MAT 121-G81 Test 2 – Fall, 2013 Name__________________

100 Points Covers Chapter 2 NO GRAPHING CALCULATORS!!!

1. (5 pts) Find an equation of the line through the points (-1, 2) and (3, 5).

$(x_1, y_1) \quad (x_2, y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-2}{3-(-1)} = \frac{3}{4}$$

$$y = m(x - x_1) + y_1 \qquad \boxed{y = \frac{3}{4}(x - (-1)) + 2} \quad \text{STOP!}$$

$$y = \frac{3}{4}x + \frac{3}{4} + \frac{8}{4}$$

$$y = \frac{3}{4}x + \frac{11}{4}$$

2. (5 pts) Sketch the graph of the line whose equation you found in #1, above. Show *x*-and *y*-intercepts.

$$y = 0 \Longrightarrow \frac{3}{4}x + \frac{11}{4} = 0$$

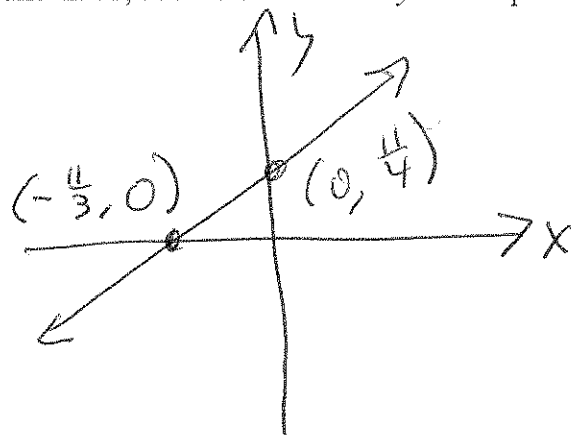
$$\frac{3}{4}x = -\frac{11}{4}$$

$$x = -\frac{11}{4} \cdot \frac{4}{3} = -\frac{11}{3}$$

$$\left(-\frac{11}{3}, 0\right)$$

$$x = 0 \Longrightarrow y = \frac{11}{4}$$

$$\left(0, \frac{11}{4}\right)$$



3. (5 pts) Is the linear function in #s 1 and 2 increasing or decreasing?

Increasing

4. Suppose *y* varies jointly as *x* and the square of *z* and inversely as the square root of *w*.

   a. (5 pts) Write an equation representing the relationship.

$$y = k\frac{xz^2}{\sqrt{w}}$$

   b. (5 pts) Suppose $y = 24$ when $x = 1$, $z = 2$ and $w = 4$. What, then, is *y* when $x = 2$, $z = 3$ and $w = 4$?

$$24 = \frac{(1)(2)^2}{\sqrt{4}}k = 2k$$

$$\Longrightarrow 12 = k$$

$$\Longrightarrow y = 12 \cdot \frac{2 \cdot 3^2}{\sqrt{4}} = 12 \cdot 9 = \boxed{108,}$$ when $x = 2$, $z = 3$, and $w = 4$.

5. Compute the discriminant for each of the following quadratic and tell me the nature of solutions, specifically, how many distinct solutions there are and whether they're real or non-real. *Do not solve the equations.* I'll throw a couple extra points of bonus your way if you distinguish between rational and irrational solutions.

a. (5 pts) $x^2 - 6x - 19 = 0$

$b^2 - 4ac = (-6)^2 - 4(1)(-19) = 36 + 76 = 112$ $\boxed{2 \text{ real}}$

b. (5 pts) $9x^2 - 30x + 53 = 0$

$b^2 - 4ac = (-30)^2 - 4(9)(53) = -1008$ $\boxed{2 \text{ nonreal}}$

c. (5 pts) $6x^2 - 25x + 14 = 0$

$b^2 - 4ac = (-25)^2 - 4(6)(14) = 289 = 17^2 \rightarrow$ $\boxed{2 \text{ real}}$

2 RATIONAL 2B

6. Solve by any method, but *show all work*!!!

a. (5 pts) $x^2 - 6x - 19 = 0$

$x^2 - 6x = 19$

$x^2 - 6x + 3^2 = 19 + 9$

$(x-3)^2 = 28$

$x - 3 = \pm\sqrt{28}$

$x = 3 \pm 2\sqrt{7}$

$2|\underline{112}$
$2|\underline{56}$
$2|\underline{28}$
$2|\underline{14}$
7

$b^2 - 4ac = (-6)^2 - 4(1)(-19)$
$= 36 + 76 = 112$
$\sqrt{112} = 4\sqrt{7}$

$x = \dfrac{6 \pm 4\sqrt{7}}{2}$

$\boxed{= 3 \pm 2\sqrt{7}}$ ✓

b. (5 pts) ~~$9x^2 - 30x + 53 = 0$~~

$b^2 - 4ac = -1008$, by #5

$x = \dfrac{30 \pm 12i\sqrt{7}}{2(9)}$

$= \dfrac{30 \pm 12i\sqrt{7}}{18}$

$= \dfrac{15 \pm 6\sqrt{7}\,i}{9}$

$= \boxed{\dfrac{5 \pm 2\sqrt{7}\,i}{3}}$

$2|\underline{1008}$
$2|\underline{504}$
$2|\underline{252}$
$2|\underline{126}$
$3|\underline{63}$
$3|\underline{21}$
7

c. (5 pts) $6x^2 - 25x + 14 = 0$

$(6)(14) = (2)(3)(7)(2)$ want a sum of -25

$6x^2 - 21x - 4x + 14 = 0$

$3x(2x-7) - 2(2x-7) = 0$

$(2x-7)(3x-2) = 0$

$\boxed{x = \frac{7}{2}, \frac{2}{3}}$

7. (5 pts) Solve $x^2-6x-55=0$ by completing the square.

