MAT 121 Online, FINAL TEST, Spring, 2014
200 Points — Name: KEY

This is our final learning opportunity together, and I'm hoping to take full advantage. Read the questions carefully. It's possible to earn points on a problem by *knowing* that you did something wrong and clearly *explaining how* you know and what you're *trying* to accomplish, and how you're going about it. More points for solid terminology and English.

1. Solve the equation $x^2 + 6x - 16 = 0$ in two different ways:

part a (10 pts) Factoring

$(x+8)(x-2)=0$

$x \in \{-8, 2\}$

part b (10 pts) Completing the square

$x^2+6x = 16$

$x^2+6x+3^2 = 16+9$

$(x+3)^2 = 25$

$x+3 = \pm 5$

$x = -3 \pm 5$; $-3+5=2$; $-3-5=-8$

$\boxed{x \in \{-8, 2\}}$

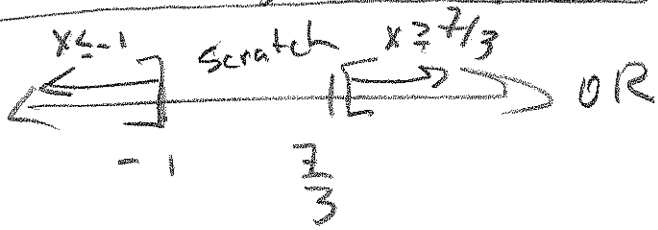
2. Solve the absolute value inequality. Give your answer in set-builder *and* interval notation.

part a (10 pts) $|3x-2| \geq 5$

$3x-2 \geq 5$ OR $3x-2 \leq -5$

$3x \geq 7$ OR $3x \leq -3$

$\boxed{\{x \mid x \geq \frac{7}{3} \text{ OR } x \leq -1\} = (-\infty, -1] \cup [\frac{7}{3}, \infty)}$

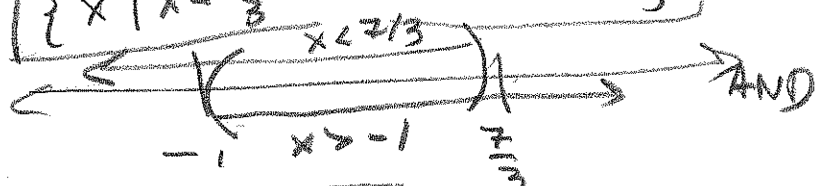


part b (10 pts) $|3x-2| < 5$

$3x-2 < 5$ AND $3x-2 > -5$

$3x < 7$ AND $3x > -3$

$\boxed{\{x \mid x < \frac{7}{3} \text{ AND } x \geq -1\}}$



$\boxed{= (-1, 7/3)}$

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3. Find the domain of each of the following:

part a (10 pts) $f(x)=\sqrt{\dfrac{(x-3)^4(x-5)^7}{(x-1)^3}}$

NEED $\dfrac{(x-3)^4(x-5)^7}{(x-1)^3} \geq 0$

$x=3, m=2$ DON'T CHANGE SIGN

$y=5, m=7$ ODD? CHANGE SIGN

$x=1, m=3$ CHANGE SIGN

ALSO NEED $(x-1)^3 \neq 0$

so $x \neq 1$ .3 part of f.

+ | − | − | +

Yes | NO | NO |

1 3 5

☆ $=0$ $=0$

No Yes Yes

TEST $x=0$

$$\frac{(0-3)^4(0-5)^7}{(0-1)^3} = \frac{(-3)^4(-5)^7}{(-1)^3}$$

$$= \frac{3^4(-5^7)}{-1^3} = \frac{-3^4 5^7}{-1^3}$$

$$= \frac{3^4 5^7}{1^3} \text{ is } +$$

$\boxed{(-\infty, 1) \cup \{3\} \cup [5, \infty)}$

part b (10 pts) $\ln(x^2-5x-24)$

Need $x^2-5x-24 > 0$

$(x-8)(x+3) > 0$

+ | − | +

Yes | No | Yes

−3 8

$=0$ $=0$

Yes Yes

Test $x=0$

$(0-8)(0+3)$

$= -24$ −

$x \in \boxed{(-\infty, -3] \cup [8, \infty)}$

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4. (20 pts) Let $f(x) = x^2 + 5x$. Simplify the difference quotient $\dfrac{f(x+h) - f(x)}{h}$.

$$= \frac{(x+h)^2 + 5(x+h) - (x^2+5x)}{h}$$

$$= \frac{x^2 + 2xh + h^2 + 5x + 5h - x^2 - 5x}{h}$$

$$= \frac{2xh + h^2 + 5h}{h}$$

$$= \frac{h(2x+h+5)}{h} = \boxed{2x+h+5}$$

5. (15 pts) Form a polynomial (in factored form) that will have *real* coefficients after expanding (which you shouldn't bother to do!) that has the following zeros with the respective multiplicities:

