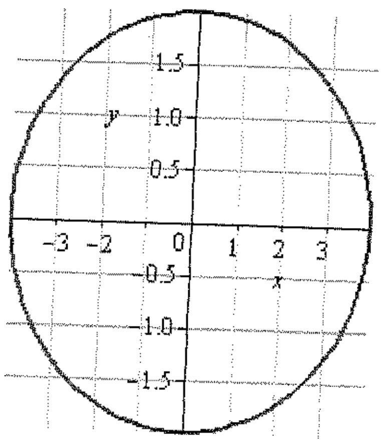
MAT 121, Spring, 2013
100 Points

**Test 1**

Name KEY

1. (5 pts) State whether the relation graphed below represents a function (Yes/No). If not, why not? What is the domain and what is the range of the relation?




No. (0,2) and (0,−2) are both members of the relation vertical line test failure, for another thing

$\mathcal{D} = [-4,4]$, $\mathcal{R} = [-2,2]$

2. (5 pts) Determine whether the equation $x^2 + 4y^2 = 16$ defines $y$ as a function of $x$. If it does *not*, show/explain why not, either by a general argument, or by finding an $x$-value in the domain that corresponds to more than one $y$-value in the range.

$4y^2 = 16 - x^2$

$y^2 = \frac{1}{4}(16 - x^2)$

$y = \pm \frac{1}{2}\sqrt{16 - x^2}$

$x = 0$ corresponds to

$y = \pm \frac{1}{2}\sqrt{16} = \pm \frac{1}{2}(4) = \pm 2.$

2 y's, one x.

3. Let $f(x) = x^2 - 6x + 8$ and $g(x) = \sqrt{3x - 6}$.

a. Determine each of the following functions. Do not simplify.

i. (5 pts) $(f + g)(x) = x^2 - 6x + 8 + \sqrt{3x-6}$

ii. (5 pts) $(f \cdot g)(x) = (x^2 - 6x + 8)\sqrt{3x-6}$

iii. (5 pts) $\left(\frac{f}{g}\right)(x) = \dfrac{x^2 - 6x + 8}{\sqrt{3x-6}}$

b. (5 pts) What is the domain of $\left(\frac{f}{g}\right)(x)$?

Need $3x - 6 \geq 0$ and $\sqrt{3x-6} \neq 0$ $\Big\} \Rightarrow 3x - 6 > 0$

$3x > 6$

$\boxed{\{x \mid x > 2\} = \mathcal{D}}$

4. (5 pts) Let $f(x) = x^2 + 5$. Simplify the difference quotient $\frac{f(x+h) - f(x)}{h}$.

$$= \frac{(x+h)^2+5 - (x^2+5)}{h} = \frac{x^2+2xh+h^2+5-x^2-5}{h}$$

$$= \frac{2xh+h^2}{h} = \frac{h(2x+h)}{h} = \boxed{2x+h}$$

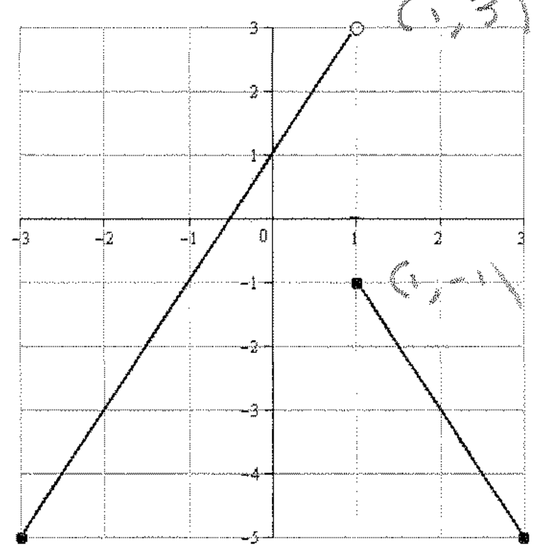
5. (5 pts) Find the average rate of change of $f$ from $x = 2$ to $x = 3$. (Hint: Let $h = 1$ and use your work from the previous problem, for an appropriate choice of $x$.)

