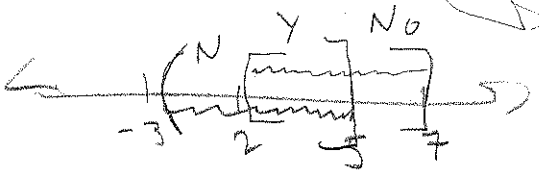
121 Test 1 Chapter 1 FALL 2013 KEY
G81

① $\mathcal{D}(f) = [2,7]$ & $\mathcal{D}(g) = (-3,5]$ ⟶

$\mathcal{D}(f+g) = \mathcal{D}(f) \cap \mathcal{D}(g) = \boxed{[2,5]}$

$\cap$ → AND



② $f(x) = \dfrac{x+1}{x^2-7x-8}$ ⟶

Scratch:
$(x-8)(x+1)$
$= x^2 - 7x - 8$ ✓

$\mathcal{D}(f) = \{x \mid x^2 - 7x - 8 \neq 0\}$

$= \boxed{\{x \mid x \neq -1 \text{ and } x \neq 8\}}$

③ $R = \{(2,3), (5,-7), (3,12), (6,3), (5,11)\}$

⟶ (a) R is Not a Function, b/c $(5,-7)$ & $(5,11)$ are members.

(c) $\boxed{\mathcal{R}(R) = \{-7, 3, 11, 12\}}$ = Range

(b) $\boxed{\mathcal{D}(R) = \{2,3,5,6\}}$ = Domain

④ $g(x) = x^2 + 5$ ⟶ m_{AVG} over $[2,4]$; $\dfrac{g(4)-g(2)}{4-2}$

$= \dfrac{4^2+5-(2^2+5)}{2} = \dfrac{21-9}{2} = \dfrac{12}{2} = \boxed{6 = m_{AVG}}$

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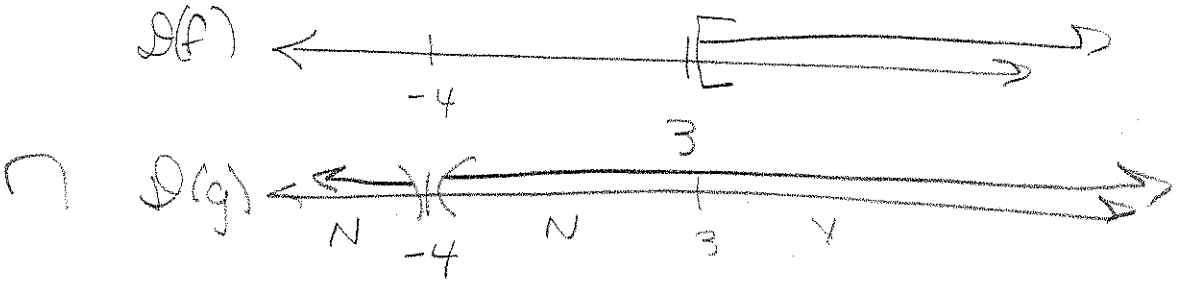
(5) $f(x) = \sqrt{x-3}$ & $g(x) = \frac{x-2}{x+4}$ ⟹

(a) $\mathcal{D}(f) = \{x \mid x-3 \geq 0\} = \{x \mid x \geq 3\} = \boxed{[3, \infty)}$

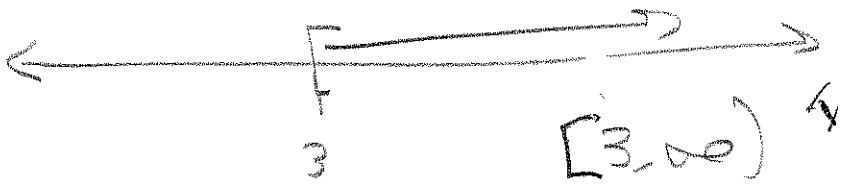
(b) $\mathcal{D}(g) = \{x \mid x+4 \neq 0\} = \{x \mid x \neq -4\} = \boxed{(-\infty, -4) \cup (-4, \infty)}$

(c)(i) $(f-g)(x) = \sqrt{x-3} - \frac{x-2}{x+4}$

$\mathcal{D}(f-g) = \mathcal{D}(f) \cap \mathcal{D}(g) : \{x \mid x \geq 3\} = [3, \infty)$



$= \mathcal{D}(f) \cap \mathcal{D}(g)$



Include $x=3$, since.

