MAT 121-G81 Test 3 – Fall, 2013 Name_________________
PRACTICE TEST FOR THE TAKE-HOME I JUST GAVE YOU.

Use separate paper to do the work on this take-home test. Start a fresh sheet of paper to show work on #4. Use paper without lines. Use only one side of each sheet of paper. *I will not grade work written on the backs of pages*. Write clearly and make sure your pencil work is *dark*. It's a struggle for me to read faint print.

Let $f(x) = 3x^5 - 10x^4 - 22x^3 + 88x^2 + 56x - 160$. We will say everything we can about this polynomial.

1. (2 pts) Describe the end behavior of the graph of f with a simple graphic.
2. (2 pts) Use Descartes' Rule of Signs to determine the *possible* number of positive and negative zeroes of f.
3. (2 pts) Use the Rational Zeroes Theorem to determine the possible rational zeroes of f.
4. (2 pts) Informed by your work, above, use synthetic division to find the zeroes. Each time you find a zero, it *should* reduce (depress) the question by one degree. Each time you find a zero, you should thereafter be working with a *depressed polynomial* that is of lesser degree.
5. (2 pts) From you work, above, factor f over the real numbers. This will involve an irreducible quadratic factor.
6. (2 pts) From your work above, factor f over the complex numbers. This should split f into linear factors.
7. (2 pts) Give a rough sketch of f that shows all intercepts.
8. (2 pts) Sketch the graph of $\dfrac{x^2+3x+2}{x^2+2x-8} = \dfrac{(x+2)(x+1)}{(x+4)(x-2)}$. Factored and expanded form given for convenience. Show all asymptotes, intercepts and any holes.
9. (2 pts) The graph of $g(x) = \dfrac{x^3-x^2-4x+4}{x^3-12x+16} = \dfrac{(x+2)(x+1)(x-3)}{(x+4)(x-2)(x-3)}$ differs from the graph of *f*, in #8, in only one small detail. Sketch the graph of *g*, showing all asymptotes, intercepts and holes.
10. (2 pts) Sketch the graph of $h(x) = \dfrac{x^2+3x+2}{x+4}$, showing all asymptotes, intercepts and holes.

121 Ⓣ3 Take-Home / Writing Project

Practice / Sample write-up.

$f(x) = 3x^5 - 10x^4 - 22x^3 + 88x^2 + 56x - 160$

① $3x^5$ controls

② Descartes: 3 sign changes $\Longrightarrow$

3 or 1 positive zeros (one, at least)

$f(-x) = -3x^5 - 10x^4 + 22x^3 + 88x^2 - 56x - 160$

2 sign changes $\Longrightarrow$

2 or 0 negative zeros

$$\begin{array}{r|l} 2 & 160 \\ 2 & 80 \\ 2 & 40 \\ 2 & 20 \\ 2 & 10 \\ & 5 \end{array}$$

ouch!

③ $a_n = 3$, $a_0 = -160$

Denoms, Numerators

$\frac{p}{q}$: $\pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}, \pm 4, \pm \frac{4}{3}, \pm 8, \pm \frac{8}{3}, \pm 16, \pm \frac{16}{3}$

$\pm 32, \pm \frac{32}{3}, \pm 5, \pm \frac{5}{3}, \pm \frac{10}{1}, \pm \frac{10}{3}, \pm 20, \pm \frac{20}{3},$

$\pm 40, \pm \frac{40}{3}, \pm 80, \pm \frac{80}{3}, \pm 160, \pm \frac{160}{3}$

Lucky for you, I made yours easier.

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(4) $3x^5-10x^4-22x^3+88x^2+56x-160$

$x=1$:

$$\begin{array}{r|rrrrrr} 1 & 3 & -10 & -22 & 88 & 56 & -160 \\ & & 3 & -7 & -29 & 59 & 115 \\ \hline & 3 & -7 & -29 & 59 & 115 & \neq 0 \end{array}$$

Seeing if $x-1$ is factor of $x=1$ is zero.

