MAT 121 – G81 Fall, 2012

Test 3, Chapter 3 80 Points Name KEY

1.(5 pts) Form a polynomial in factored form with *real* coefficients with the given zeros and degree.
Do not expand the polynomial.

Zeros: x = 7, multiplicity 3; 2 + *i*, multiplicity 1. Degree 5.

$$\boxed{(x-7)^3(x-(2+i))(x-(2-i))}$$

2. (10 pts) Expand $(x+(2+5i))(x+(2-5i))$

$= x^2+(2-5i)x+(2+5i)x+(2+5i)(2-5i)$

$= x^2+2x-5ix+2x+5ix+4+25$

$= \boxed{x^2+4x+29}$

$$\begin{array}{r} 116 \\ 3 \\ \hline 348 \end{array}$$

3. (5 pts) Use synthetic division to find $P(3)$ if $P(x)=2x^5-5x^4+12x^3-10x^2+11x-5$.

$$\begin{array}{r|rrrrrr} 3 & 2 & -5 & 12 & -10 & 11 & -5 \\ & & 6 & 3 & 45 & 105 & 348 \\ \hline & 2 & 1 & 15 & 35 & 116 & \boxed{343 = P(3)} \end{array}$$

4. (5 pts) Divide $P(x)=x^5-3x^4+4x^3-4x^2-62x+5$ by $d(x)=x^2-5$. Then write the result in the form *Dividend* = *Divisor* · *Quotient* + *Remainder*.

$$\begin{array}{r} x^3 -3x^2 +9x -19 \\ x^2-5 \overline{\big) x^5 - 3x^4 + 4x^3 - 4x^2 - 62x + 5} \\ -(x^5 \qquad -5x^3) \\ \hline -3x^4 + 9x^3 - 4x^2 - 62x + 5 \\ -(-3x^4 \qquad +15x^2) \\ \hline 9x^3 - 19x^2 - 62x + 5 \\ -(9x^3 \qquad -45x) \\ \hline -19x^2 - 17x + 5 \\ -(-19x^2 \qquad +95) \\ \hline -17x - 90 \end{array}$$

$\frac{x^5}{x^2} = x^3$ $\frac{9x^3}{x^2} = 9x$

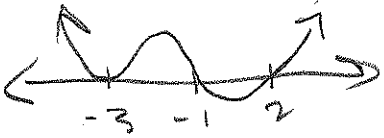
$\frac{-3x^4}{x^2} = -3x^2$ $\begin{array}{r} 62 \\ -45 \\ \hline 17 \end{array}$

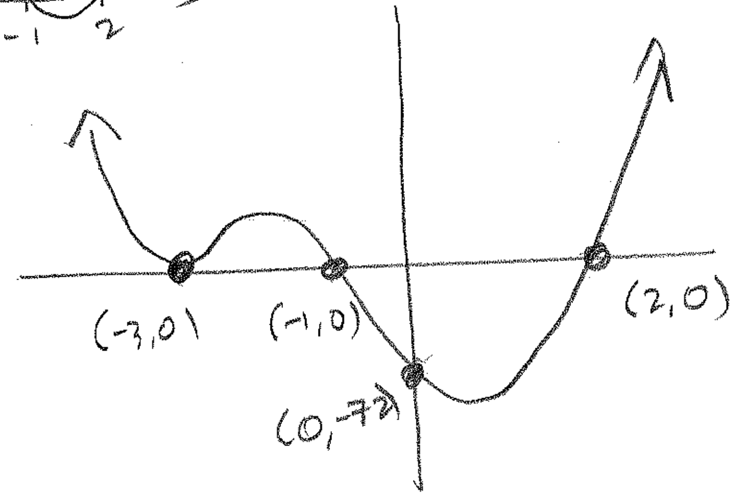
$\frac{-19x^2}{x^2} = -19$ $\begin{array}{r} {}^{4}19 \\ 5 \\ \hline 95 \end{array}$

