

Test 3 TH MU

(8) $f(x) = 4x^5 - 16x^4 + 49x^3 + 11x^2 - 35x - 13$

(a) $P = 13$
 $Q = 4 \Rightarrow \frac{P}{Q} = \pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 13, \pm \frac{13}{2}, \pm \frac{13}{4}$

(b) Descartes: 3 or 1 positive zeros.

$f(-x) = -4x^5 - 16x^4 - 49x^3 + 11x^2 + 35x - 13$

2 or 0 negative zeros.

(c) $x = 4, 13$ are upper bd on real zeros:

$$\begin{array}{r} 4 \overline{) 4 \quad -16 \quad 49 \quad 11 \quad -35 \quad -13} \\ \underline{16 \quad 0 \quad 196 \quad 828 \quad 3172} \end{array}$$

$$4 \quad 0 \quad 49 \quad 207 \quad 793 \quad 3154$$

Bottom row all nonnegative.

(d) We find all real zeros of f .

$$\begin{array}{r} 1 \overline{) 4 \quad -16 \quad 49 \quad 11 \quad -35 \quad -13} \\ \underline{4 \quad -12 \quad 37 \quad 48 \quad 13} \end{array}$$

$$\begin{array}{r} -\frac{1}{2} \overline{) 4 \quad -12 \quad 37 \quad 48 \quad 13 \quad 0} \\ \underline{-2 \quad 7 \quad -22 \quad -13} \end{array}$$

$$\begin{array}{r} -\frac{1}{2} \overline{) 4 \quad -14 \quad 44 \quad 26} \\ \underline{-2 \quad 8 \quad -26} \\ 4 \quad -16 \quad 52 \quad 0 \end{array}$$

$a = 4, b = -16, c = 52$

$b^2 - 4ac = (-16)^2 - 4(4)(52)$

$= 256 - 832$

$= -576$

No more real zeros.

$f(x) = (x-1)(x+\frac{1}{2})^2(4x^2-16x+52)$

real zeros:

$x = 1, x = -\frac{1}{2}, m = 2$