$x=-1$:

$$\begin{array}{r|rrrrrr} -1 & 3 & -10 & -22 & 88 & 56 & -160 \\ & & -3 & 13 & -9 & 79 & -135 \\ \hline & 3 & -13 & 9 & 79 & 135 & \end{array}$$

Nope

$x+1$ Nope

$x=2$

$$\begin{array}{r|rrrrrr} 2 & 3 & -10 & -22 & 88 & 56 & -160 \\ & & 6 & -8 & -60 & 36 & 184 \\ \hline & 3 & -4 & -30 & 18 & 92 & \end{array}$$

Nope

$x=-2$:

$$\begin{array}{r|rrrrrr} -2 & 3 & -10 & -22 & 88 & 56 & -160 \\ & & -6 & 32 & -20 & -136 & 160 \\ \hline & 3 & -16 & 10 & 68 & -80 & 0 \end{array}$$

Yes!

This says we have

$f(x)=(3x^4-16x^3+10x^2+68x-80)(x+2)$

$x=-2$ worked once. Maybe it works again!

$$\begin{array}{r|rrrrr} -2 & 3 & -16 & 10 & 68 & -80 \\ & & -6 & 44 & -108 & 80 \\ \hline & 3 & -22 & 54 & -40 & 0 \end{array}$$

Yes!

Depressed polynomial Use!

$f(x)=(3x^3-22x^2+54x-40)(x+2)^2$

is where we are!

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Try $x=-2$ again!

$$\begin{array}{r|rrrl} -2 & 3 & -22 & 54 & -40 \\ & & -6 & 56 & -220 \\ \hline & 3 & -28 & 110 & \text{Nope.} \end{array}$$

Guess again. I bet the teacher wants me to have to do a fraction

$\frac{-120+47}{3} =$

$$\begin{array}{r|rrrl} \frac{1}{3} & 3 & -22 & 54 & -40 \\ & & 1 & -7 & 47/3 \\ \hline & 3 & -21 & 47 & \text{Nope} \end{array}$$

$\frac{54}{1} \cdot \frac{3}{3} - \frac{23}{3}$

$= \frac{162-23}{3} = \frac{139}{3}$

$$\begin{array}{r|rrrl} -\frac{1}{3} & 3 & -22 & 54 & -40 \\ & & -1 & -\frac{23}{3} & -\frac{139}{9} \\ \hline & 3 & -23 & \frac{139}{3} & \text{NOPE} \end{array}$$

$\frac{162-40}{3} = \frac{122}{3}$

$$\begin{array}{r|rrrl} \frac{2}{3} & 3 & -22 & 54 & -40 \\ & & 2 & -\frac{40}{3} & \frac{\text{[illegible]}}{9} \\ \hline & 3 & -20 & \frac{122}{3} & \text{Nope} \end{array}$$

$$\begin{array}{r|rrrl} -\frac{2}{3} & 3 & -22 & 54 & -40 \\ & & -2 & 16 & -\frac{140}{3} \\ \hline & 3 & -24 & 70 & \text{Nope} \end{array}$$

Now we have it down to a quadratic!

$$\begin{array}{r|rrrr} \frac{4}{3} & 3 & -22 & 54 & -40 \\ & & 4 & -24 & 40 \\ \hline & 3 & -18 & 30 & 0 \end{array}$$

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Sweet!

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We now have it broken-down to this point:

$$f(x) = (3x^2 - 18x + 30)(x+2)^2\left(x - \frac{4}{3}\right)$$

Just have to find zeros of $3x^2 - 18x + 30$ & we're done.

$a = 3,\ b = -18,\ c = 30$

$b^2 - 4ac = (-18)^2 - 4(3)(30) = 324 - 360 = -36$

$\Rightarrow$ No real zeros. This means $3x^2 - 18x + 30$ is irreducible over the reals

(This relates to #5.)

Continuing over the complexes:

$\sqrt{-36} = 6i$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{18 \pm 6i}{2(3)} = \frac{6(3 \pm 2i)}{6} = \boxed{3 \pm 2i}$$

FINALLY! zeroes of $f(x)$ are

$x = -2$ (multiplicity 2)

$x = \frac{4}{3}$

$x = 3 \pm 2i$

Degree 5

5 zeroes

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One page writeup of #4:

$$
\begin{array}{r|rrrrrr}
-2 & 3 & -10 & -22 & 88 & 56 & -160 \\
 & & -6 & 32 & -20 & -136 & 160 \\
\hline
-2 & 3 & -16 & 10 & 68 & -80 & 0 \\
 & & -6 & 44 & -108 & 80 & \\
\hline
4/3 & 3 & -22 & 54 & -40 & 0 & \\
 & & 4 & -24 & 40 & & \\
\hline
 & 3 & -18 & 30 & 0 & &
\end{array}
$$

