9:52 – 10:14

1. For each of the following functions, state the domain in set-builder notation and in interval notation.

a. (5 pts) $f(x) = \sqrt{9x - 2}$

Need $9x - 2 \geq 0$

$9x \geq 2$

$\left\{ x \mid x \geq \frac{2}{9} \right\} = \left[\frac{2}{9}, \infty \right)$

b. (5 pts) $g(x) = \dfrac{x^2 + 13}{9x - 2}$

Need $9x - 2 \neq 0$

$9x \neq 2$

$\left\{ x \mid x \neq \frac{2}{9} \right\}$

$= \left(-\infty, \frac{2}{9} \right) \cup \left(\frac{2}{9}, \infty \right)$

c. (5 pts) $h(x) = \dfrac{x^2 + 13}{\sqrt{9x - 2}}$

Need $9x - 2 \geq 0$

AND $\implies$ Need $9x - 2 > 0$

$\sqrt{9x - 2} \neq 0$

$9x > 2$

$\left\{ x \mid x > \frac{2}{9} \right\} = \left(\frac{2}{9}, \infty \right)$

2. (5 pts) If $f(x) = |3x + 7|$, what is $f(-8)$?

$f(-8) = |3(-8) + 7| = |-24 + 7| = |-14| = 14 = f(-8)$

3. (10 pts) What is the average rate of change of the function $f(x) = x^2 - 3$ from $x = 2$ to $x = 4$?

$\dfrac{f(4) - f(2)}{4 - 2} = \dfrac{4^2 - 3 - (2^2 - 3)}{2} = \dfrac{16 - 3 - (4 - 3)}{2}$

$= \dfrac{13 - 1}{2} = \dfrac{12}{2} = 6 = m_{AVG}$

4. (5 pts) Consider the relation $\mathbf{R} = \{(2,3), (3,-2), (5,7), (6,2)\}$ and fill in the following:

a. Domain $\{2, 3, 5, 6\}$

b. Range $\{3, -2, 7, 2\}$

c. Is $\mathbf{R}$ a function? Yes

d. If $\mathbf{R}$ is a function, is it a 1-to-1 function? (Yes, No or DNA) Yes

5. Let $f(x) = \dfrac{x-2}{x-5}$ and $g(x) = \sqrt{2x+5}$.

   a. (5 pts) What is the domain of f?

$$\{x \mid x \neq 5\}$$
$$= (-\infty, 5) \cup (5, \infty)$$

   b. (5 pts) What is the domain of g?

$$\{x \mid 2x+5 \geq 0\}$$
$$= \{x \mid x \geq -\tfrac{5}{2}\}$$
$$= [-\tfrac{5}{2}, \infty)$$

   c. (5 pts) Write the function $(f-g)(x)$. Do not simplify. What is its domain?

$$\frac{x-2}{x-5} - \sqrt{2x+5}$$

Need $x \neq 5$ AND $x \geq -5/2$

$= \{x \mid x \neq 5 \text{ and } x \geq -\tfrac{5}{2}\}$

$= [-\tfrac{5}{2}, 5) \cup (5, \infty)$

   d. (5 pts) Write the function $\left(\dfrac{g}{f}\right)(x)$. Do not simplify. What is its domain?

$$\frac{\sqrt{2x+5}}{\frac{x-2}{x-5}}$$

Need previous AND
need $\frac{x-2}{x-5} \neq 0$, i.e., $x \neq 2$.

$\{x \mid x \neq 2, x \neq 5 \text{ and } x \geq -\tfrac{5}{2}\} = [-\tfrac{5}{2}, 2) \cup (2, 5) \cup (5, \infty)$

   e. (5 pts) Write the function $(f \circ g)(x)$. Do not simplify. What is its domain?

$$\frac{\sqrt{2x+5} - 2}{\sqrt{2x+5} - 5}$$
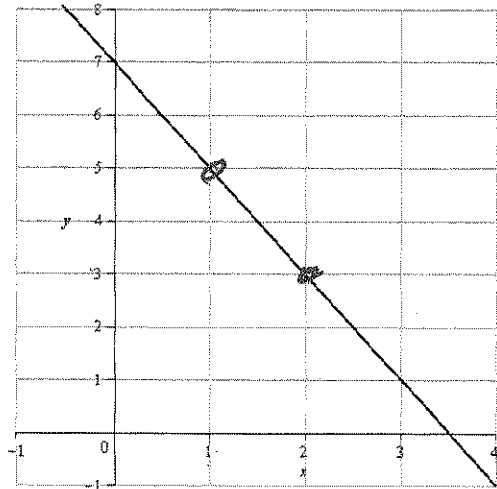
Need $x \in D(g)$ AND $g(x) \in D(f)$

