MAT 121 Online, FINAL TEST, Fall, 2013
200 Points   Name KEY

1. Solve the equation $x^2 + 4x - 21 = 0$ in two different ways:

part a (10 pts) Factoring

$(x+7)(x-3) = 0$

$x \in \{-7, 3\}$

part b (10 pts) Completing the square

$x^2 + 4x = 21$

$x^2 + 4x + 2^2 = 21 + 4$

$(x+2)^2 = 25$

$\sqrt{(x+2)^2} = \sqrt{25}$

$|x+2| = 5$

$x+2 = \pm 5$

$x = -2 \pm 5 \to 3, -7$

$x \in \{-7, 3\}$

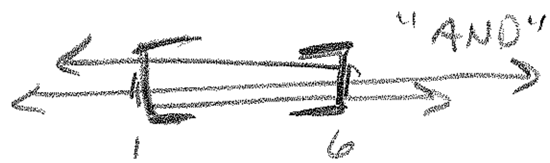
2. Solve the absolute value inequality. Give your answer in set-builder *and* interval notation:

part a (10 pts) $|2x - 7| \le 5$

$2x-7 \le 5$ and $2x-7 \ge -5$

$2x \le 12$ $2x \ge 2$

$\{x \mid x \le 6 \text{ and } x \ge 1\}$



$= [1, 6]$

part b (10 pts) $|3x - 2| > 5$

$3x-2 > 5$ OR $3x-2 < -5$

$3x > 7$ OR $3x < -3$

$\{x \mid x > \frac{7}{3} \text{ OR } x < -1\}$

$= (-\infty, -1) \cup (\frac{7}{3}, \infty)$

part c (5 pts) $|7x + 2| \ge -4$

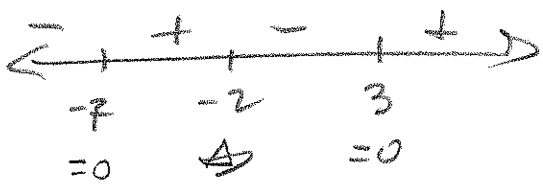
$\mathbb{R}$

part d (5 pts) $|2x - 7| < -4$

$\emptyset$

3. (10 pts) What is the domain of $f(x)=\sqrt{\dfrac{x^2+4x-21}{x+2}}$?

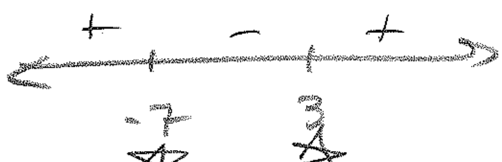
$$\frac{(x+7)(x-3)}{x+2} \geq 0$$



$$\boxed{[-7,2) \cup [3,\infty)}$$

4. (10 pts) What is the domain of $\log_5\left(x^2+4x-21\right)$?

$$(x+7)(x-3) > 0$$



$$(-\infty,-7) \cup (3,\infty)$$

5. (10 pts) Form a polynomial (in factored form) that will have *real* coefficients after expanding (which you shouldn't bother to do!) that has the following zeros and respective multiplicities. What is its degree?

$x=3$, multiplicity $=2$;
$x=-5$, multiplicity $=1$; and,
$x=3+2i$, multiplicity $=1$.

$$(x-3)^2(x+5)(x-(3+2i))(x-(3-2i))$$

Degree $=5$

6. Let $f(x)=4x^4-16x^3-31x^2+94x-195$.

part a (10 pts) Use synthetic division to determine if $x+3$ is a factor of f.

Interpret the your work by filling in the *quotient* and *remainder* in the statement $4x^4-16x^3-31x^2+94x-195=(x+3)\cdot$ *quotient* + *remainder* .

Hopefully, your remainder is zero. It's how you get it and how you interpret it that matter to me.

-3	4	-16	-31	94	-195
		-12	84	-159	195
	4	-28	53	-65	0

$(x+3)(4x^3-28x^2+53x-65)+0$

part b (10 pts) Show that $x=5$ is a root of f by dividing your *quotient* in **part a** by $x-5$. The quotient from **part a** is the so-called *depressed polynomial*, of degree 3. This question, in itself, ought to give you a very clear idea of what your conclusion ought to have been in part a.

5	4	-28	53	-65
		20	-40	65
	4	-8	13	0

part c (10 pts) Compute the discriminant of $4x^2-8x+13$. Then find the two nonreal roots of $4x^2-8x+13$, by any method (short of copying from someone else). This question *should* give you a very good idea of how things went for you, above.

