121 Test 3 Take-Home/Makeup

(1) $P(x) = 5x^4 + 2x^3 - 3x + 121$

2	5	2	0	-3	121
		10	24	48	90
	5	12	24	45	211 = P(2)

(2) $(x-1)^2 (x-3)^5 (x+7)^{13} (x-(2-3i))(x-(2+3i))$

(From $x=1, m=2$; $x=3, m=5$; $x=-7, m=13$; $x=2-3i, m=1$ and coefficients on expansion are real)

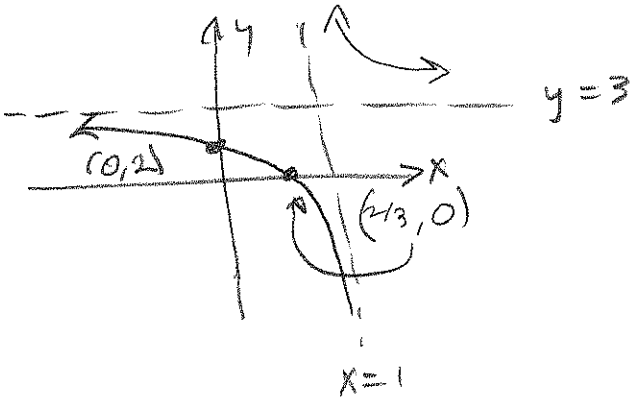
(3) $(x-(3-7i))(x-(3+7i))$

$= x^2 - (3+7i)x - (3-7i)x + (3-7i)(3+7i)$

$= x^2 - 3x - 7ix - 3x + 7ix + 3^2 + 7^2$

$= \boxed{x^2 - 6x + 58}$

(4) $y = \frac{3x-2}{x-1} = f(x)$

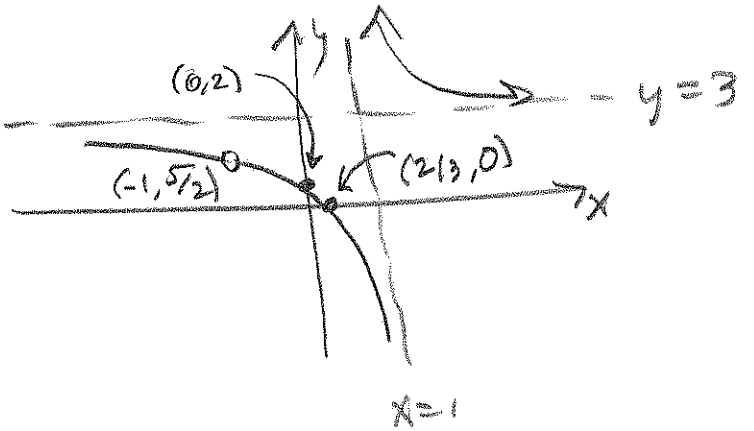


121 T3 T-H-M

(5) Same as #4, but hole @ $x=-1$:

$$g(x)=\left(\frac{3x-2}{x-1}\right)\left(\frac{x+1}{x+1}\right)$$

$\frac{3(-1)-2}{-1-1}=\frac{-3-2}{-2}=\frac{-5}{-2}=\frac{5}{2}$ ⇝ $(-1,\frac{5}{2})$ is hole.



(6) $x^2-7x-11>0$

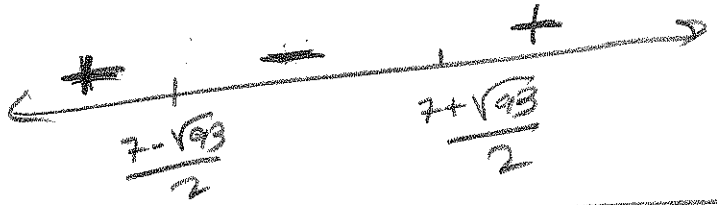
$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$a=1, b=-7, c=-11$

$b^2-4ac=(-7)^2-4(1)(-11)$
$=49+44$
$=93$

$=\frac{7\pm\sqrt{93}}{2}$

Want >0 "+"



$x\in(-\infty,\frac{7-\sqrt{93}}{2})\cup(\frac{7+\sqrt{93}}{2},\infty)$

(7) $f(x)=\frac{x-3}{x-5}$, $g(x)=\sqrt{x-7}$ ⟹ $(f\circ g)(x)=\frac{\sqrt{x-7}-3}{\sqrt{x-7}-5}$

