AN EXAMPLE OF STUDENT WORK, GRADED.
MY SOLUTIONS AT THE END.

MAT 121-G11 – Fall, 2011 Test 3 Take-Home
Chapter 3 – 30 pts
Due Wednesday, October 26th

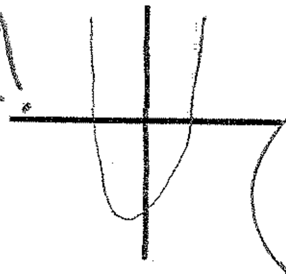
Name ______

29/30

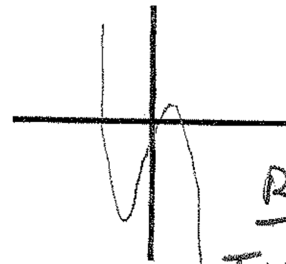
This test shows evidence of graphing calculator used as crutch. Graphers are OK on take-home, but will earn an 'F' on the sit-down test.

1. (5 pts) For each of the following polynomials, draw the end behavior on the graph.

a. $p(x) = 5x^4 - 4x^3 + 2x - 5$



b. $g(x) = -5x^5 + 3x^4 - 5x^2 + 7x - 2$

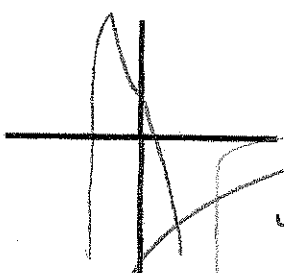


END BEHAVIOR!

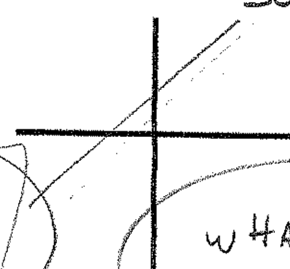
↖ ... ↗ IS WHAT I WANT

NOT REALLY WHAT I'M AFTER. SEE SOLUTIONS.

c. $h(x) = -x^6 - 4x^3 + 2x^2 - 8x + 1$



d. $f(x) = 2x + 7$



WHAT I WANT: ↙ ... ↘

WHAT I WANT: ↙ ... ↗

Let $f(x) = x^5 - 6x^4 + x^3 + 56x^2 - 60x - 208$ for problems 2 - 6. $-x - 6x^4 - x^3 + 56x^2 + 60x$ (−208

2. (5 pts) What does Descartes Rule of Signs tell you about this function?

There are either 3 or Ø positive real zeros and there are either 2 or Ø negative real zeros.

3. (5 pts) Use the Rational Zeros (Roots) Theorem to list the possible rational zeros of f.

p(-208): ±1, ±2, ±4, ±8, ±13, ±16, ±26, ±52, ±104, ±208
q(1): ±1

p/q: same

5

4

4. (5 pts) Find all real and complex zeros of $f(x) = x^5 - 6x^4 + x^3 + 56x^2 - 60x - 208$, using the rational zero candidates you have from the previous problem. Put your work NEATLY in the space below. **This means doing your work on separate paper, organizing it, and transferring it to the space, below, after you've eliminated the bad guesses. No credit for sloppy work.**

3 or 1 pos

$$\begin{array}{r|rrrrrr} -2 & 1 & -6 & 1 & 56 & -60 & -208 \\ & & -2 & 16 & -34 & -44 & 208 \\ \hline & 1 & -8 & 17 & 22 & -104 & 0 \end{array}$$

$-x^5 - 6x^4 - x^3 + 56x^2 + 60x - 208$

2 or 0 neg

$(x+2)(x^4 - 8x^3 + 17x^2 + 22x - 104)$

$$\begin{array}{r|rrrrr} 4 & 1 & -8 & 17 & 22 & -104 \\ & & 4 & -16 & 4 & 104 \\ \hline & 1 & -4 & 1 & 26 & 0 \end{array}$$

$(x+2)(x-4)(x^3 - 4x^2 + x + 26)$

5

$$\begin{array}{r|rrrr} -2 & 1 & -4 & 1 & 26 \\ & & -2 & 12 & -26 \\ \hline & 1 & -6 & 13 & 0 \end{array}$$