$a=4$
$b=-8$
$c=13$

$b^2-4ac=(-8)^2-4(4)(13)$
$=64-208$
$\boxed{=-144}$

$\sqrt{-144}=12i$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\dfrac{8\pm 12i}{2(4)}=\dfrac{4(2\pm 3i)}{8}$

$=\boxed{\dfrac{2\pm 3i}{2}=x}$

part d (10 pts) Write f as the product of linear factors. You can still earn the **part d** points without any of **a**, **b**, or **c** by *making up* plausible answers and incorporating them into the answer to this question. It should have 2 real and 2 nonreal zeros represented by the factors.

$4(x+3)(x-5)\left(x-\left(\frac{2+3i}{2}\right)\right)\left(x-\left(\frac{2-3i}{2}\right)\right)$

7. (10 pts) Determine a, r and n for the finite geometric series $3+6+12+....+1536$

Use a, r, and n to determine the sum by the formula $\sum_{k=1}^{n} a \cdot r^{k-1} = a\left(\frac{1-r^n}{1-r}\right)$.

$\boxed{a=3,\ r=2}$ $n = ?$

$ar^{n-1} = 3 \cdot 2^9 = 3 \cdot 2^{n-1}$

$9 = n-1$

$\boxed{10 = n}$

$\therefore\ S_{10} = 3\left(\frac{1-2^{10}}{1-2}\right) = 3\left(\frac{-1023}{-1}\right) = \boxed{3069}$

$2\,|\,1536$
$2\,|\,768$
$2\,|\,384$
$2\,|\,192$
$2\,|\,96$
$2\,|\,48$
$2\,|\,24$
$2\,|\,12$
$2\,|\,6$
3

8. (10 pts) Use Pascal's Triangle (Binomial Theorem) to expand the binomial power $(x-2)^6$.

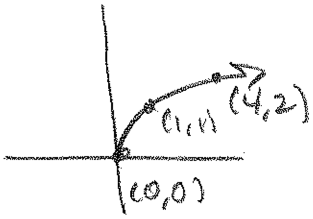
Expanding without using a recognizable version of this technique will earn at most 2 points.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

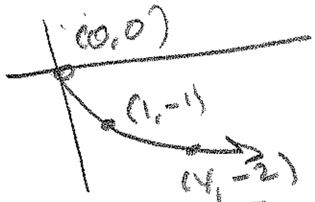
$x^6 + 6x^5(-2) + 15x^4(-2)^2 + 20x^3(-2)^3 + 15x^2(-2)^4 + 6x(-2)^5 + x^0(-2)^6$

$= x^6 - 12x^5 + 60x^4 - 160x^3 + 240x^2 - 192x + 64$

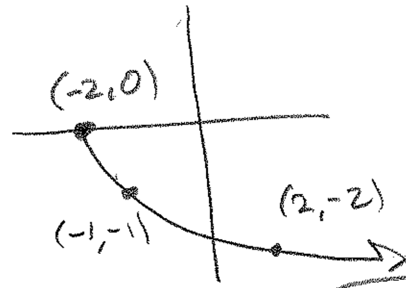
9. (10 pts) Graph $g(x) = -\sqrt{x+2} + 5$ using the techniques of shifting and reflecting. Start with the graph of the basic function $f(x) = \sqrt{x}$ and show all stages. In the final graph, indicate (label as ordered pairs) the x- and y- intercepts.



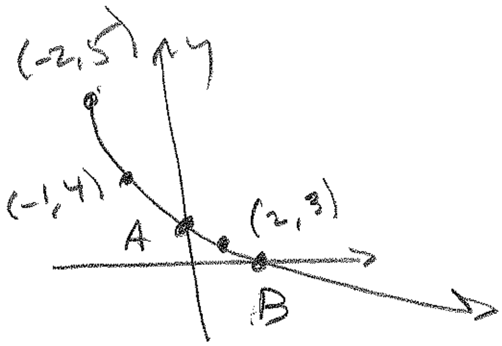
$f(x) = \sqrt{x}$



$-f(x) = -\sqrt{x}$



$-f(x+2) = -\sqrt{x+2}$



$A: g(0) = -\sqrt{2} + 5$

≈ 3.585786438

$$\boxed{\begin{array}{l} A = (0, 5-\sqrt{2}) \\ \approx (0, 3.585786438) \end{array}}$$

B:

$-\sqrt{x+2} + 5 = 0$

$-\sqrt{x+2} = -5$

$\sqrt{x+2} = 5$

$(\sqrt{x+2})^2 = 5^2$

$x + 2 = 25$

$x = 23$

$$\boxed{(23, 0) = B}$$

10. (15 pts) Solve the equation $A_0 e^{5300k} = \frac{1}{2} A_0$ for the decay rate, k.

