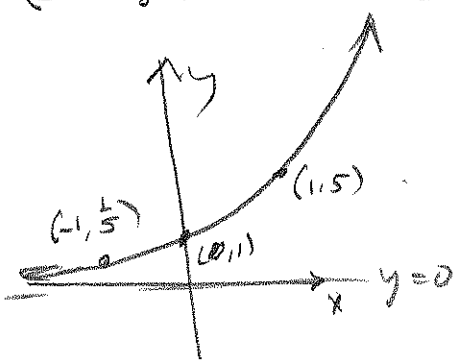
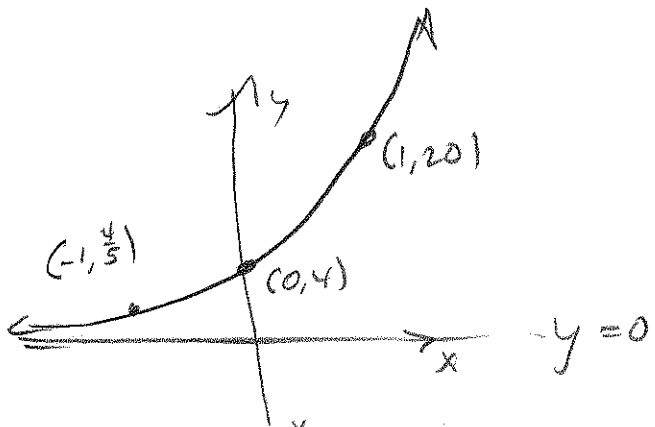
121 TEST 4 Spring '13

① $g(x) = 4 \cdot 5^{x+1} - 3$



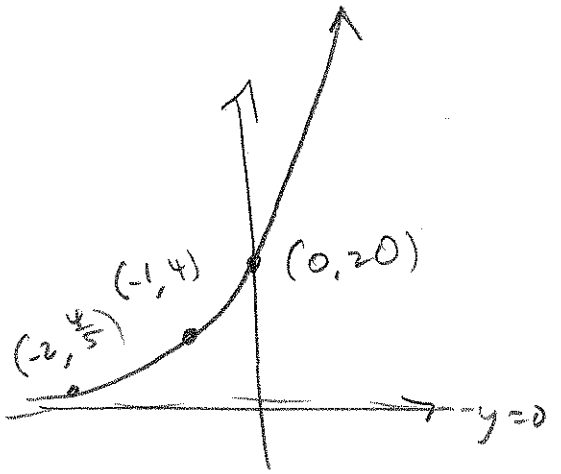
$f(x) = 5^x$

①



$4f(x) = 4 \cdot 5^x$

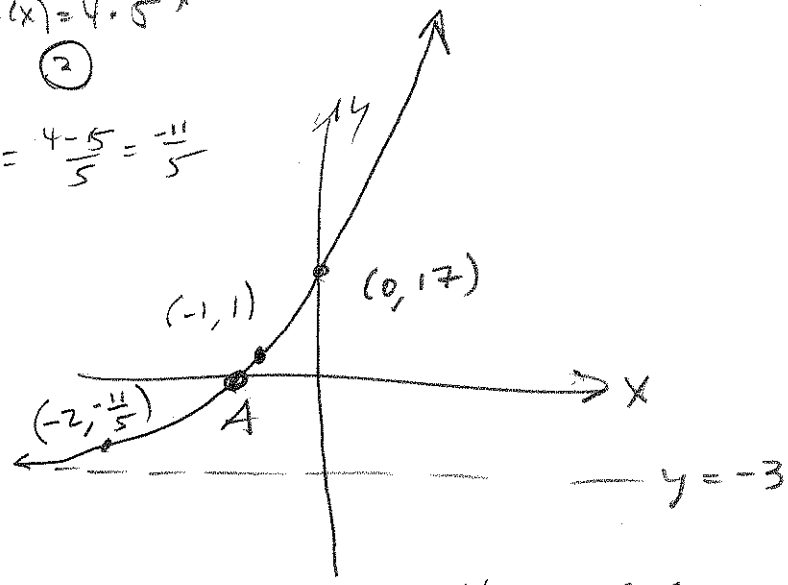
②



$4f(x+1) = 4 \cdot 5^{x+1}$

③

$\frac{4}{5} - 3 = \frac{4-5}{5} = \frac{-11}{5}$



$4 \cdot f(x+1) - 3 = 4 \cdot 5^{x+1} - 3 = g(x)$

$g(x) = 0$

$4 \cdot 5^{x+1} - 3 = 0$

$4 \cdot 5^{x+1} = 3$

$5^{x+1} = \frac{3}{4}$

$x + 1 = \log_5(3/4)$

$x = -1 + \log_5(3/4)$

$A = (-1 + \log_5(3/4), 0) \approx (-1.178746922, 0)$

(2) $f(x) = \sqrt{2x+1}$, $g(x) = \frac{1}{x-2}$ ⟹

(8pts) (a) $\mathcal{D}(f) = \{x \mid 2x+1 \geq 0\} = \{x \mid x \geq -\frac{1}{2}\} = [-\frac{1}{2}, \infty)$