Let $x = 2$, $h = 1$, then

$$m_{AVG} = \frac{f(2+h) - f(2)}{h} = \boxed{2(2) + 1 = 5} \text{ by \#4}$$

OR: $\frac{f(3)-f(2)}{3-2} = \frac{14-9}{1} = 5$

6. (5 pts) The graph of a piecewise-defined function is given. Write its definition.




$$m_1 = \frac{3-(-5)}{1-(-3)} = \frac{8}{4} = 2$$

$$y_1 = 2(x-(-3)) - 5 = 2x+6-5$$

$$y_1 = 2x+1$$

$$m_2 = \frac{-5-(-1)}{3-1} = \frac{-4}{2} = -2$$

$$y = -2(x-1) - 1 = -2x+2-1 = -2x+1$$

$$y_2 = -2x+1$$

$$\boxed{f(x) = \begin{cases} 2x+1 & \text{if } -3 \le x < 1 \\ -2x+1 & \text{if } 1 \le x \le 3 \end{cases}}$$

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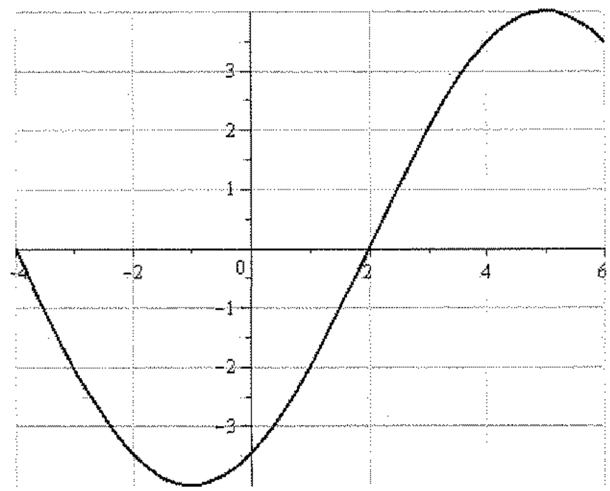
7. Use the graph of the function $f$, below, to answer the following questions. Assume you're seeing the *entire* function, and don't worry about what it might be doing off the edges.




a. (5 pts) $x$-intercept(s):

(-4,0), (2,0)

b. (5 pts) $y$-intercept(s):

(0, -3.5)

c. (5 pts) The domain and range:

D = [-4,6]

R = [-4,4]

d. (5 pts) $f$ has local minimum of -4 at x = -1.

e. (5 pts) $f$ has a local maximum of y = 4 at x = 5.

f. (5 pts) $f$ is increasing on (-1, 5).

g. (5 pts) $f$ is decreasing on (-4, -1) ∪ (5, 6).

8. (6 pts) Sketch the graph of $f(x) = \begin{cases} 3x+9 & \text{if } -5 \le x < -1 \\ 1 & \text{if } x = -1 \\ -x & \text{if } x > 1 \end{cases}$. Show all intercepts.

3(-5)+9 = -6   (-5, -6) •

3(-1)+9 = 6   (-1, 6) o

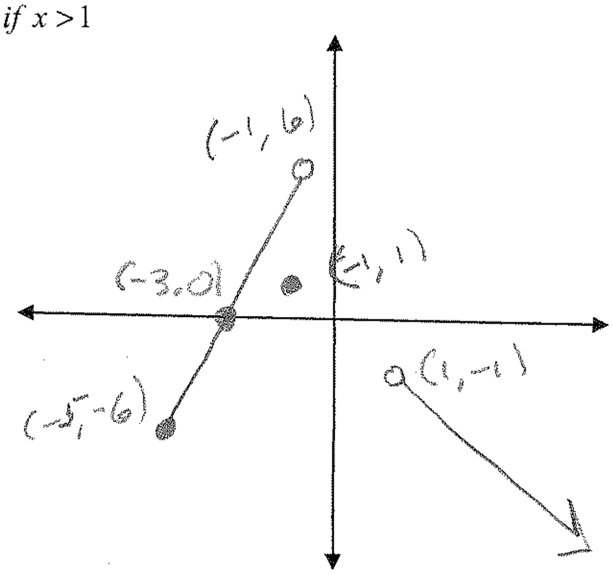
(-1, 1) •

(1, -1) o

3x+9 = 0

3x = -9

x = -3 → (-3, 0)