$(x+2)(x+2)(x-4)(x^2 - 6x + 13) \rightarrow \dfrac{6 \pm \sqrt{-16}}{2} = 3 \pm 2i$

$(x+2)(x+2)(x-4)(x-3+2i)(x-3-2i)$

zeros: $-2, 4, 3+2i, 3-2i$

5

5. Now that you've done all the prep work, write f in factored form, in two ways:

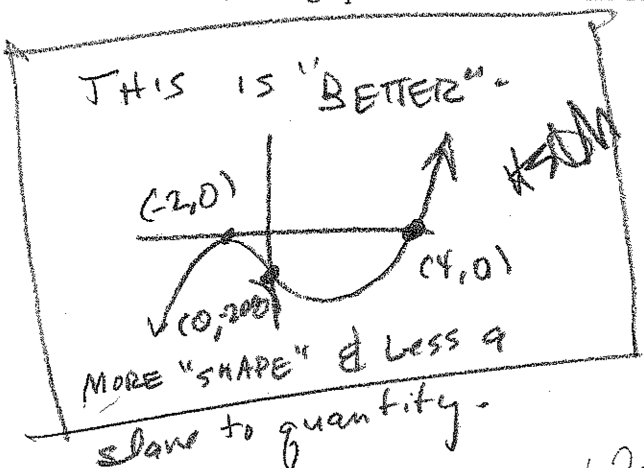
a. (3 pts) Factor f over the REAL number field (Involves an *irreducible quadratic* factor.).