(ii) $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x-3}}{\frac{x-2}{x+4}}$. $\mathcal{D}(f/g) = \mathcal{D}(f) \cap \mathcal{D}(g) \cap \{x \mid g(x) \neq 0\}$

$= \{x \mid x \geq 3\} \cap \{x \mid \frac{x-2}{x+4} \neq 0\}$

$= \{x \mid x \geq 3\} \cap \{x \mid x \neq 2\}$

$= \{x \mid x \geq 3\} = [3, \infty)$

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(f∘g)(x)

⑤ (c) (i) (ii) $(f\circ g)(x) = f(g(x)) = \sqrt{g-3} = \sqrt{\frac{x-2}{x+4} - 3}$

$\mathcal{D}(f\circ g) = \{x \mid x \in \mathcal{D}(g) \text{ AND } g(x) \in \mathcal{D}(f)\}$

$= \{x \mid x \neq -4 \text{ AND } g(x) \geq 3\}$

$= \{x \mid x \neq -4 \text{ AND } \frac{x-2}{x+4} \geq 3\}$

Here's how $\frac{x-2}{x+4} \geq 3$ goes:

→ Something we haven't been trained to handle! Poorly-posed question. BONUS IF YOU GET IT, NO HARM IF NOT

$\frac{x-2}{x+4} - 3 \geq 0$

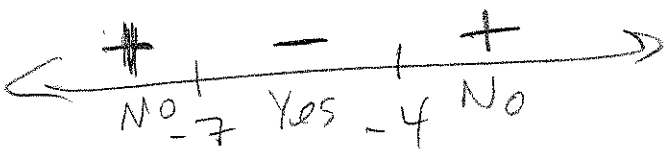
$\frac{x-2}{x+4} - \frac{3(x+4)}{x+4} \geq 0$

$\frac{x-2-3x-12}{x+4} \geq 0$

$\frac{-2x-14}{x+4} \geq 0$

$\frac{-2(x+7)}{x+4} \geq 0$

$\frac{x+7}{x+4} \leq 0$ → This says we want the negative, which is between −7 & −4, with $x = -4$ illegal: $-7 \leq x < -4$



Test $x = -8$: $\frac{-1}{-4} = \frac{1}{4}$ +

$x = -6$: $\frac{1}{-2} = -\frac{1}{2}$ −

$x = 0$: $\frac{7}{4}$ +

$\frac{x-2}{x+4} \geq 3$

$\therefore \mathcal{D}(f\circ g) = \{x \mid x \neq -4 \text{ and } -7 \leq x < -4\}$

FINAL ANSWER: $= \{x \mid -7 \leq x < -4\} = [-7, -4)$

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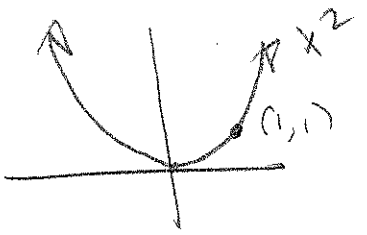
(6) $f(x) = x^2 + 3x \longrightarrow \dfrac{f(x+h) - f(x)}{h}$

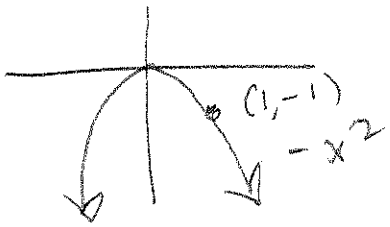
$$= \frac{(x+h)^2 + 3(x+h) - [x^2 + 3x]}{h}$$

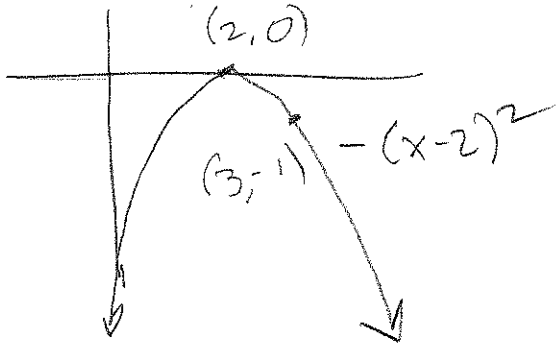
$$= \frac{x^2 + 2xh + h^2 + 3x + 3h - x^2 - 3x}{h}$$