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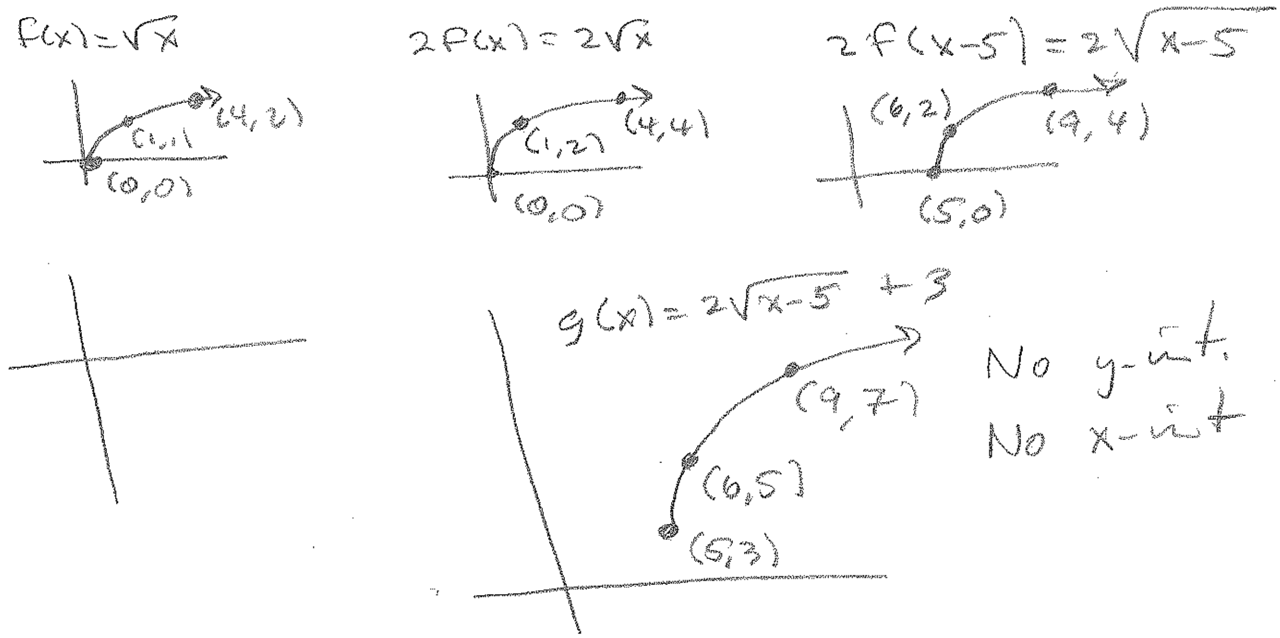
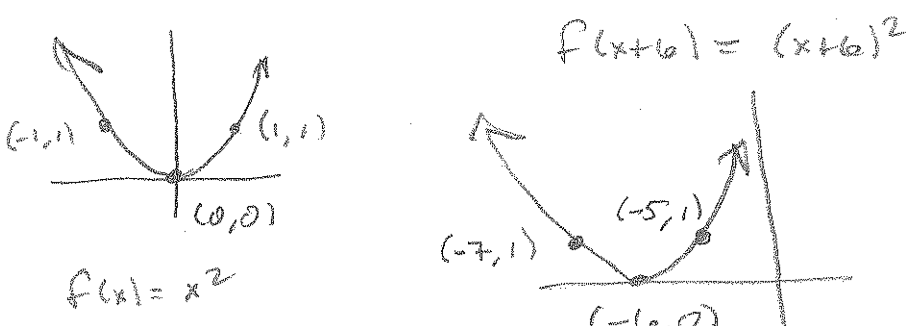
9. Graph each of the following functions using the techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function and show all stages in separate sketches. Track 3 key points through the transformations, and show the $y$-intercept in the final sketch.

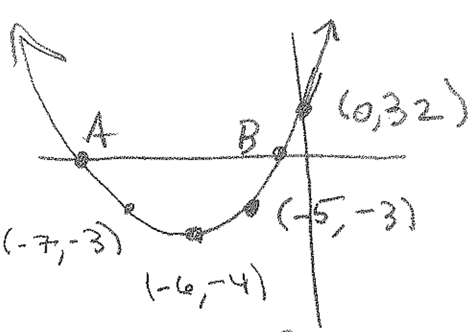
a. (7 pts) $g(x) = -2\sqrt{x-5} + 3$. (**2 pts bonus** – Show $x$-intercepts in final graph.)




