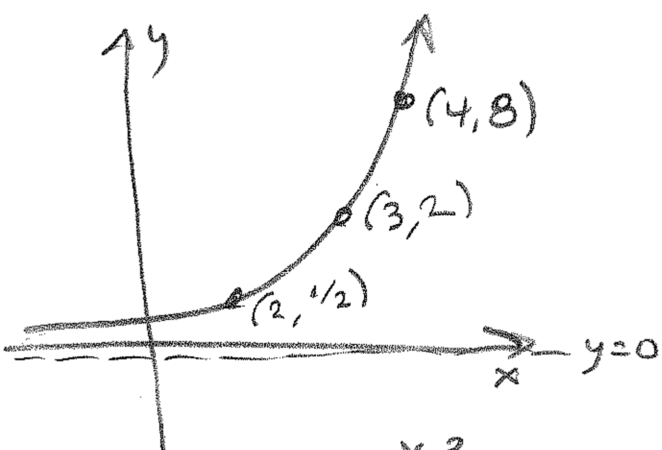
1. (20 pts) Starting with $f(x)=4^x$, sketch the graph of $g(x)=2\cdot 4^{x-3}-9$ in 4 steps (counting $f(x)=4^x$ as the first step). Use $x = -1$, $x = 0$, and $x = 1$ to find 3 points in the first graph, and show how these 3 points are moved around by each step in the transformation to $g(x)$. Finding the x- and y-intercepts is a separate problem, so don't worry about them, on this page.



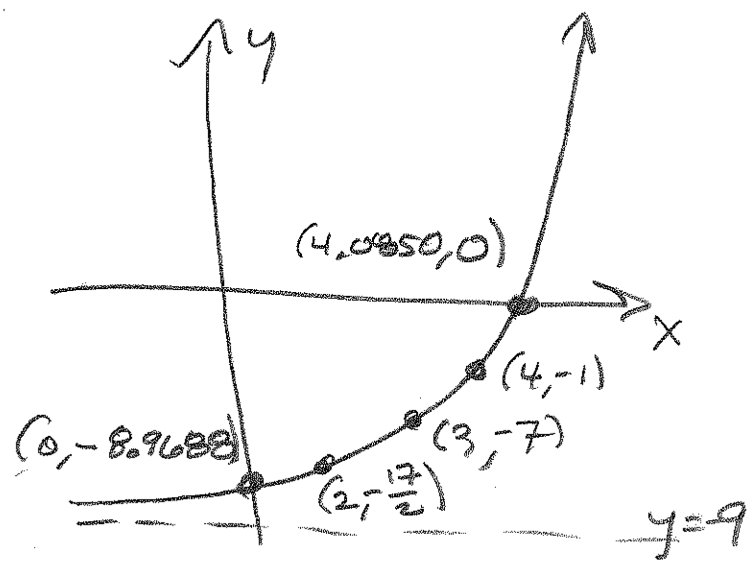
$f(x)=4^x$



$2f(x)=2\cdot 4^x$



$2\cdot f(x-3)=2\cdot 4^{x-3}$



See page 3 for work on finding x- & y-intercepts to 4 decimal places.

2. Let $f(x)=\sqrt{2x+4}$ and $g(x)=\dfrac{x-2}{x-7}$.

a. (5 pts) What is the domain of f?

Need $2x+4 \geq 0$

$2x \geq -4$

$\mathcal{D} = \{x \mid x \geq -2\}$

$= [-2, \infty)$

b. (5 pts) What is the domain of g?

Need $x-7 \neq 0$

$\mathcal{D} = \{x \mid x \neq 7\}$

$= (-\infty, 7) \cup (7, \infty)$

c. (5 pts) Write the function $\dfrac{f}{g}$. Do not simplify.

$$\frac{\sqrt{2x+4}}{\frac{x-2}{x-7}}$$

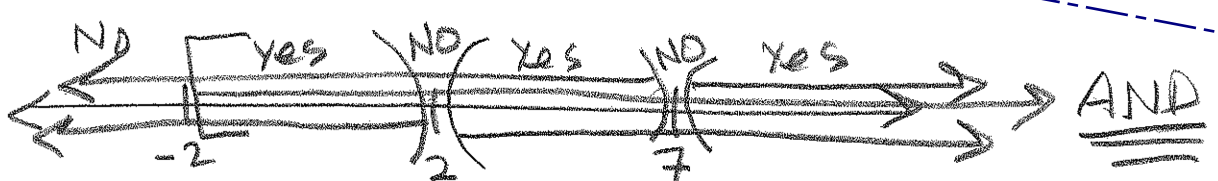
d. (5 pts) Write the function $f \circ g$. Do not simplify.

$$\sqrt{2\left(\frac{x-2}{x-7}\right)+4}$$

e. (10 pts) What is the domain of $\dfrac{f}{g}$?

