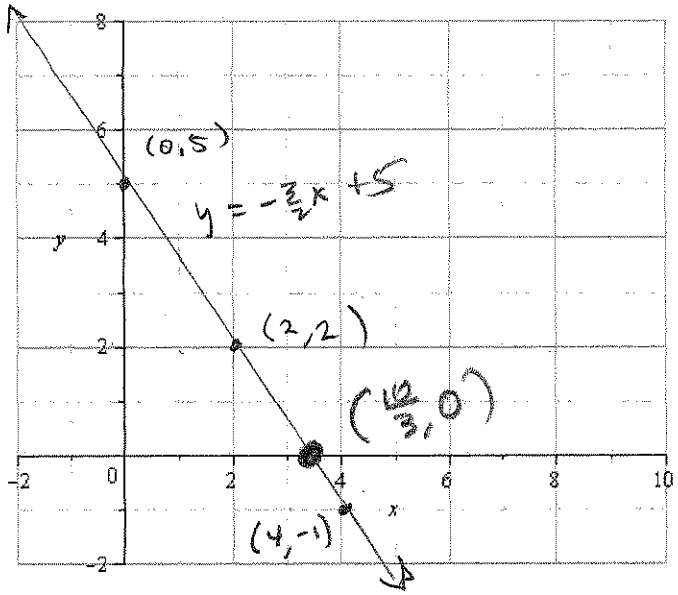
MAT 121-G81, Spring, 2013 Test 2
100 Points Name KEY

1. Let $f(x) = -\frac{3}{2}x + 5$ in the following:

a. (4 pts) Determine the slope and y-intercept of f.

$m = -\frac{3}{2}$, y-int: $(0,5)$

b. (4 pts) Use the slope and y-intercept to graph f here:



Down 3
Right 2

c. (4 pts) What's the x-intercept of f?

$-\frac{3}{2}x + 5 = 0$

$-\frac{3}{2}x = -5$

$x = \frac{10}{3} \rightsquigarrow \left(\frac{10}{3}, 0\right)$

d. (4 pts) Is f increasing, decreasing or constant?

Decreasing

2. Compute the discriminant for the following quadratic functions. Find how many zeroes does h have, and whether they are real, nonreal, one of each, or what have you.

a. (4 pts) $h(x) = 4x^2 - 12x + 13$

$b^2 - 4ac = (-12)^2 - 4(4)(13) = 144 - 208 = -64 < 0$

No real zeros, Two nonreal complex zeros

b. (4 pts) $h(x) = 3x^2 - 5x - 5$

$b^2 - 4ac = (-5)^2 - 4(3)(-5) = 25 + 60 = 85 > 0$

Two real zeros

3. Let $f(x)=15x^2+8x-12$.

a. (4 pts) Find the zeros of f by factoring.

$3|15$ $2|12$ Find a grouping
5 $2|6$ that adds to $+8$
3 $(3)(3)(2)-(5)(2)=8$

$15x^2+18x-10x-12$

$=3x(5x+6)-2(5x+6)$

$=(5x+6)(3x-2)=0$ ⟹

$x\in\{-\frac{6}{5},\frac{2}{3}\}$

b. (4 pts) Find the zeros of f by quadratic formula.

$b^2-4ac=8^2-4(15)(-12)$

$=64+720=784$

$\sqrt{784}=\sqrt{2^4\cdot 7^2}=2^2\cdot 7=28$

$2|784$ $2|392$ $2|196$ $2|98$ $7|49$ 7

$x=\frac{-8\pm 28}{2(15)}=\frac{-4\pm 14}{15}$

$-\frac{6}{5}=-\frac{18}{15}$ $\quad\frac{10}{15}=\frac{2}{3}$

$x\in\{-\frac{6}{5},\frac{2}{3}\}$ ✓

(4 pts) Find the zeros of $f(x)=x^2+6x-35$ by completing the square.

$x^2+6x=+35$

$x^2+6x+3^2=35+9$

$(x+3)^2=44$

$x+3=\pm\sqrt{44}=\pm 2\sqrt{11}$

$x=-3\pm 2\sqrt{11}$

Check: $b^2-4ac=6^2-4(1)(-35)$

$=36+140=176$

$\sqrt{176}=\sqrt{2^4\cdot 11}=2^2\sqrt{11}=4\sqrt{11}$

$x=\frac{-6\pm 4\sqrt{11}}{2}=-3\pm 2\sqrt{11}$ ✓

$2|176$ $2|88$ $2|44$ $2|22$ 11

4. (20 pts) Complete the square for $f(x) = x^2 - 6x + 2$, and re-write it in the form $a(x-h)^2 + k$. Sketch its graph, based on your work. Label the vertex, axis of symmetry, and x- and y-intercepts on your graph. State the range of f.