$a=3,\ b=-18,\ c=30$

$b^2-4ac=(-18)^2-4(3)(30)$

$=-36$

$\sqrt{-36}=6i$

$x=\dfrac{18\pm 6i}{2(3)}=\dfrac{6(3\pm 1i)}{6}=3\pm i$

zeros: $\boxed{\begin{array}{l} x=-2,\ m=2 \\ x=\frac{4}{3} \\ x=3\pm i \end{array}}$

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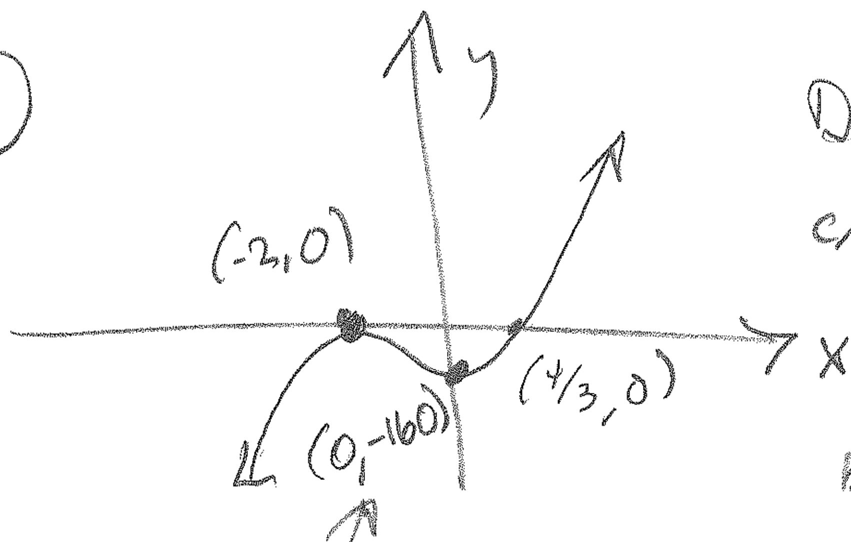
(5) $f(x) = (x+2)^2\left(x-\frac{4}{3}\right)(3x^2-18x+30)$
is factored over reals

(6) $f(x) = 3(x+2)^2(x-4/3)(x-(3+i))(x-(3-i))$

Don't forget leading coefficient!

(7)



Don't get crazy - Just use the info given by your work

Label makes it right. Trying to be true-to-scale just makes it ugly and loses its essence.

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8 $y = \frac{x^2+3x+2}{x^2+2x-8} = \frac{(x+2)(x+1)}{(x+4)(x-2)}$