$\mathcal{D} = \{x \mid x \in \mathcal{D}(f) \text{ and } x \in \mathcal{D}(g) \text{ and } g(x) \neq 0\}$

$= \{x \mid x \geq -2 \text{ and } x \neq 7 \text{ and } x \neq 2\}$

=


$= [-2, 2) \cup (2, 7) \cup (7, \infty)$

"And" means we need all 3 conditions satisfied, simultaneously.

Need $g(x) \neq 0$

$$\frac{x-2}{x-7} \neq 0$$

$x-2 \neq 0$ Only way for frac to be zero is if numerator is zero.

$x \neq 2$

3. (10 pts) Let $g(x) = 2\cdot 4^{x-3} - 9$. Find the x- and y-intercepts for this function, rounded to 4 decimal places. For 5 **bonus** points, label these intercepts on your final graph on page 1.

y-int

$g(0) = 2\cdot 4^{-3} - 9$

$= 2\left(\frac{1}{64}\right) - 9$

$= \frac{1}{32} - \frac{9}{1}\cdot\frac{32}{32}$

$= \frac{1-288}{32} = \frac{-287}{32} \approx -8.96875 \approx -8.9688$

$\boxed{(0, -8.9688)}$

x-int

$g(x) = 0$

$2\cdot 4^{x-3} - 9 = 0$

$2\cdot 4^{x-3} = 9$

$4^{x-3} = \frac{9}{2}$

$x - 3 = \log_4\left(\frac{9}{2}\right)$

$x = \log_4\left(\frac{9}{2}\right) + 3$

$= \frac{\ln(9/2)}{\ln(4)} + 3 \approx 4.084962501$

≈ 4.0850

$\boxed{(4.0850, 0)}$

4. Find the domain:

a. (5 pts) $\sqrt{\frac{(x-2)(x+3)^2}{(x-7)^4(x+5)}}$. (Sign Pattern!)

Need $\frac{(x-2)(x+3)^2}{(x-7)^4(x+5)} \geq 0$

Yes + No − Yes No − Yes Yes + No Yes +

−5: $\not\exists$; −3: $= 0$; 2: $= 0$; 7: $\not\exists$

$(-\infty, -5) \cup \{-3\} \cup [2, 7) \cup (7, \infty)$

$(-\infty, -5) \cup \{-3\} \cup [2, 7) \cup (7, \infty)$

b. (5 pts) $\log_3\left(\frac{(x-2)(x+3)^2}{(x-7)^4(x+5)}\right)$ (Reinterpret previous sign pattern in the current context!)

Yes No No No No No Yes No + No Yes +

−5: $\not\exists$; −3: $= 0$; 2: $= 0$; 7: $\not\exists$

Same, only "> 0" instead of "≥ 0"

5. (10 pts) Solve $\log_7(x-4) + \log_7(x+2) = 1$.

$\log_7((x-4)(x+2)) = 1$

$(x-4)(x+2) = 7$

$x^2 - 2x - 8 = 7$

$x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

$x \in \{\cancel{-3}, 5\}$, but

$x = -3 \notin D$

Domain:

$x - 4 > 0$ and $x + 2 > 0$

$x > 4$ and $x > -2$

$\{x \mid x > 4\}$

This is why $x = -3$ is out.

6. (10 pts) Solve $2^{x^2-8} \cdot 2^{-3x} = 4$.

$2^{x^2-8-3x} = 2^2$

$x^2-3x-8=2$

$x^2-3x-10=0$

$(x-5)(x+2)=0$

$\boxed{x \in \{-2,5\}}$

$A = A(t) =$ Remaining Amount of radioactive isotope, as a function of $t =$ time, in years.

$A(t) = A_0 e^{kt}$

7. (10 pts) The half-life of a radioactive isotope is 700 years. Find how old a sample is, if 95% of the isotope in an ancient manuscript has decayed (i.e., if only 5% of the radioactive isotope remains.). Give this answer to the nearest year.