$e^{5300K} = \frac{1}{2}$

$\ln(e^{5300K}) = \ln(1/2)$

$5300K = \ln(1/2)$

$K = \frac{\ln(1/2)}{5300} = \boxed{-\frac{\ln 2}{5300} = K}$

11. (10 pts) Suppose the half-life of Carbon-14 is 5300 years. (It isn't, but just suppose…). How old is a sample of wood from a fire pit if only 30% of the original amount of Carbon-14 remains?

$A_0 e^{kt} = .35 A_0$

$e^{kt} = .35$

$\ln(e^{kt}) = \ln(.35)$

$kt = \ln(.35)$

$t = \frac{\ln(.35)}{K} = \frac{\ln(.35)}{-\frac{\ln 2}{5300}} = -\frac{5300 \ln(.35)}{\ln 2}$

≈ 8027.237813

$\approx \boxed{8{,}027 \text{ years old}}$

12. (15 pts) Solve the system of linear equations:

$3x + 2y = 6$

$x - 3y = 12$

$x = 3y + 12$

$3(3y+12) + 2y = 6$

$9y + 36 + 2y = 6$

$11y = -30$

$\boxed{y = -\frac{30}{11}}$

$x = 3\left(-\frac{30}{11}\right) + 12$

$= -\frac{90}{11} + \frac{132}{11} = \boxed{\frac{42}{11} = x}$

$(x, y) = \left(\frac{42}{11}, -\frac{30}{11}\right)$

$= (3.\overline{81}, -2.\overline{72})$

BONUS

1) The population of a bee colony in 2008 is 800 bees. The population of that colony grows to 900 in 2012. The population is a function of time in the exponential model $P(t) = P_0 e^{kt}$ where $t = 0$ represents the year 2008.

a) Define the variables given this information and identify the two ordered pairs to use as points.

Let t = time, in years after 2008

P = population, as function of t.

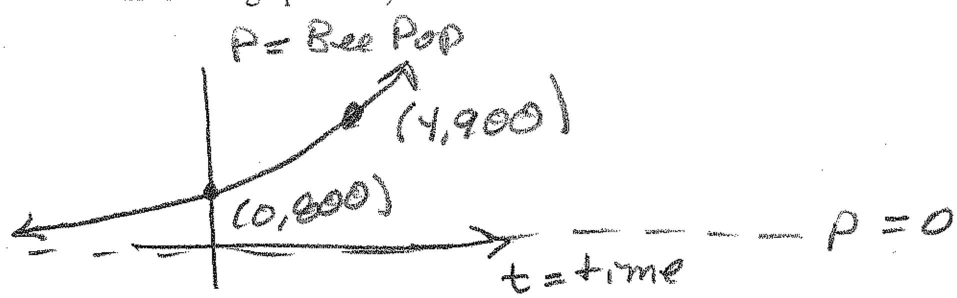
We are given $(0, 800)$ and $(4, 900)$

$2008 - 2008 = 0$ $2012 - 2008 = 4$

b) Graphing

i) Label the axes appropriately for the context of the problem.

ii) Graph (plot) the 2 points. (*Just* the two points, in correct (relative) position. We will finish the graph later.)



c) Pop is 900 when $t = 4$:

i) Find the rate of growth. Show your work. Round to 4 decimal places.

ii) Find the equation of the exponential function which models the situation.

$800e^{4k} = 900$

$e^{4k} = \frac{9}{8}$

$4k = \ln(9/8)$

$k = \frac{1}{4} \ln\left(\frac{9}{8}\right) \approx \boxed{.0294 = k}$ (i)

$\boxed{P(t) = 800e^{.0294t}}$ (ii)

d) Graph the equation of the curve on the same graph as the two points in part b. (I'd *rather* you just did the graph of the thing (correct *shape*) and then stuck the two points on it in relatively correct position.)

DONE

e) Use your equation (with k rounded to 4 places) to find the estimated population in 2017. Show your work.

$2017-2008=9$

$P(9)=800e^{.0294(9)} \approx 1032.988904 \approx \boxed{1033 \text{ Bees!}}$

f) Use the equation to calculate in what year the population will reach 1000 if the growth continues at this same rate. Show your work.

$P(t) \overset{SET}{=} 1000$

$800e^{.0294t} = 1000$

$e^{.0294t} = \frac{10}{8} = \frac{5}{4}$

$.0294t = \ln(5/4)$

$t = \frac{\ln(5/4)}{.0294} \approx 7.589916711$

It'll happen during year 7, although 7.6 rounds up to 8!

g) What would be the effect to the population if the rate had the opposite sign? Use complete sentences in your explanation.

It would be exponential decay!

Pop. would be SHRINKING.

h) List two real-life factors which may affect the population such that this model would not prove valid. Use complete sentences.

Climate, destruction of habitate, planting more flowers - All these things can take us out of our nice, tightly-constructed box!