D: $x \neq -4,\ x \neq 2$

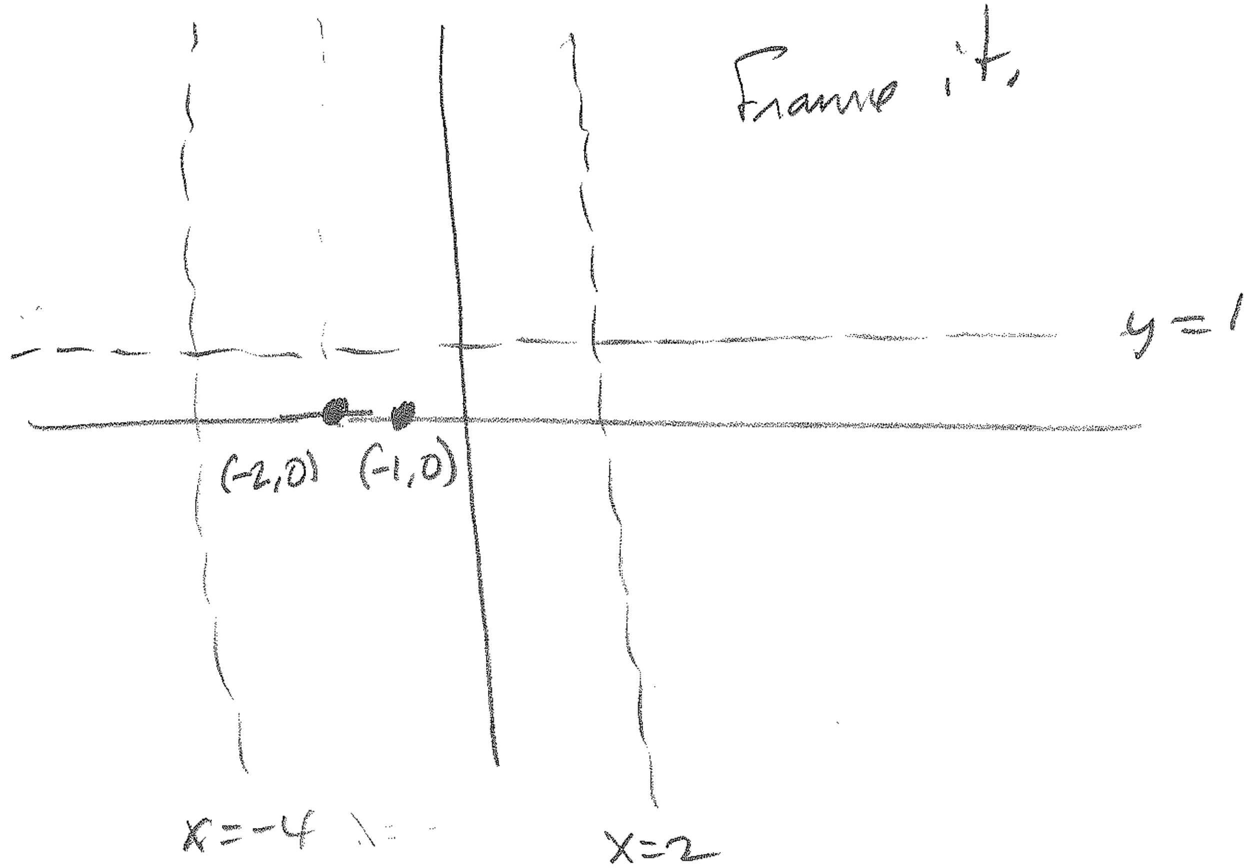
V.A. $x=-4,\ x=2$

H.A.: $\frac{x^2}{x^2} = 1 = y$

y-int: $(0, -\frac{2}{8}) = (0, -\frac{1}{4})$

x-int: $(-2,0), (-1,0)$

Frame it.



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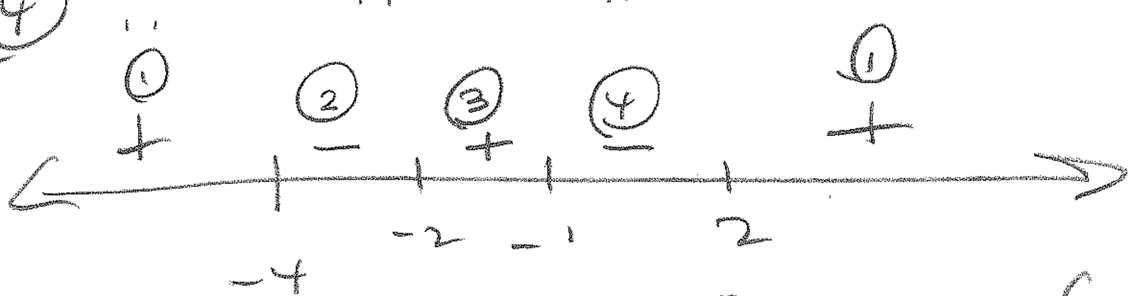
To finish, analyze sign and sign changes

① $y=1$ says + to far right ~~& far left~~

② Changes sign at $x=-4$ because $(x+4)$ is to 1^{st} power (odd power)