No y-int.
No x-int

b. (7 pts) $g(x) = (x+6)^2 - 4$ (**2 pts bonus** – Show $x$-intercept(s) in final graph.)




$g(x) = (x+6)^2 - 4$

$g(0) = 6^2 - 4 = 36 - 4 = 32$

$(x+6)^2 - 4 = 0$

$(x+6)^2 = 4$

$x + 6 = \pm\sqrt{4} = \pm 2$

$x = -6 \pm 2$

$\boxed{\begin{array}{l}(-4, 0) = A \\ (-8, 0) = B\end{array}}$

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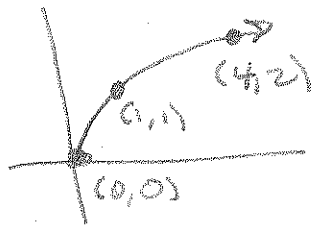
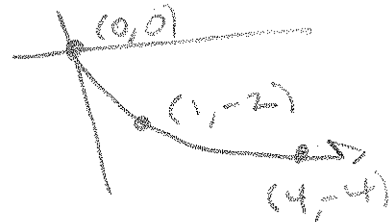
$g(x) = -2\sqrt{x-5} + 3$

$f(x) = \sqrt{x}$



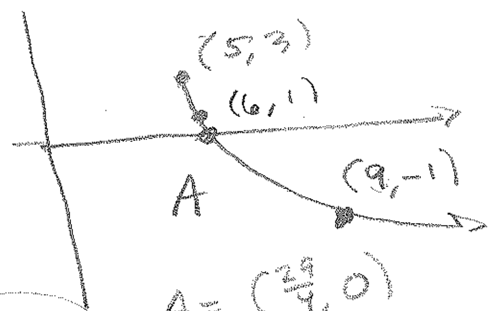

$-2f(x) = -2\sqrt{x}$




$-2f(x-5) = -2\sqrt{x-5}$


$-2f(x-5) + 3 = g(x) = -2\sqrt{x-5} + 3$

7pts




$A = (\frac{29}{4}, 0)$ OR $(7.25, 0)$

No y-int.

$-2\sqrt{x-5} + 3 = 0$

$-2\sqrt{x-5} = -3$

$\sqrt{x-5} = \frac{3}{2}$

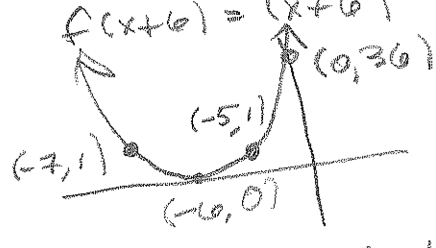
$x-5 = \frac{3^2}{2^2} = \frac{9}{4}$

$x = 5 + \frac{9}{4} = \frac{29}{4}$

$g(x) = (x+6)^2 - 4$

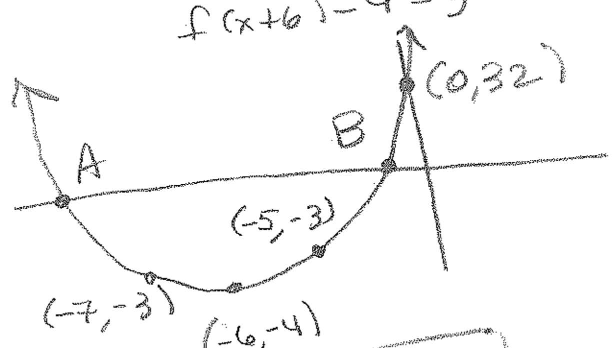
$f(x) = x^2$


$f(x+6) = (x+6)^2$




7pts

$f(x+6) - 4 = g(x) = (x+6)^2 - 4$




$A = (-8,0)$
$B = (-4,0)$

$(x+6)^2 - 4 = 0$

$(x+6)^2 = 4$

$\sqrt{(x+6)^2} = \sqrt{4}$

$|x+6| = 2$

$x+6 = \pm 2$

$x = -6 \pm 2$ → $(-8,0)$, $(-4,0)$