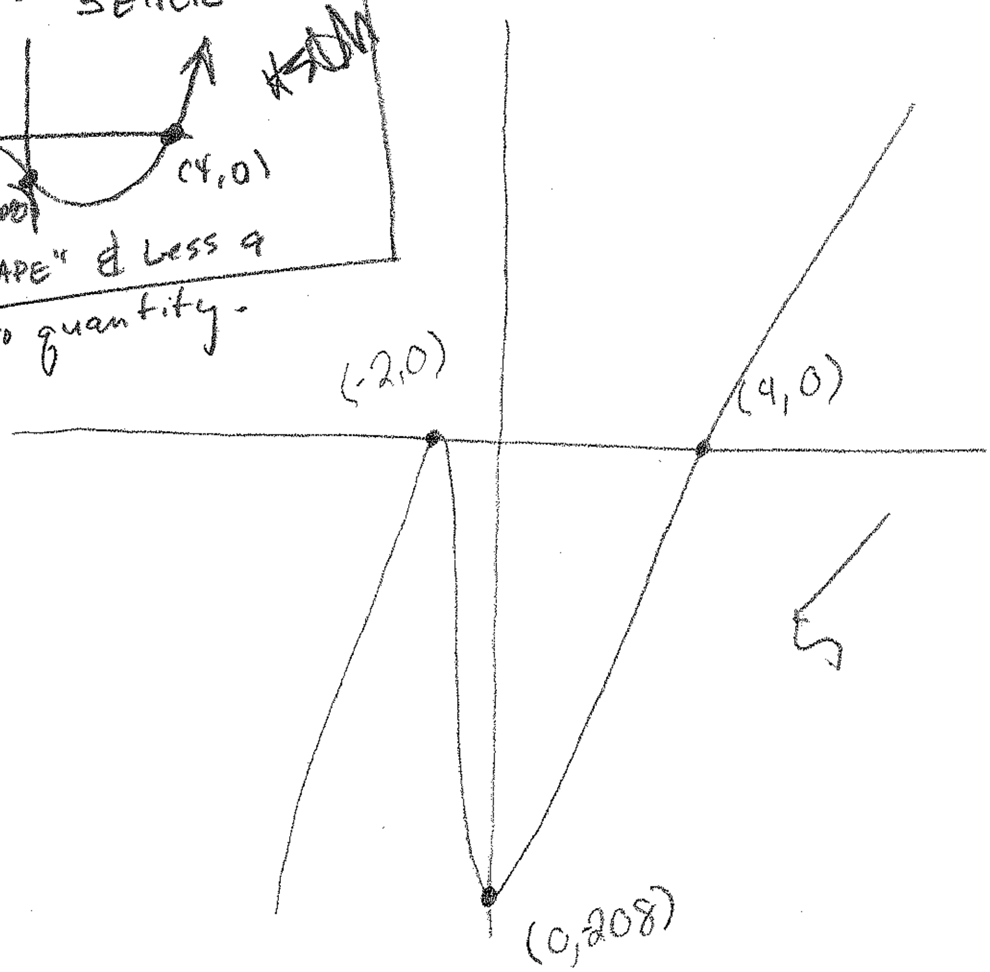
$(x+2)^2(x-4)(x^2-6x+13)$ ✓

b. (2 pts) Factor f over the COMPLEX number field. (All *linear* factors.).

$(x+2)^2(x-4)(x-3+2i)(x-3-2i)$ 5

6. (5 pts) Now that you've factored it, I want you to sketch the graph of $f(x)=x^5-6x^4+x^3+56x^2-60x-208$, showing all intercepts. A *smooth* graph is the goal, here, not a graph that's a slave to the scale.





10

(21) T3 Take-Home

5pts (1) (a) $P(x) = 5x^4 + \text{smaller}$ ↖ … ↗

(b) $g(x) = -5x^5 + \text{smaller}$ ↖ … ↘

(c) $h(x) = -x^4 + \text{smaller}$ ↙ … ↘

(d) $f(x) = 2x + 7$ ↙ … ↗

5pts (2) $f(x) = x^5 - 6x^4 + x^3 + 56x^2 - 60x - 208$

Descartes: 3 or 1 positive zeros (But 1, for sure)

$f(-x) = -x^5 - 6x^4 - x^3 + 56x^2 + 60x - 208$

ZERO negative (maybe none!)

5pts (3)

$$\begin{array}{l} 2\,\underline{|208} \\ 2\,\underline{|104} \\ 2\,\underline{|52} \\ 2\,\underline{|26} \\ \quad 13 \end{array}$$

$\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 13, \pm 26,$

~~$\pm 2 \cdot 13 = \pm 26$~~

$\pm 4 \cdot 13 = \pm 52, \pm 8 \cdot 13 = \pm 104$

$\pm 16 \cdot 13 = \pm 208,$

These are all the possible $\frac{p}{q}$'s p divides 208, q divides 1

5pts (4) We find all real & complex zeros

121 T3 TH = Test 3 Take-Home

$$\begin{array}{r|rrrrrr} -2 & 1 & -6 & 1 & 56 & -60 & -208 \\ & & -2 & 16 & -34 & -44 & 208 \\ \hline -2 & 1 & -8 & 17 & 22 & -104 & 0 \\ & & -2 & 20 & -74 & 104 & \\ \hline 4 & 1 & -10 & 37 & -52 & 0 & \\ & & 4 & -24 & 52 & & \\ \hline & 1 & -6 & 13 & 0 & & \end{array}$$

$(x+2)(x^4-8x^3+17x^2+22x-104)$

$(x+2)^2(x^3-10x^2+37x-52)$

$(x+2)^2(x-4)(x^2-6x+13)$

5a over $\mathbb{R}$ 3pts

$x^2-6x+13=0 \Longrightarrow$

$x^2-6x = -13$

$x^2-6x+3^2=-13+9$

$(x-3)^2=-4$

$x-3=\pm\sqrt{-4}=\pm 2i$

$x=3\pm 2i$

zeros:

4 5pts

$x=-2$ $m=2$

$x=4$ $m=1$

$x=3+2i$ $m=1$

$x=3-2i$ $m=1$

TOT = 5 ✓

$x+2$

5b 2pts

$(x+2)^2(x-4)(x-(3+2i))(x-(3-2i))$ over $\mathbb{C}$

6 5pts

$(x+2)^2(x-4)(x^2-6x+13)$

$x=-2$ touch

$x=4$ cross

x^5 + smaller $\Longrightarrow$ ↙ ↗, so

$f(0)=-208$

$\rightsquigarrow (0,-208)$