

1. In each of the following, form a polynomial with *real* coefficients that has the given zeros and degree. Please do not expand the polynomial.

- a. (5 pts) Zeros: -4, multiplicity 2; 2, multiplicity 3. Degree 5.

$$(x+4)^2(x-2)^3$$

- b. (5 pts) Zeros: 2, multiplicity 1; 5, multiplicity 2; $7-8i$, multiplicity 1. Degree 5.

$$(x-2)(x-5)^2(x-(7-8i))(x-(7+8i))$$

2. (5 pts) Expand $(x-(3+6i))(x-(3-6i)) = (x-3-6i)(x-3+6i)$

$$= x^2 - 3x + 6ix - 3x + 9 - 18i - 6ix + 18i - 36i^2$$

$$= x^2 - 6x + 9 + 36 = x^2 - 6x + 45$$

3. (5 pts) Use synthetic division to find $P(2)$ if $P(x) = x^4 - 5x^3 + 11x^2 - 12x + 13$.

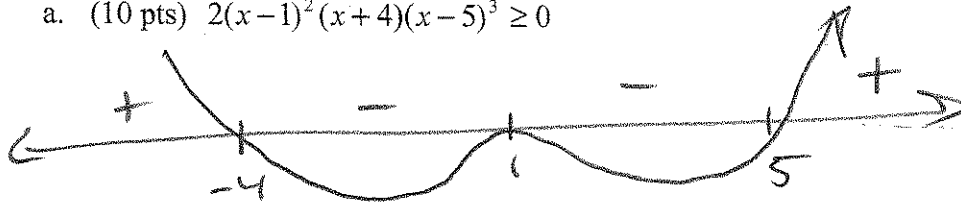
$$\begin{array}{r|rrrrr} 2 & 1 & -5 & 11 & -12 & 13 \\ & & 2 & -6 & 10 & -4 \\ \hline & 1 & -3 & 5 & -2 & 9 = P(2) \end{array}$$

4. (5 pts) Divide $f(x) = 2x^4 - 3x^3 + x - 3$ by $f(x) = x^2 - 1$

$$\begin{array}{r} 2x^2 - 3x + 2 \quad r = -2x - 1 \\ x^2 - 1 \overline{) 2x^4 - 3x^3 + 0x^2 + x - 3} \\ \underline{-(2x^4 \quad - 2x^2)} \\ -3x^3 + 2x^2 + x - 3 \\ \underline{-(-3x^3 \quad + 3x)} \\ 2x^2 - 2x - 3 \\ \underline{-(2x^2 \quad - 2x)} \\ -2x - 1 \end{array}$$

6. Solve the inequalities (Hint: You already laid the foundations for *both* of these in the previous problem.).

a. (10 pts) $2(x-1)^2(x+4)(x-5)^3 \geq 0$



$$x \in (-\infty, -4] \cup \{1\} \cup [5, \infty)$$

- b. (5 pts) $\frac{(x+4)(x-1)^2}{(x-5)^3} \geq 0$ (Hint: This one differs only *slightly* from the previous one.)

$$x \in (-\infty, -4] \cup \{1\} \cup (5, \infty)$$

8. (10 pts) Graph the function $R(x) = \frac{2x^3 - 3x^2 - 2x + 3}{x^3 - 4x^2 + x + 6} = \frac{(x-1)(2x-3)(\cancel{x+1})}{(x+1)(x-2)(x-3)}$. Key features are asymptotes, holes (if any) and intercepts. I was kind enough to factor it for you.