$x = 4$, multiplicity = 3
$x = -5$, multiplicity = 2
$x = 4 - 5i$, multiplicity = 1

$$(x-4)^3(x+5)^2(x-(4-5i))(x-(4+5i))$$

$$3 + 2 + 1 + 1 = 7$$

6. (5 pts) What's the minimum possible degree for the polynomial described? $\boxed{7}$

7. How many ways to…

part a (5 pts) … choose 4 trumpet players for the jazz band, if there are 20 players trying out?

$$\binom{20}{4} = {}_{20}C_4 = C(20,4) = \frac{20!}{4!\,16!} = \frac{\cancel{20}^{5}\cdot 19\cdot \cancel{18}^{3}\cdot 17}{\cancel{4\cdot 3\cdot 2}} = 5\cdot 19\cdot 3\cdot 17 = \boxed{4845}$$

part b (5 pts) … choose 4 trumpet players for the jazz band, if there are 20 players trying out and you will then arrange them into 1st, 2nd, 3rd and 4th chair?

$$P(20,4) = {}_{20}P_4 = \frac{20!}{16!} = 20\cdot 19\cdot 18\cdot 17 = \boxed{116{,}280}$$

check?

$$= C(20,4)\cdot 4! = 4845\cdot 4\cdot 3\cdot 2 = 116{,}280 \checkmark$$

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8. Let $f(x) = 4x^4 - \cancel{?x^2}\ {-63x^2} + 159x - 130$.

part a (20 pts) Use synthetic division to determine if $x+5$ is a factor of f.

Interpret the your work by filling in the quotient and remainder in the statement
$4x^4 - 163x^2 + 159x - 130 = (x+5) \cdot \textit{quotient} + \textit{remainder}$.

Hopefully, your remainder is zero. It's how you get it and how you interpret it that matter to me.

$$\begin{array}{r|rrrrr} -5 & 4 & 0 & -63 & 159 & -130 \\ & & -20 & 100 & -185 & 130 \\ \hline & 4 & -20 & 37 & -26 & 0 \end{array}$$

$f(x) = (x+5)(4x^3 - 20x^2 + 37x - 26) + 0$

part b (10 pts) Show that $x = 2$ is a root of f by dividing your *quotient* in **part a** by $x-2$. This question, in itself ought to give you a very clear idea of what your conclusion ought to have been in **part a**.

$$\begin{array}{r|rrrr} 2 & 4 & -20 & 37 & -26 \\ & & 8 & -24 & 26 \\ \hline & 4 & -12 & 13 & 0 \end{array}$$

part c (5 pts) Based on your work, factor $f(x)$ over the real number field. This involves an irreducible quadratic factor.

$f(x) = (x+5)(x-2)(4x^2 - 12x + 13)$

part d (10 pts) Compute the discriminant of $4x^2 - 12x + 13$. Then find the two nonreal roots of $4x^2 - 12x + 13$, by any method (other than copying from someone else).

$b^2 - 4ac = (-12)^2 - 4(4)(13) = 144 - 208 = -64$

$x = \dfrac{-(-12) \pm \sqrt{-64}}{2(4)} = \dfrac{12 \pm 8i}{8} = \dfrac{3 \pm 2i}{2}$

$\boxed{\dfrac{3+2i}{2}, \dfrac{3-2i}{2}}$

2⌊64
2⌊32
2⌊16
2⌊8
2⌊4
2

$64 = 2^6$

$\sqrt{-64} = 2^3 i = 8i$

part e (5 pts) Based on your work, split $f(x)$ into the product of linear factors.

$\boxed{4(x+5)(x-2)\left(x - \left(\frac{3+2i}{2}\right)\right)\left(x - \left(\frac{3-2i}{2}\right)\right)}$

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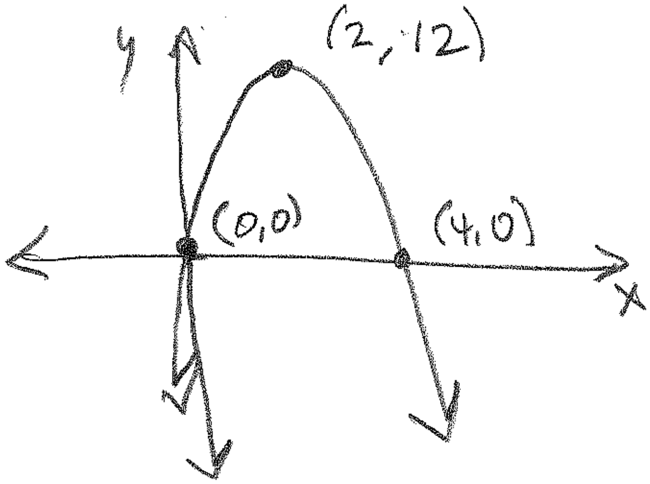
9. (10 pts) Graph $g(x) = -3(x-2)^2 + 12$ using the techniques of shifting, stretching and reflecting. Label the vertex and all intercepts.