$\frac{1}{2}$-life is 700

$A_0 e^{700K} = \frac{1}{2}A_0$

$e^{700K} = \frac{1}{2}$

$700K = \ln(1/2)$

$K = \frac{\ln(1/2)}{700} = -\frac{\ln 2}{700} = K$

There is only 5% of radioactive isotope left

$A_0 e^{Kt} = .05A_0$

$e^{Kt} = .05$

$Kt = \ln(.05)$

$t = \frac{\ln(.05)}{K} = \frac{\ln(0.05)}{\frac{-\ln 2}{700}}$

≈ 3025.349666

$\approx$ $\boxed{3025 \text{ yrs old}}$

8. (10 pts) Solve the equation $5 \cdot (1.08)^x = 2^x$. Give an exact answer and a decimal answer, rounded to 4 places.

$\ln(5 \cdot 1.08^x) = \ln(2^x)$

$\ln 5 + \ln(1.08^x) = \ln(2^x)$

$\ln 5 + (\ln(1.08))x = (\ln(2))x$

$A + Bx = Cx$

$Bx - Cx = -A$

$(B-C)x = -A$

$\boxed{x \approx 2.6119}$

$x = \frac{-A}{B-C} = \frac{-\ln 5}{\ln 1.08 - \ln 2}$

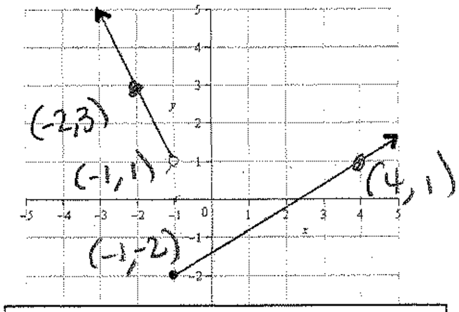
$\approx 2.611934624 \approx \boxed{2.6119 \approx x}$

Solve any two (2) Bonus problems for up to 10 points. I'll grade the first two I come to.

1. BONUS (5 pts) Solve the absolute value inequality $|2x-7| \geq 8$

2. **BONUS** (5 pts) Find the inverse function for $f(x)=\sqrt{2x-6}+1$. Then state the domain and range for both f and f^{-1}.

3. BONUS (5 pts) Re-write the function $g(x)=5x^2+10x-19$ in the form $g(x)=a(x-h)^2+k$. State the vertex of this parabola.



This is the picture for **Bonus #4**

4. BONUS (5 pts) Write the formula for the piecewise-defined function shown, above right.

① $|2x-7| \geq 8$

$2x-7 \geq 8$ OR $2x-7 \leq -8$

$2x \geq 15$ OR $2x \leq -1$

$\{x \mid x \geq \frac{15}{2}$ OR $x \leq -\frac{1}{2}\}$

$-\frac{1}{2}$ $\frac{15}{2}$

$(-\infty, -\frac{1}{2}] \cup [\frac{15}{2}, \infty)$

② $\sqrt{2y-6}+1 = x$

$\sqrt{2y-6} = x-1$

$2y-6=(x-1)^2$

$2y=(x-1)^2+6$

$y=\boxed{\frac{1}{2}(x-1)^2+3 = f^{-1}(x)}$

OR $\frac{1}{2}x^2-x+\frac{7}{2}$

$\mathcal{D}(f)=[3,\infty)=\mathcal{R}(f^{-1})$

$\mathcal{R}(f)=[1,\infty)=\mathcal{D}(f^{-1})$

③ $5x^2+10x-19$

$=5(x^2+2x+1^2)-5(1)^2-19$

$=\boxed{5(x+1)^2-24}$

$\boxed{(h,k)=(-1,-24)}$

ALTERNATE:

$-\frac{b}{2a} = -\frac{10}{2(5)} = -1 = h$

$f(-\frac{b}{2a}) = f(-1) = 5(-1)^2+10(-1)-19$

$=5-29=-24=k$

$\therefore f(x)=5(x-(-1))^2-24 = 5(x+1)^2-24$

④ $m_1 = \frac{1-3}{-1+2} = \frac{-2}{1} = -2$

$y=-2(x-(-1))+1$

$=-2x-2+1 = -2x-1$

$m_2 = \frac{-2-1}{-1-4} = \frac{-3}{-5} = \frac{3}{5}$

$y=\frac{3}{5}(x-(-1))-2$

$=\boxed{\frac{3}{5}x-\frac{7}{5}}$

$f(x)=\begin{cases} -2x-1 & \text{if } x<-1 \\ \frac{3}{5}x-\frac{7}{5} & \text{if } x \geq -1 \end{cases}$