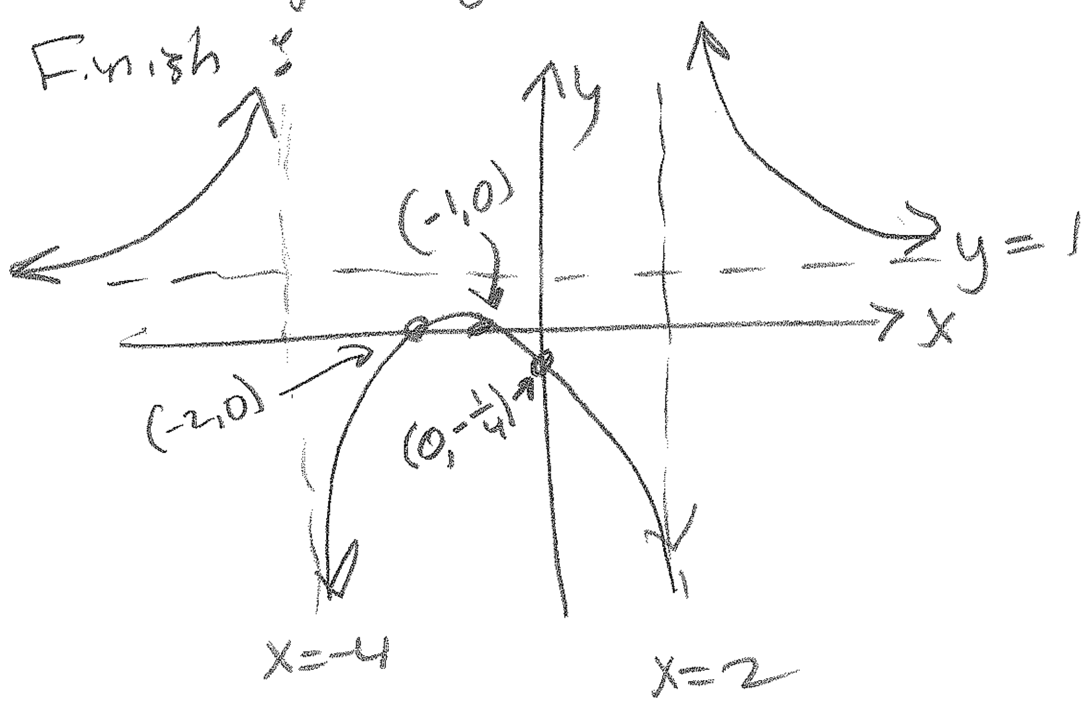
③ Changes sign at $x=-2$, b/c $(x+2)^1$ ODD

④ " " $x=-1$ " $(x+1)^1$



Check: Changes sign @ $x=2$ b/c $(x-2)^1$

Finish:



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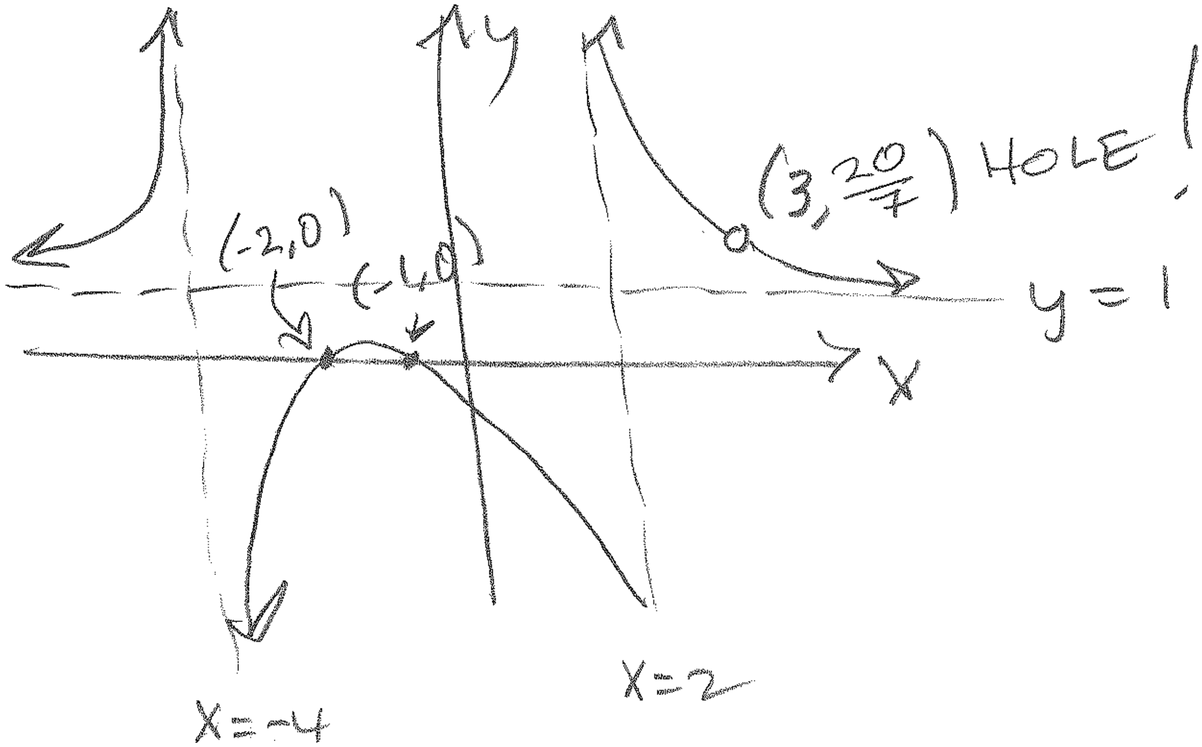
(9) Same as # 8, but $\frac{\cancel{x-3}}{\cancel{x-3}}$ makes