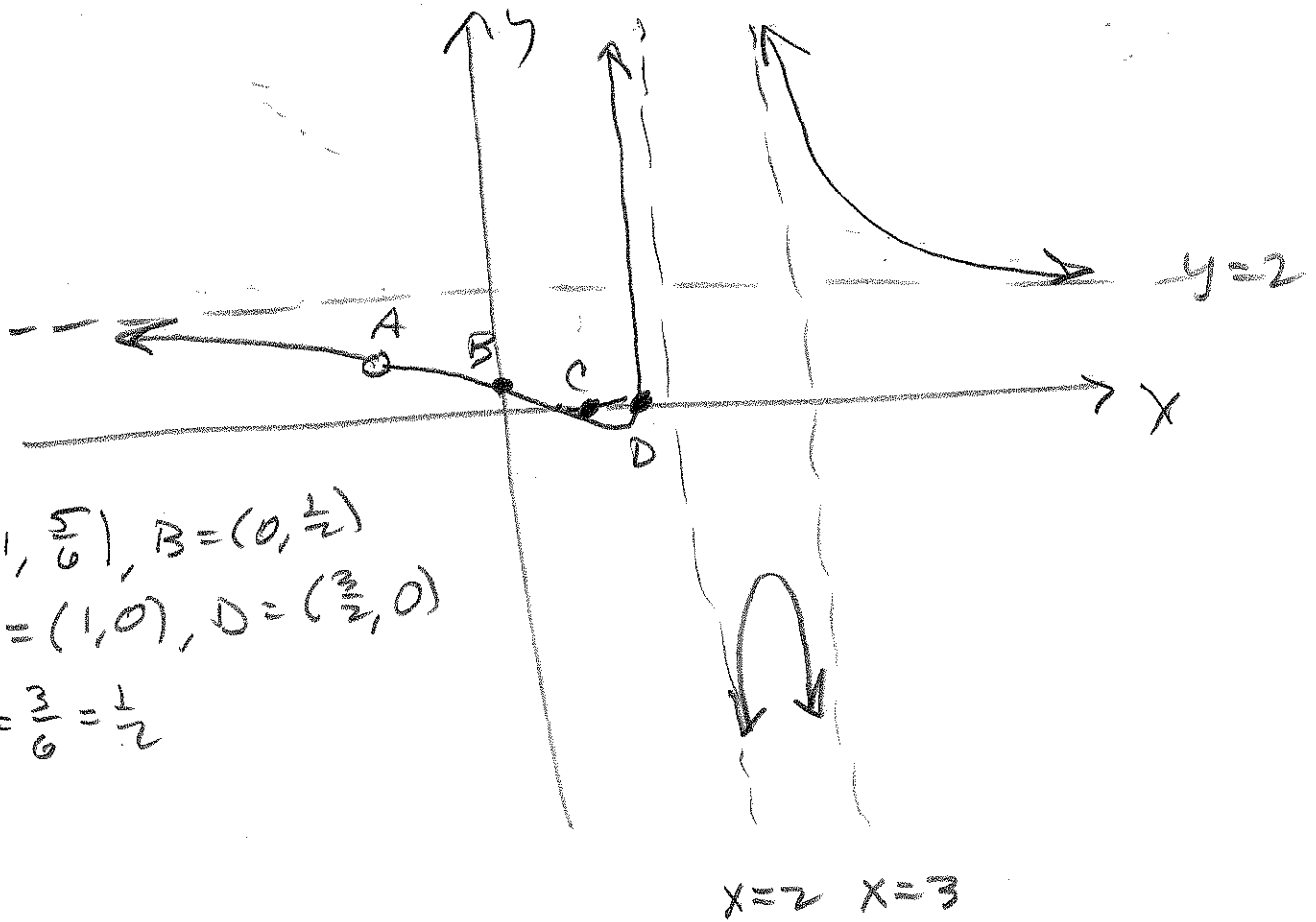
$D = \mathbb{R} \setminus \{-1, 2, 3\}$ ($x+1$'s cancel, so)

$x = -1$ is a hole $\frac{(-1-1)(2(-1)-3)}{(-1-2)(-1-3)} = \frac{-2(-5)}{-3(-4)} = \frac{10}{12} = \frac{5}{6}$

$x = 2, x = 3$ are vertical asymptotes. HOLE: $(-1, \frac{5}{6})$

End Behavior $\frac{n=3}{n=3} \Rightarrow y = \frac{2x^3}{x^3} = 2$ is Horizontal Asymptote

Zeros: $x = 1, x = \frac{3}{2}$



$A = (-1, \frac{5}{6}), B = (0, \frac{1}{2})$
 $C = (1, 0), D = (\frac{3}{2}, 0)$

$R(0) = \frac{3}{6} = \frac{1}{2}$

$x=2 \quad x=3$