$x^2-6x \quad = 55$

$x^2-6x+3^2=55+9$

$(x-3)^2=64$

$x-3=\pm 8$

$x=3\pm 8$ → 11, −5

"Stylish" solution

$\boxed{x\in\{-5,11\}}$

8. (10 pts) Complete the square for $f(x)=x^2-6x-55$, and re-write it in the form $f(x)=a(x-h)^2+k$. This is very similar to what you just did in #7, <u>but you're manipulating an expression, rather than solving an equation</u>, here. Use your work to sketch a graph of $f(x)$ that includes vertex, x- and y-intercepts, labeled as ordered pairs. I refuse to count tickmarks on the x- or y-axis.

$f(x)=x^2-6x-55$

$=x^2-6x+3^2-9-55$

$=(x-3)^2-64$

$(h,k)=(3,-64)$

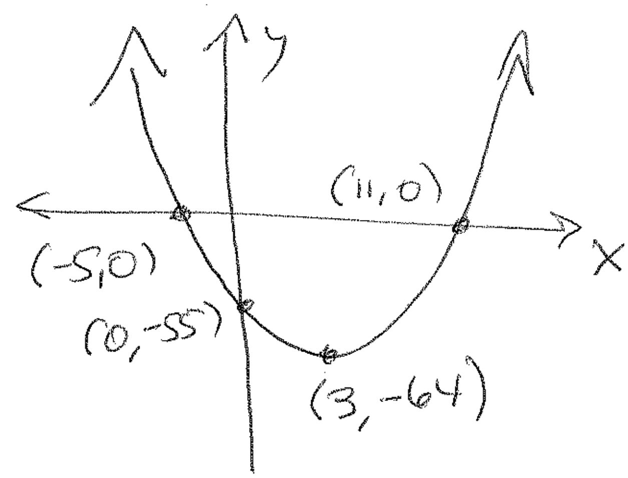
By previous work,

$x=-5, 11$ solves $f(x)=0$

(x-ints!)

$f(0)=-55 \rightsquigarrow (0,-55)$

is y-int



9. (5 pts) Based on your work on #8, state the domain and range of $f(x)$.

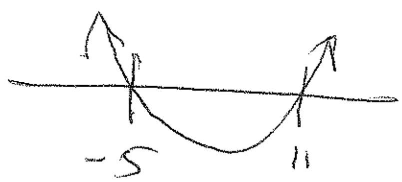
$\mathcal{D}=\mathbb{R}$, $\mathcal{R}=[-64,\infty)$

10. (5 pts) State intervals of increase and decrease for $f(x)$ from #s 8 and 9.

$\boxed{\text{Inc: } [3,\infty) \quad \text{Dec: } (-\infty,3]}$

11. (10 pts) Well, you've done so much with $f(x)=x^2-6x-55$, now I want you to solve the inequality $3x^2+2x-20 \le 2x^2+8x+35$. That was a hint, by the way.

$x^2-6x-55 \le 0$



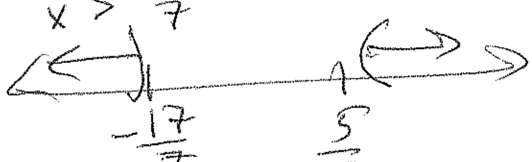
$\longrightarrow \boxed{x \in [-5, 11]}$

12. (5 pts) Solve $|7x+6|>11$. Give your answer in set-builder *and* interval notation.

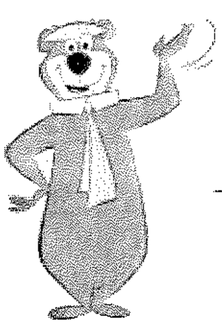
$7x+6>11$ OR $7x+6<-11$

$7x>5$ $\quad 7x<-17$

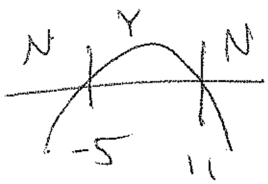
$x>\frac{5}{7}$ OR $x<-\frac{17}{7}$



$(-\infty,-\frac{17}{7})\cup(\frac{5}{7},\infty)=\{x \mid x<-\frac{17}{7} \text{ OR } x>\frac{5}{7}\}$



Bonus Now, tell me what the domain of $g(x)=\sqrt{-x^2+6x+55}$ is.



$-x^2+6x+55 \ge 0$

$\longrightarrow x \in [-5,11]=\{x \mid -5 \le x \le 11\}$