(7pts) (b) $\mathcal{D}(g) = \{x \mid x \neq 2\} = (-\infty, 2) \cup (2, \infty)$

(c) (i) (5pts) $(f \circ g)(x) = \sqrt{2\left(\frac{1}{x-2}\right)+1}$

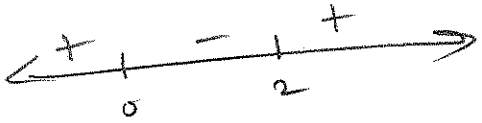
(ii) (5pts) $(g \circ f)(x) = \frac{1}{\sqrt{2x+1}-2}$

(d) (5pts) $\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ AND } g(x) \in \mathcal{D}(f)\}$

Need $x \neq 2$ and $\frac{1}{x-2} \geq -\frac{1}{2}$ ⟶

$\frac{1}{x-2} + \frac{1}{2} \geq 0$

$\frac{2 + x - 2}{2(x-2)} = \frac{x}{2(x-2)} \geq 0$



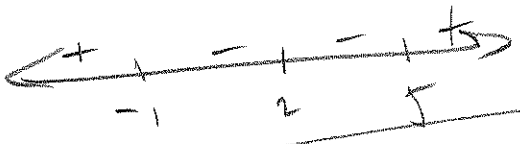
⟹ $x \neq 2$ and $(x \leq 0$ OR $x \geq 2)$

⟹ $\mathcal{D}(f \circ g) = \{x \mid x \leq 0 \text{ OR } x > 2\} = (-\infty, 0] \cup (2, \infty)$

121-G81 TEST 4

(3) (5pts) $f(x) = \sqrt{\dfrac{x+1}{(x-2)^2(x-5)}}$ →

$$\mathcal{D}(f) = \left\{ x \;\middle|\; \frac{x+1}{(x-2)^2(x-5)} \geq 0 \text{ AND } x \neq 2 \text{ AND } x \neq 5 \right\}$$



→ $\boxed{\mathcal{D}(f) = (-\infty, -1] \cup (5, \infty) = \{ x \mid x \leq -1 \text{ OR } x > 5 \}}$

(4) (5pts) $\boxed{f(x) = \ln x \text{ and } g(x) = x^2 - 3x + 2}$ →

$(f \circ g)(x) = \ln(x^2 - 3x + 2) = h(x)$

(5) (5pts) $y = e^{2x-5} - 7$;

$e^{2y-5} - 7 = x$

$e^{2y-5} = x + 7$

$2y - 5 = \ln(x+7)$

$2y = \ln(x+7) + 5$

$y = \boxed{\frac{1}{2}\left[\ln(x+7) + 5\right] = f^{-1}(x)}$

121-981 Test 4

(6) (a) $3-6+12-24+\cdots+192$

(5pts) $a=3,\ r=-2$

$192 = ar^{n-1} = 3\cdot 2^6 \longrightarrow$

$n-1=6 \longrightarrow n=7$

$S_7 = a\left(\frac{1-r^n}{1-r}\right) = 3\left(\frac{1-(-2)^7}{1-(-2)}\right)$

$= 3\left(\frac{1+2^7}{1+2}\right) = 3\left(\frac{129}{3}\right) = \boxed{129}$

$192 = 2\cdot 96 = 2\cdot 2\cdot 48 = 2\cdot 2\cdot 2\cdot 24 = 2\cdot 2\cdot 2\cdot 2\cdot 12 = 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 6 = 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 3$

(6) (5pts) $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^{n-1}$ $\quad a=1,\ r=\frac{1}{3}$

$S = \frac{a}{1-r} = \frac{1}{1-\frac{1}{3}} = \frac{1}{\frac{2}{3}} = \boxed{\frac{3}{2}}$

(7) (5pts) $\ln(x+1) + \ln(x-4) = \ln 6$

$\ln\left((x+1)(x-4)\right) = \ln 6$

$(x+1)(x-4) = 6$

$x^2 - 3x - 4 - 6 = 0$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x = 5$ or $x = -2$

$\ln(6) + \ln(5-4) = \ln 6$ ✓

$\boxed{x \in \{5\}}$

No. Not in domain

121 G-81 Test 4

8 a 10pts $A(t) = A_0 e^{kt}$

Given $A(7000) = A_0 e^{7000k} = \frac{1}{2} A_0$

$\Rightarrow e^{7000k} = \frac{1}{2}$

$\Rightarrow 7000k = \ln(1/2) = -\ln 2$

$\Rightarrow k = -\frac{\ln 2}{7000} \Rightarrow A(t) = A_0 e^{-\frac{\ln 2}{7000} t}$

$\approx A_0 e^{-.000099021025\,79t}$

b 5pts Want 100 grams to decay to 27 grams.

$100 e^{kt} = 27$ Solve for t.

$e^{kt} = \frac{27}{100} = .27$

$kt = \ln(.27)$

$t = \frac{\ln(.27)}{k} = \left(\frac{\ln(.27)}{\ln 2}\right)(-7000)$

$\approx 13,222.78081$

$\approx$ 13,223 years