MAT 121-G81-G82 Test 3 – Fall, 2014 Name_________________

70 Points Covers Chapter 3 NO GRAPHING CALCULATORS!!!

1. (10 pts) Form a polynomial of *minimial degree* in *factored form* that has real coefficients (after expanding) and will have the given zeros. Do *not* expand your polynomial. Leave it factored! If you run out of room, you're doing it wrong!

$x = 3$, multiplicity 2;
$x = 3 - 7i$, multiplicity 1;
$x = 2$, multiplicity 4.

2. (10 pts) Use synthetic division to find $P(3)$ if $P(x) = 5x^5 - 2x^3 + 3x^2 - 4x + 3$.

3. (5 pts) Represent the work you just did on the previous problem by writing $P(x)$ in the form $Dividend = Divisor \bullet Quotient + Remainder$.

4. Suppose $f(x) = (x+3)^2(x-3)(x-6) = x^4 - 3x^3 - 27x^2 + 27x + 162$. I'm showing you both factored and expanded form to help you answer the following:
   a. (10 pts) Provide a rough sketch of *f*, using its zeros, their respective multiplicities and its end behavior. Include *x*- and *y*-intercepts. Your graph should be smooth. Un-exaggerate the vertical for a better quality graph.

   b. Solve the inequalities (You've done the work. Now, INTERPRET.):
      i) (5 pts) $f(x) = (x+3)^2(x-3)(x-6) < 0$
      ii) (5 pts) $f(x) = \dfrac{(x+3)^2}{(x-3)(x-6)} \geq 0$

5. (10 pts) Find the *real* zeros of $f(x)=2x^5-4x^4-11x^3+41x^2-43x+15$. Then factor *f* over the set of **real numbers**. This should involve an irreducible quadratic factor.

6. (5 pts) Find the remaining (nonreal) zeros of *f* and factor *f* over the set of **complex numbers**. This step requires breaking down the quadratic piece that's irreducible over the real numbers. The fundamental theorem tells us that *nothing* is irreducible over the complex numbers.

7. (5 pts) You don't need to graph $R(x)=\dfrac{2x^3+6x^2+4x}{x^2-4}$, here, but I do want to see you graph its asymptotes.

8. (10 pts) Sketch the graph of $R(x)=\dfrac{3x^2-13x-4}{x^2-3x-10}$. Show all asymptotes and intercepts.

ANSWER ANY TWO (2) OF THE FOLLOWING.

Bonus: (5 pts) Form a polynomial of *minimial degree* in *factored form* that has ***rational*** coefficients (after expanding) and will have the given zeros. Do *not* expand your polynomial. Leave it factored! If you run out of room, you're doing it wrong!

Zeros: $x = 2+\sqrt{3}$, multiplicity 1; $x = 2+3i$, multiplicity 2; $x = -5$, multiplicity 17.

Bonus: (5 pts) What is the domain of $f(x) = \sqrt{\dfrac{(x+3)^2}{(x-3)(x-6)}}$? Hint: See previous work.

Bonus: (5 pts) Sketch the graph of $R(x) = \dfrac{2x^3+6x^2+4x}{x^2-4}$. Hint: See previous work.