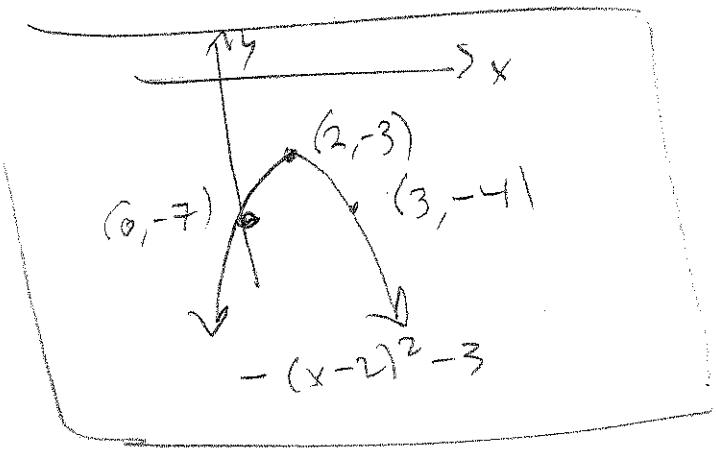
$$= \frac{2xh + h^2 + 3h}{h} = \boxed{2x + h + 3}$$

(7) $g(x) = -(x-2)^2 - 3$









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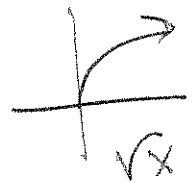
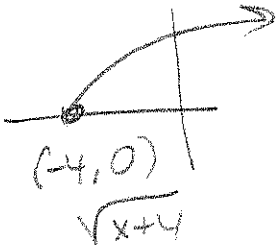
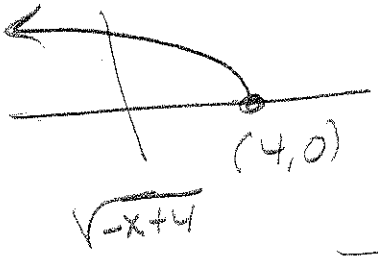
(7) (b) $g(x) = \sqrt{4-x} + 7 = \sqrt{-x+4} + 7 = \sqrt{-(x-4)} + 7.$