$f(x) = x^2 - 6x + 2$

$= x^2 - 6x + 3^2 - 9 + 2$

$= \boxed{(x-3)^2 - 7}$ SET $= 0$

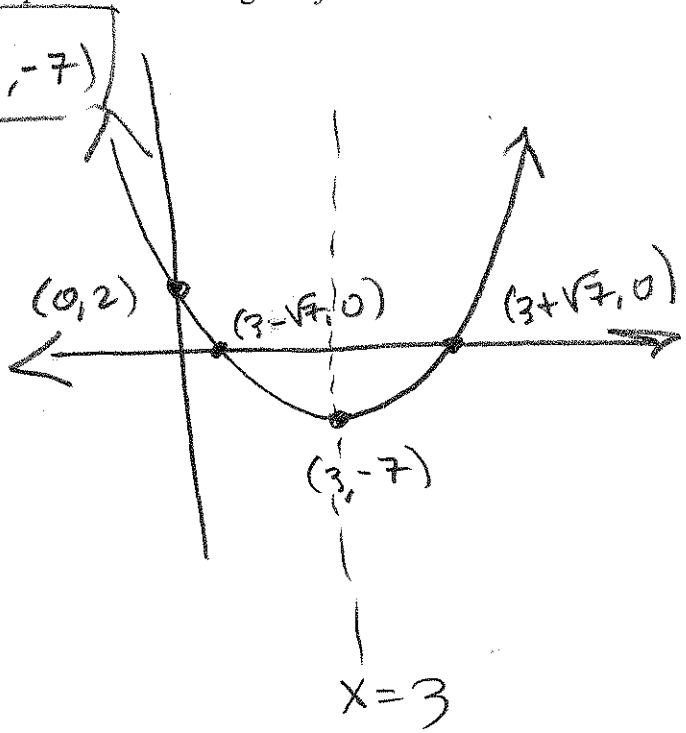
$x - 3 = \pm\sqrt{7}$

$\boxed{x = 3 \pm \sqrt{7}}$ zeros

$f(0) = 2 \rightarrow \boxed{(0,2)}$ y-int

$\boxed{(3-\sqrt{7}, 0),\ (3+\sqrt{7}, 0)}$ x-int

$(h,k) = (3,-7)$



5. (10 pts) Find the complex zeros of $f(x) = 9x^2 - 12x + 13$. Leave your answer in simplified radical form (no calculator stuff). **(5 bonus points if you solve it by completing the square)**

$b^2 - 4ac = (-12)^2 - 4(9)(13)$

$= 144 - 468 = -324$

$x = \dfrac{12 \pm \sqrt{-324}}{2(9)} = \dfrac{12 \pm 18i}{18}$

$= \boxed{\dfrac{2 \pm 3i}{3}} = \dfrac{2}{3} \pm i$

$9\left(x^2 - \frac{12}{9}x + \frac{13}{9}\right) = 0$

$x^2 - \frac{4}{3}x + \left(\frac{2}{3}\right)^2 = -\frac{13}{9} + \frac{4}{9}$

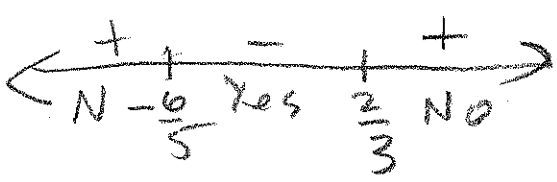
$\left(x - \frac{2}{3}\right)^2 = -1$

$x - \frac{2}{3} = \pm\sqrt{-1} = \pm i$

$\boxed{x = \frac{2}{3} \pm i}$ ✓

6. (10 pts) Solve $15x^2 < -8x + 12$. Express your answer in both set-builder and interval notation. You've already done about half the work on this one, in #3, on page 2.

$15x^2 + 8x - 12 < 0$



$\Rightarrow x \in \left(-\frac{6}{5}, \frac{2}{3}\right)$

$= \left\{x \mid -\frac{6}{5} < x < \frac{2}{3}\right\}$

Solve the absolute value equations and inequalities. (4 pts each). Same work for 7 – 9. Just interpret the results, differently.

7. $|7x+4| = 3$

$7x + 4 = \pm 3$

$7x = -4 \pm 3$

$x = \frac{-4 \pm 3}{7}$ → $\frac{-7}{7} = -1$, $\frac{-1}{7}$

$x \in \left\{-1, -\frac{1}{7}\right\}$

8. $|7x-5| \le 1$

$-1 \le 7x - 5 \le 1$

$4 \le 7x \le 6$

$\left\{x \mid \frac{4}{7} \le x \le \frac{6}{7}\right\}$

9. $|7x-5| > 1$

$7x - 5 > 1$ OR $7x - 5 < -1$

$7x > 6$ OR $7x < 4$

$\left\{x \mid x > \frac{6}{7} \text{ OR } x < \frac{4}{7}\right\}$

10. $|7x-5| = -1$

$\emptyset$

NEVER!

11. $|7x-5| \le -1$

$\emptyset$

NEVER!

12. $|7x-5| \ge -1$

$\mathbb{R} = (-\infty, \infty)$

Always!