$x^5-3x^4+4x^3-4x^2-62x+5 = (x^2-5)(x^3-3x^2+9x-19)-17x-90$

5. Let $f(x) = x^6 + x^5 - 15x^4 - 5x^3 + 70x^2 - 12x - 72$, and suppose its factored form is given by $f(x) = (x+1)(x-2)^3(x+3)^2$.

a. Sketch a graph of *f*, using its zeros, their multiplicities and anything else you can bring to bear, such as end behavior. Your graph should include all *x*- and *y*-intercepts, and be *smooth*, because *polynomials are smooth.*





b. Use your sketch from the part a. to help you solve the following inequalities

i. (5 pts) $7(x+1)(x-2)^3(x+3)^2 \le 0$

$\Rightarrow \boxed{x \in [-1, 2]}$

ii. (5 pts) $\dfrac{(x+1)}{(x-2)^3(x+3)^2} \le 0$ (A very different-looking function, but not so very different, when it comes to solving inequalities).

Can't let x = -2 OR -3

$\Rightarrow \boxed{x \in (-2, 1]}$

6. (10 pts) Find the *real* zeros of $f(x) = x^5 - 6x^4 + 10x^3 + 20x^2 - 51x + 26$. Then factor f over the set of **real numbers**. The more knowledge of the theory you display, the more partial credit will be awarded, if your guesses don't work out so well. (Factor Theorem, Descartes'..., Rational Zeros, etc.).

$\pm 1, \pm 2, \pm 13, \pm 26$

4, 2, or zero positive zeros

$f(-x) = -x^5 - 4x^4 - 10x^3 + 20x^2 + 51x + 26$

Exactly one negative zero.

1| 1 -6 10 20 -51 26
1 -5 5 25 -26

1| 1 -5 5 25 -26 0
1 -4 1 26

1| 1 -4 1 26 0
1 -3

1 -3 -2 Nope

$(x-1)^2(x^3-4x^2+x+26)$

-1| 1 -4 1 26
-1 5

1 -5 6 No

2| 1 -4 1 26
2 -4

1 -2 -3 No

-2| 1 -4 1 26
-2 12 -26

1 -6 13 0

$(x-1)^2(x+2)(x^2-6x+13)$, so far.

Let's see about $x^2-6x+13$:

$b^2-4ac = (-6)^2-4(1)(13)$
$= 36-52 = -16$

$\sqrt{-16} = 4i$

$x = \frac{6 \pm 4i}{2} = \frac{3 \pm 2i}{1} = 3 \pm 2i$

$\boxed{x = 1\ (m=2), 2,}$

$\boxed{(x-1)^2(x-2)(x^2-6x+13)}$

7. (5 pts) Find the remaining (nonreal) zeros of f and factor f over the set of **complex numbers**.

$(x-1)^2(x+2)(x^2-6x+13)$

$= (x-1)^2(x+2)(x-(3+2i))(x-(3-2i))$

$\boxed{x = 1\ (m=2), -2, 3 \pm 2i}$

8. (10 pts) Suppose $R(x)=\dfrac{x^3+3x^2-x-3}{x^3-2x^2-5x+6}$ can be factored into

$R(x)=\dfrac{(x+3)(x+1)(x-1)}{(x-1)(x+2)(x-3)}$. (It can.) Sketch the graph of R showing all intercepts, asymptotes and holes (if any).

$R(x)=\dfrac{(x+3)(x+1)}{(x+2)(x-3)}$ $x \neq 1$

$R(1)=\dfrac{(1+3)(1+1)}{(1+2)(1-3)}=\dfrac{(4)(2)}{(3)(-2)}$

$=\dfrac{8}{-6}=-\dfrac{4}{3}$

HOLE: $(1,-\frac{4}{3})$

$\boxed{\text{H.A.: } y=1}$

$\boxed{\text{V.A: } x=-2, x=3}$

x-int: $x=-3$, $x=-1$

$\boxed{(-3,0), (-1,0)}$

y-int: $(0,-\frac{3}{6})=(0,-\frac{1}{2})$