$\{x \mid x \geq -\tfrac{5}{2} \text{ and } \sqrt{2x+5} \neq 5\}$

$\sqrt{2x+5} \neq 5$
$2x+5 \neq 25$
$2x \neq 20$
$x \neq 10$

$= \{x \mid x \geq -\tfrac{5}{2} \text{ and } x \neq 10\}$
$= [-\tfrac{5}{2}, 10) \cup (10, \infty)$

6. (10 pts) Determine the equation of the line from its graph.
   Give the equation in…
   a. … point-slope form and
   b. … slope-intercept form.

$(1,5) = (x_1, y_1)$; $(2,3) = (x_2, y_2)$

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3-5}{2-1} = \dfrac{-2}{1} = -2$
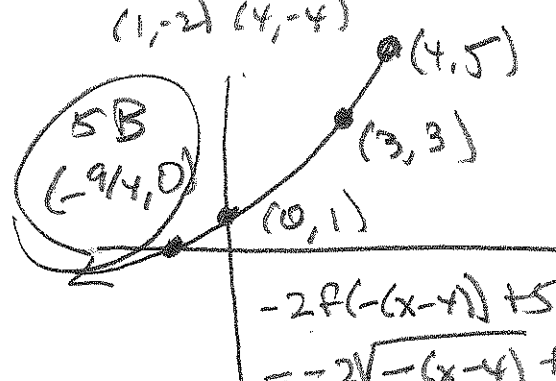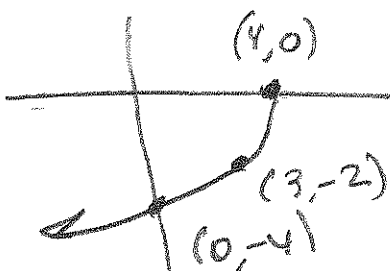
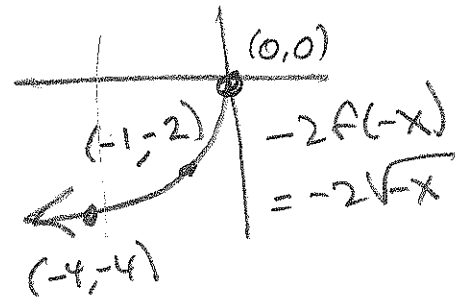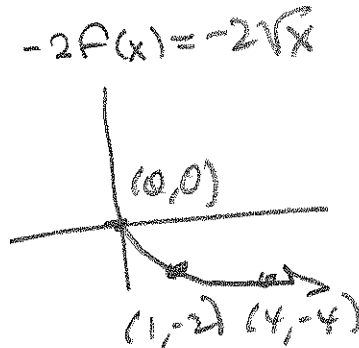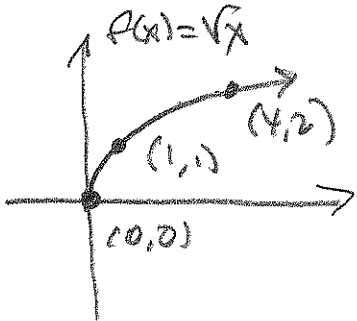$y = m(x - x_1) + y_1 = \boxed{-2(x-1) + 5 = y}$ (a)

$= -2x + 2 + 5 = \boxed{-2x + 7 = y}$ (b)

7. Graph each of the following by the techniques of shifting, stretching, compressing or reflecting. Start with the graph of a basic function and show all steps *as demonstrated in Videos*. I expect to see 3 points labeled in the first sketch, and to see where those points are moved to in each subsequent step.