$x-7\neq 25$
$x\neq 32$

$D(f\circ g)=\{x \mid x\in D(g)$ and $g(x)\in D(f)\}$

$=\{x \mid x\geq 7$ and $\sqrt{x-7}\neq 5\}$

$=\{x \mid x\geq 7$ & $x\neq 32\}=[7,32)\cup(32,\infty)$

Test 3 TH MU

(8) $f(x) = 4x^5 - 16x^4 + 49x^3 + 11x^2 - 35x - 13$

(a) $p: 13$, $q: 4$ $\Rightarrow \frac{p}{q}: \pm 1, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm 13, \pm\frac{13}{2}, \pm\frac{13}{4}$

(b) Descartes: 3 or 1 positive zeros.

$f(-x) = -4x^5 - 16x^4 - 49x^3 + 11x^2 + 35x - 13$

2 or 0 negative zeros.

(c) $x = 4$ is an upper bd on real zeros:

4	4	-16	49	11	-35	-13
		16	0	196	828	3172
	4	0	49	207	793	3159

Bottom row all nonnegative.

(d) We find all real zeros of f.

1	4	-16	49	11	-35	-13
		4	-12	37	48	13
	4	-12	37	48	13	0
$-\frac{1}{2}$		-2	7	-22	-13	
	4	-14	44	26		
$-\frac{1}{2}$		-2	8	-26		
	4	-16	52	0		

$f(x) = (x-1)(x+\frac{1}{2})^2(4x^2 - 16x + 52)$

real zeros:
$x = 1$, $x = -\frac{1}{2}$, $m = 2$

$a = 4, b = -16, c = 52$

$b^2 - 4ac = (-16)^2 - 4(4)(52)$

$= 256 - 832$

$= -576$

No more real zeros.

(c) $b^2-4ac=-576$

$\sqrt{-576}=24i$

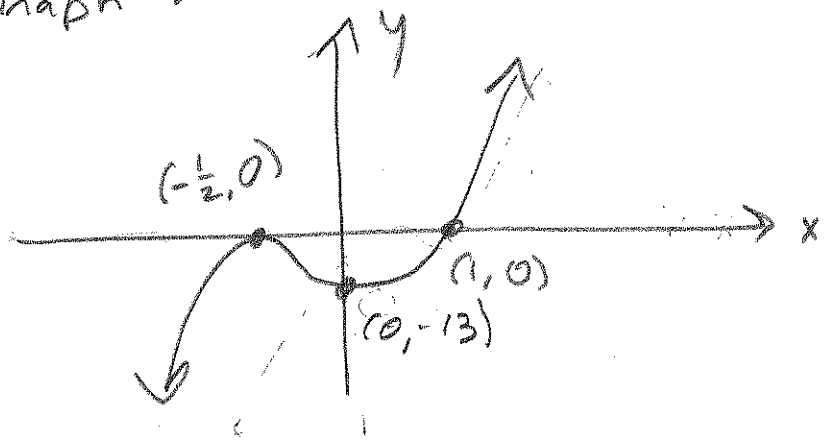
$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\dfrac{16\pm24i}{2(4)}=\dfrac{8(2\pm3i)}{8}=2\pm3i$

$x=2\pm3i$

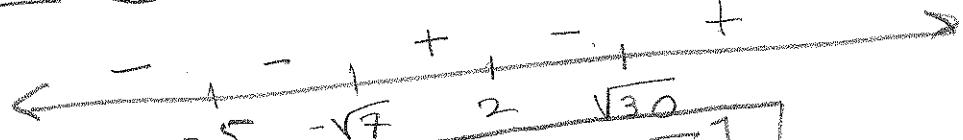
$f(x)=4(x-1)\left(x+\frac{1}{2}\right)^2(x-(2+3i))(x-(2-3i))$

(9) Graph:



BONUS:

B1 (5pts) $(x-2)(x+5)^2(x-\sqrt{30})(x+\sqrt{7})\le 0$



$x\in(-\infty,-\sqrt{7}]\cup[2,\sqrt{30}]$

B2 (5pts) $\dfrac{(x-\sqrt{30})(x+\sqrt{7})}{(x-2)(x+5)^2}\le 0$

By previous work and $x\neq2$ and $x\neq-5$, we have

$x\in(-\infty,-5)\cup(-5,-\sqrt{7}]\cup(2,\sqrt{30}]$