$$g(0) = -3(-2)^2 + 12$$
$$= -3(4) + 12$$
$$= -12 + 12 = 0$$

$$-3(x-2)^2 + 12 = 0$$
$$-3(x-2)^2 = -12$$
$$(x-2)^2 = 4$$
$$x-2 = \pm\sqrt{4} = \pm 2$$
$$x = 2 \pm 2 \quad 4, \; 0$$

10. (10 pts) Evaluate the sum $\sum_{k=1}^{\infty} 7 \cdot \left(\frac{3}{5}\right)^{k-1}$ $a = 7, r = \frac{3}{5}$

$$S = a\left(\frac{1}{1-r}\right) = 7\left(\frac{1}{1-\frac{3}{5}}\right) = 7\left(\frac{1}{\frac{2}{5}}\right) = 7\left(\frac{5}{2}\right) = \boxed{\frac{35}{2}}$$

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11. The population of a town in 2008 is 15,000 people. The population of that town grows to 16,500 in 2012. The population is a function of time in the exponential model $P(t)=P_0e^{kt}$ where t=0 nrepresents the year 2008.

part a (2 pts) Define the variables given this information and identify the two ordered pairs to use as points.

Let t = time, in years, after 2008

P = Pop. as a function of t

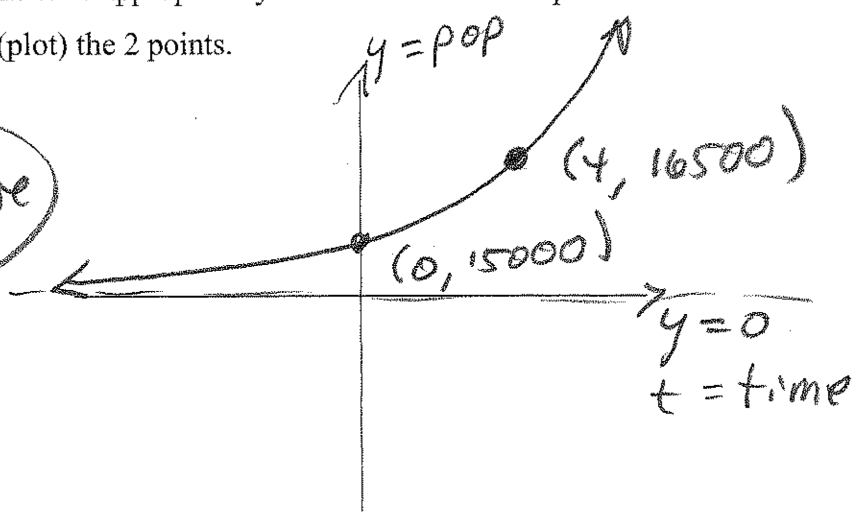
2008 - 2008 = 0 → (0, 15,000)

part b Graphing 2012 - 2008 = 4 → (4, 16,500)

i. (2 pts) Label the axes appropriately for the context of the problem.

ii. (2 pts) Graph (plot) the 2 points.

2pts for curve



part c i. (2 pts) Find the rate of growth. Show your work. Round to 4 decimal places.

$P_0e^{kt} = 15000e^{kt}$ and

$P(4) = 16500$

$15000e^{4k} = 16500$

$e^{4k} = \frac{16500}{15000} = 1.1$

$e^{4k} = 1.1$

$4k = \ln(1.1)$

$k = \frac{\ln(1.1)}{4}$

$\approx \boxed{.0238}$

.02382754$\cancel{5}$

ii. (2 pts) Find the equation of the exponential function which models the situation.

$P(t) = 15000e^{.0238t}$

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part d (2 pts) Graph the equation of the curve on the same graph as the two points in **part b**.

part e (2 pts) Use the equation to find the estimated population in 2015. Show your work.

$P(t) = 15000e^{.0238t}$

$2015 \longmapsto 2015 - 2008 = 7 = t$

$P(7) = 15000e^{.0238(7)} \approx 17,719.22487$

$\boxed{P(7) \approx 17,719}$

part f (2 pts) Use your equation to calculate in what year the population will reach 20000 if the growth continues at this same rate. Show your work.

$P(t) = 20000$

$15000e^{.0238t} = 20000$

$e^{.0238t} = \frac{20}{15} = \frac{4}{3} = 1.\overline{3}$

$.0238t = \ln(1.\overline{3})$

$t = \frac{\ln(1.\overline{3})}{.0238} \approx 12.08748204 \approx 12$

$2008 + 12 = \boxed{2020}$

part g (2 pts) What would be the effect to the population if the rate had the opposite sign? Use complete sentences in your explanation.

If k were negative, this would be exponential decay.

part h (2 pts) List two real-life factors which may affect the population such that this model would not prove valid. Use complete sentences.

Plague, immigration/emigration, poor economy, are just a few factors that could ruin our wonderful, beautiful exponential growth model.

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