

Test 3, Chapter 3 80 Points

Name \_\_\_\_\_

1.(5 pts) Form a polynomial in factored form with *real* coefficients with the given zeros and degree.

**Do not expand the polynomial.**

Zeros:  $x = 7$ , multiplicity 3;  $2 + i$ , multiplicity 1. Degree 5.

2. (10 pts) Expand  $(x + (2 + 5i))(x + (2 - 5i))$

3. (5 pts) Use synthetic division to find  $P(3)$  if  $P(x) = 2x^5 - 5x^4 + 12x^3 - 10x^2 + 11x - 5$ .

4. (5 pts) Divide  $P(x) = x^5 - 3x^4 + 4x^3 - 4x^2 - 62x + 5$  by  $d(x) = x^2 - 5$ . Then write the result in the form  $Dividend = Divisor \cdot Quotient + Remainder$ .

5. Let  $f(x) = x^6 + x^5 - 15x^4 - 5x^3 + 70x^2 - 12x - 72$ , and suppose its factored form is given by

$$f(x) = (x+1)(x-2)^3(x+3)^2.$$

- a. Sketch a graph of  $f$ , using its zeros, their multiplicities and anything else you can bring to bear, such as end behavior. Your graph should include all  $x$ - and  $y$ -intercepts, and be *smooth*, because *polynomials are smooth*.

- b. Use your sketch from the part a. to help you solve the following inequalities

i. (5 pts)  $7(x+1)(x-2)^3(x+3)^2 \leq 0$

ii. (5 pts)  $\frac{(x+1)}{(x-2)^3(x+3)^2} \leq 0$  (A very different-looking function, but not so very different, when it comes to solving inequalities).

6. (10 pts) Find the *real* zeros of  $f(x) = x^5 - 6x^4 + 10x^3 + 20x^2 - 51x + 26$ . Then factor  $f$  over the set of **real numbers**. The more knowledge of the theory you display, the more partial credit will be awarded, if your guesses don't work out so well. (Factor Theorem, Descartes' ..., Rational Zeros, etc.).

7. (5 pts) Find the remaining (nonreal) zeros of  $f$  and factor  $f$  over the set of **complex numbers**.

8. (10 pts) Suppose  $R(x) = \frac{x^3 + 3x^2 - x - 3}{x^3 - 2x^2 - 5x + 6}$  can be factored into

$R(x) = \frac{(x+3)(x+1)(x-1)}{(x-1)(x+2)(x-3)}$ . (It can.) Sketch the graph of  $R$  showing all intercepts, asymptotes and holes (if any).