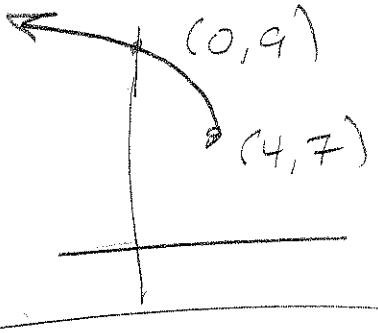
M1 M2

M1

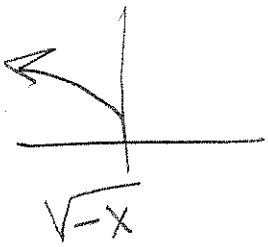


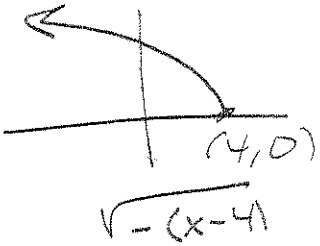


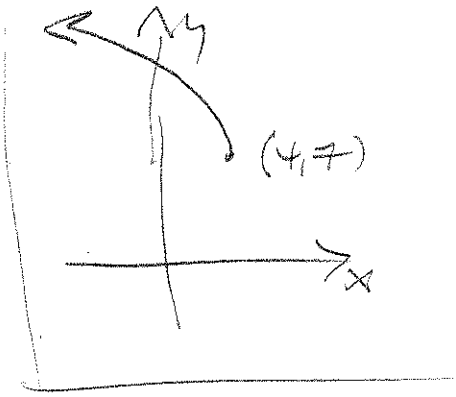




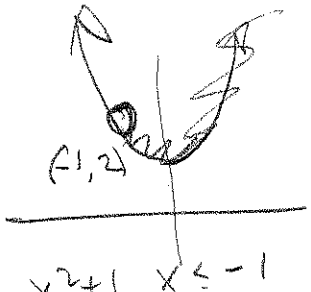
M2

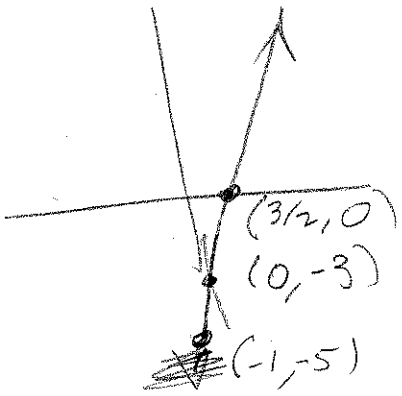




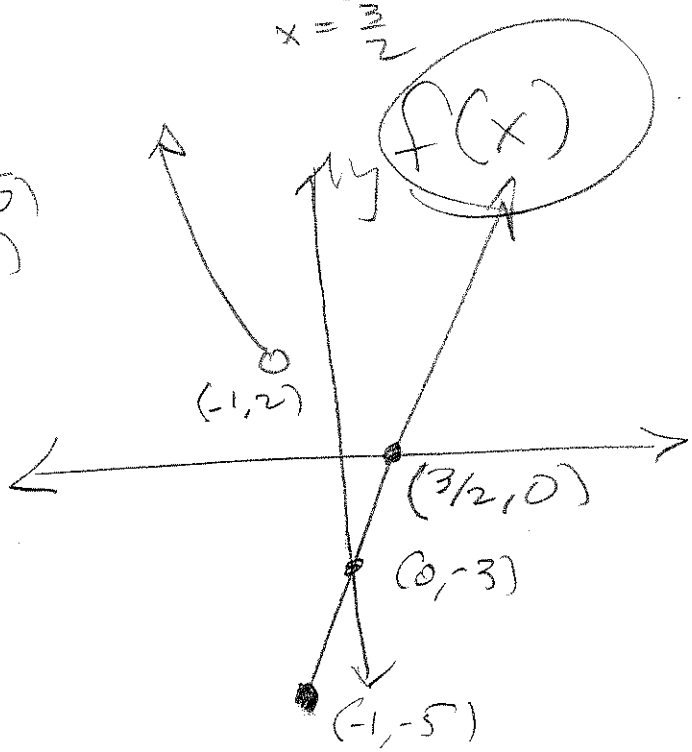


(8) $f(x) = \begin{cases} x^2+1 & \text{if } x \le -1 \\ 2x-3 & \text{if } x \ge -1 \end{cases}$





$2x-3=0$
$2x=3$
$x=\frac{3}{2}$



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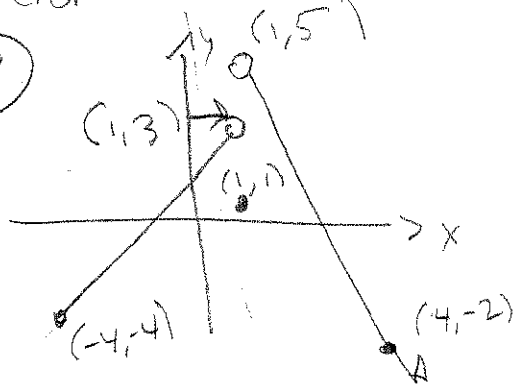
(9)



Fixed

$$f(x) = \begin{cases} \frac{7}{5}(x-1)+3 & ;f\ -4 \le x < 1 \\ 1 & ;f\ x = 1 \\ -\frac{7}{3}(x-1)+5 & ;f\ x > 1 \end{cases}$$

$-4 \le x < 1$:

$m = \frac{-4-3}{-4-1} = \frac{-7}{-5} = \frac{7}{5}$

$y = \frac{7}{5}(x-1)+1$

$=$

$1 < x$:

$m = \frac{-2-5}{4-1} = -\frac{7}{3}$

$y = -\frac{7}{3}(x-1)+5$

$y = m(x-x_1)+y_1$ is easiest!

$y = \frac{7}{5}(x-1)+3 = \frac{7}{5}x - \frac{7}{5} + \frac{15}{5}$

STOP!

$y = \frac{7}{5}x + \frac{8}{5}$

STOP!

$y = -\frac{7}{3}(x-1)+5$

$y = -\frac{7}{3}x + \frac{7}{3} + \frac{15}{3}$

$y = -\frac{7}{3}x + \frac{22}{3}$

$y = mx+b$ is fine, but $y = m(x-x_1)+y_1$ is easier, IF YOU WILL LEARN IT.

BONUS

$x^2 - 6x - 11 = 0$

$x^2 - 6x = 11$

$x^2 - 6x + 3^2 = 11 + 9$

$(x-3)^2 = 20$

$x - 3 = \pm\sqrt{20} = \pm 2\sqrt{5}$

$x = 3 \pm 2\sqrt{5}$

$x \in \{3 \pm 2\sqrt{5}\}$