$4 - x = -(x - 4)$

a. (10 pts) $g(x) = -2\sqrt{4-x} + 5$ $f(x) = \sqrt{x}$

$f(x) = \sqrt{x}$

$(1,1)$ $(4,2)$ $(0,0)$

$-2f(x) = -2\sqrt{x}$

$(0,0)$ $(1,-2)$ $(4,-4)$

$-2f(-x) = -2\sqrt{-x}$

$(0,0)$ $(-1,-2)$ $(-4,-4)$

$-2f(-(x-4))$
$= -2\sqrt{-(x-4)}$
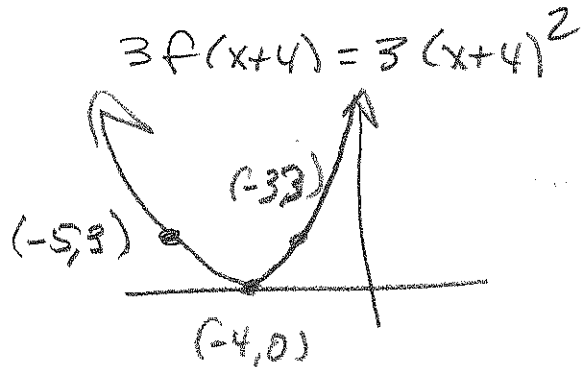
$(4,0)$ $(3,-2)$ $(0,-4)$

5B
$(-9/4, 0)$

$(4,5)$ $(3,3)$ $(0,1)$

$-2f(-(x-4)) + 5$
$= -2\sqrt{-(x-4)} + 5$
$= -2\sqrt{-x+4} + 5 = -2\sqrt{4-x} + 5$
$= g(x)$

BONUS:
$-2\sqrt{4-x} + 5 = 0$
$-2\sqrt{4-x} = -5$
$4(4-x) = 25$
$4 - x = \dfrac{25}{4}$
$-x = \dfrac{25}{4} - \dfrac{16}{4} = \dfrac{9}{4}$
$x = -9/4$

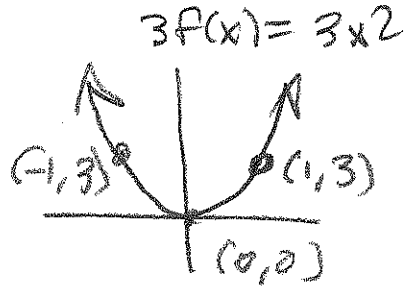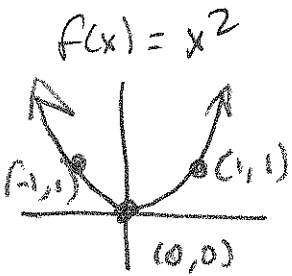**Bonus:** (5 pts) Label the *x*- and *y*-intercepts on the graph of *g*(*x*).

b. (10 pts) $g(x) = 3(x+4)^2 - 2$

$f(x) = x^2$

(-1,1)   (1,1)
        (0,0)

$3f(x) = 3x^2$

(-1,3)   (1,3)
        (0,0)

$3f(x+4) = 3(x+4)^2$

(-3,3)
(-5,3)
        (-4,0)

$3f(x+4) - 2 = 3(x+4)^2 - 2 = g(x)$

(-3,1)
(-5,1)
        (-4,-2)
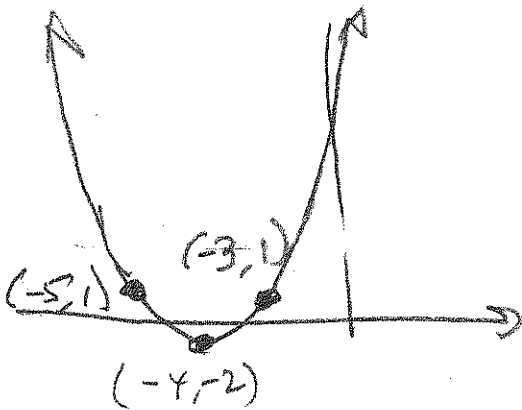
8. (10 pts) Sketch the graph of the piecewise-defined function $f(x) = \begin{cases} x - 1 & \text{if } -2 \le x \le 0 \\ x^2 - 3 & \text{if } 0 < x \end{cases}$

$x^2 = 3$

$x = \pm\sqrt{3}$

$(\sqrt{3}, 0)$

$(0, -1)$

$(-2, -3)$ $(0, -3)$