a hole @ $x = +3$.



To locate the hole, plug in $x=3$ after cancelling the $(x-3)$'s:

$$\frac{(3+2)(3+1)}{(3+4)(3-2)} = \frac{(5)(4)}{(7)(1)} = \frac{20}{7}$$

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(10) $y = \dfrac{x^2+3x+2}{x+4}$

has oblique asymptote. Find its equation by division

$$\begin{array}{r} x - 1 \;\; r\,6 \\ x+4 \,\overline{)\, x^2+3x+2} \\ -(x^2+4x\quad) \\ \hline -x+2 \\ -(\;-x-4\;) \\ \hline 6 \end{array}$$

→ Unnecessary Just need $y = x-1$.

$\boxed{y = x-1 \text{ is its equation}}$

D: $x \neq -4$

$x = -4$ is V.A.

$$\begin{array}{r|rrr} -4 & 1 & 3 & 2 \\ & & -4 & 4 \\ \hline & 1 & -1 & 6 \\ & x & c & r \end{array}$$

$x - 1 \;\; r\,6$

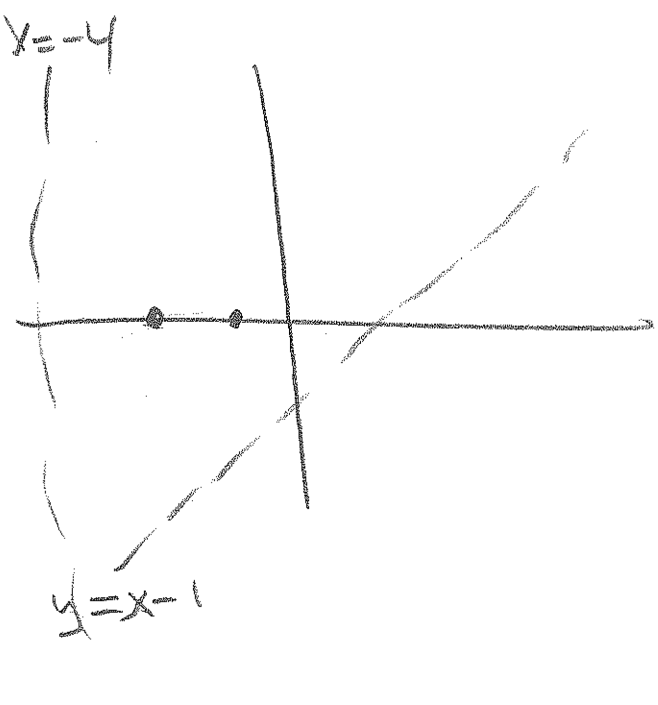
y-int: $(0, \frac{2}{4}) = (0, \frac{1}{2})$

x-int: x^2+3x+2

$= (x+1)(x+2)$

$x = -1, x = -2$

$$\begin{array}{ccccccc} & + & & - & & + & \\ \longleftarrow & | & & | & & | & \longrightarrow \\ & -4 & & -2 & & -1 & \end{array}$$



Need room...

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