MAT 121 Online, FINAL TEST, Spring, 2013
200 Points (10 pts) Name__________________________

This is our final learning opportunity together, and I'm hoping to take full advantage. Read the questions carefully. It's possible to earn points on a problem by *knowing* that you did something wrong and clearly *explaining how* you know and what you're *trying* to accomplish, and how you're going about it. More points for solid terminology and English.

1. Solve the equation $x^2 - 3x + 2 = 0$ in two different ways:

part a (10 pts) Factoring

$(x-2)(x-1) = 0$

$\boxed{x \in \{1, 2\}}$

part b (10 pts) Completing the square

$x^2 - 3x = -2$

$x^2 - 3x + \left(\frac{3}{2}\right)^2 = -2 + \frac{9}{4} = \frac{1}{4}$

$\left(x - \frac{3}{2}\right)^2 = \frac{1}{4}$

$x - \frac{3}{2} = \pm \frac{1}{2}$ $\qquad \frac{4}{2} = 2$

$x = \frac{3}{2} \pm \frac{1}{2}$ $\qquad \frac{2}{2} = 1$

$\boxed{x \in \{1, 2\}}$

2. Solve the absolute value inequality. Give your answer in set-builder *and* interval notation.

part a (10 pts) $|3x - 2| \geq 5$

$3x - 2 \geq 5$ OR $3x - 2 \leq -5$

$3x \geq 7$ OR $3x \leq -3$

$\boxed{\{x \mid x \geq \frac{7}{3} \text{ OR } x \leq -1\} = (-\infty, -1] \cup [\frac{7}{3}, \infty)}$

part b (10 pts) $|3x - 2| < 5$

$3x - 2 < 5$ AND $3x - 2 > -5$

$3x < 7$ AND $3x > -3$

$\boxed{\{x \mid x < 7 \text{ AND } x > -1\} = (-1, 7)}$

#2 cnt'd (These last two are *supposed* to be *easy* ! Conceptual.

part c (5 pts) $|7x + 2| \geq -4$

$(-\infty, \infty)$

part d (5 pts) $|2x - 7| < -4$

$\emptyset$

3. Find the domain of each of the following:

part a (10 pts) $f(x)=\dfrac{x^2-9}{x^2+5x-14}$

$x^2+5x-14=0$

$(x+7)(x-2)=0$

$x \in \{-7, 2\}$

Are the bad pts

$\mathcal{D} = \{x \mid x \neq -7 \text{ and } x \neq 2\}$

$= (-\infty,-7) \cup (-7,2) \cup (2,\infty)$

$= \mathbb{R} \setminus \{-7, 2\}$

part b (10 pts) $f(x)=\sqrt{x^2+5x-14}$

$x^2+5x-14 \geq 0$

$+ \quad - \quad +$

$-7 \quad 2$

$x \in (-\infty,-7] \cup [2,\infty) = \{x \mid x \leq -7 \text{ OR } x \geq 2\}$

part c (10 pts) $\log_3(x^2+5x-14)$

$x = -7$ and $x = 2$ are bad

$\mathcal{D} = (-\infty,-7) \cup (2,\infty)$

$= \{x \mid x < -7 \text{ OR } x > 2\}$

4. (10 pts) Let $f(x) = x^2 - 2x$. Simplify the difference quotient $\frac{f(x+h) - f(x)}{h}$.

$$\frac{(x+h)^2 - 2(x+h) - [x^2 - 2x]}{h} = \frac{x^2 + 2xh + h^2 + 2x + 2h - x^2 + 2x}{h}$$

$$= \frac{2xh + h^2 + 2h}{h}$$

$$= \boxed{2x + h + 2}$$

5. (15 pts) Form a polynomial (in factored form) that will have *real* coefficients after expanding (which you shouldn't bother to do!) that has the following zeros with the respective multiplicities:

$x = 3$, multiplicity = 2
$x = -5$, multiplicity = 1
$x = 3 - 7i$, multiplicity = 1

$$(x-3)^2(x+5)(x-(3-7i))(x-(3+7i))$$

(5 pts) What's the minimum possible degree for the polynomial described?

$$\boxed{n = 5}$$

6. Let $f(x) = \cancel{x^4 - 5x^3 - x^2 + 23x - 6}$. $x^4 - 5x^3 + 3x^2 + 19x - 30$

Hopefully, your remainder is zero. It's how you get it and how you interpret it that matter to me.

part a (10 pts) Use synthetic division to determine if $x-3$ is a factor of f. Interpret the your work by filling in the quotient and remainder in the statement $\cancel{x^4 - 5x^3 \ldots 60} = (x-3)\cdot \textit{quotient} + \textit{remainder}$.

$$\begin{array}{r|rrrrr} 3 & 1 & -5 & 3 & 19 & -30 \\ & & 3 & -6 & -9 & 30 \\ \hline & 1 & -2 & -3 & 10 & 0 \end{array}$$

$$\boxed{x^4 - 5x^3 + 3x^2 + 19x - 30 = (x-3)(x^3 - 2x^2 - 3x + 10) + 0}$$

part b (10 pts) Show that $x = -2$ is a root of f by dividing your *quotient* in **part a** by ~~x-4~~. This question, in itself ought to give you a very clear idea of what your conclusion ought to have been in part a.

$$\begin{array}{r|rrrr} -2 & 1 & -2 & -3 & 10 \\ & & -2 & 8 & -10 \\ \hline & 1 & -4 & 5 & 0 \end{array}$$

part c (15 pts) Compute the discriminant of $\cancel{x^2 - 4x + 1}$ $x^2 - 4x + 5$. Then find the two nonreal roots of $\cancel{x^2 - 4x + 1}$, by any method (short of copying from someone else).

$x^2 - 4x + 5$

$b^2 - 4ac = (-4)^2 - 4(1)(5) = 16 - 20 = -4$

$$x = \frac{4 \pm \sqrt{-4}}{2} = \frac{4 \pm 2i}{2} = 2 \pm i$$

Bonus (10 pts) Write f as the product of linear factors. Hint: If your work from #6 is up-to-snuff, then the hard part is already done, and I've given you *just* enough touchstones to help you know when you're right, or have a good reason why you aren't. You can still earn the **Bonus** without #6 by *making up* plausible answers and incorporating them into the answer to this question. It should have 2 real and 2 nonreal zeros represented by the factors.

$(x-3)(x+2)(x-(2+i))(x-(2-i))$

$2\overline{)128}$ $2\overline{)64}$ $2\overline{)32}$ $2\overline{)16}$ $2\overline{)8}$ $2\overline{)4}$ 2

7. (10 pts) Determine a, r and n for the finite geometric series $5+\frac{5}{2}+\frac{5}{4}+\ldots+\frac{5}{128}$

Use a, r, and n to determine the sum by the formula $\sum_{k=1}^{n} a\cdot r^{k-1} = a\left(\frac{1-r^n}{1-r}\right)$. A fractional answer is better, but I'll give you most of the points if you provide a decimal answer that is accurate to 4 decimal places.

$n-1=7,\ n=8$

$$5\left(\frac{1-\left(\frac{1}{2}\right)^8}{1-\frac{1}{2}}\right) = 5\left(\frac{1-\frac{1}{256}}{\frac{1}{2}}\right) = 5\left(\frac{\frac{255}{256}}{\frac{1}{2}}\right)$$

$$= 5\left(\frac{255}{256}\right)\left(\frac{2}{1}\right)$$

$$= \frac{5(255)}{128} = \frac{1275}{128} \approx \boxed{9.9609}$$

8. (10 pts) Use Pascal's Triangle (Binomial Theorem) to expand the binomial power $(x-3)^5$. Expanding without using a recognizable version of this technique will earn at most 2 points.

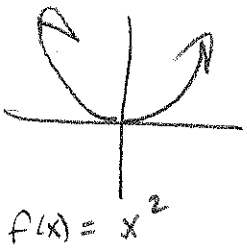
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

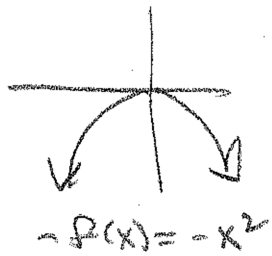
$(x-3)^5 =$

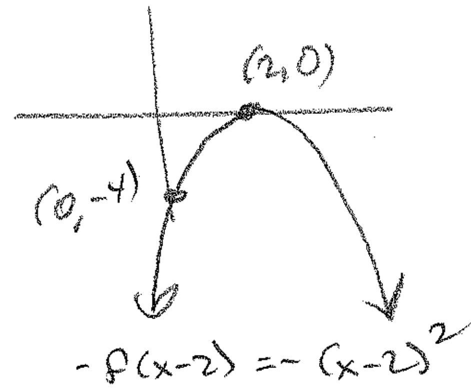
$x^5 + 5(x^4)(-3) + 10(x^3)(-3)^2$
$+ 10(x^2)(-3)^3 + 5(x)(-3)^4 + (-3)^5$

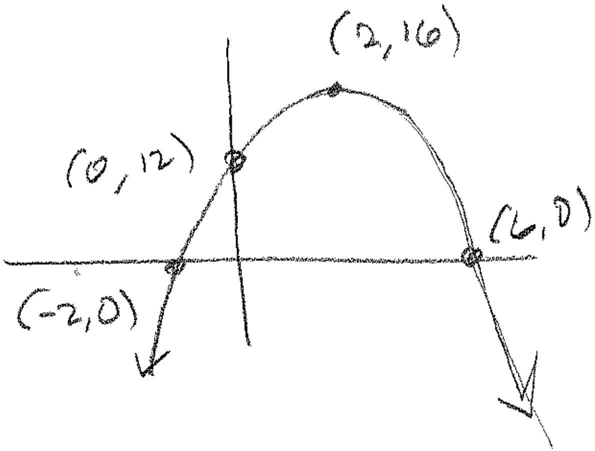
$= x^5 - 15x^4 + 90x^3 - 270x^2 + 405x - 243$

9. (10 pts) Graph $g(x) = -(x-2)^2 + 16$ using the techniques of shifting and reflecting. Start with the graph of the basic function $f(x) = x^2$ and show all stages. In the final graph, indicate (label as ordered pairs) the x- and y- intercepts.









$-(x-2)^2 = -16$

$(x-2)^2 = 16$

$x - 2 = \pm 4$

$x = 2 \pm 4$ → 6, -2

10. (15 pts) Solve the system of linear equations

$$\begin{aligned} 2x + 3y &= 7 \\ 3x - 4y &= -10 \end{aligned}$$