MAT 121 Test 3 - Fall, 2013 Name KEY

100 Points Covers Chapter 3

As this is a take-home, I expect high-quality work, so work these out on scratch paper and then write them up neatly, and with sufficiently dark text for me to read!

1. (5 pts) Use synthetic division to find $P(2)$ if $P(x)=5x^4+2x^3-3x+121$.

2	5	2	0	-3	121
		10	24	48	90
	5	12	24	45	$\boxed{211 = P(2)}$

2. (5 pts) Construct a polynomial (in factored form) of minimal degree that has *real* coefficients (if you expand it, but *don't* expand it!) and the following zeros, with the indicated multiplicities. Do *not* expand..

$x=1, m=2;\ x=3,\ m=5;\ \ x=-7, m=13;\ x=2-3i, m=1.$

$(x-1)^2(x-3)^5(x+7)^{13}(x-(2-3i))(x-(2+3i))$

3. (5 pts) Multiply (Expand) and simplify the product: $(x-(3-7i))(x-(3+7i))$.

$= x^2-(3+7i)x-(3-7i)x+(3-7i)(3+7i)$

$= x^2-3x-7ix-3x+7ix+3^2+7^2 \;=\; x^2-6x+9+49 = \boxed{x^2-6x+58}$

4. (5 pts) Sketch the graph of $\dfrac{3x-2}{x-1}$. Show all asymptotes and intercepts.

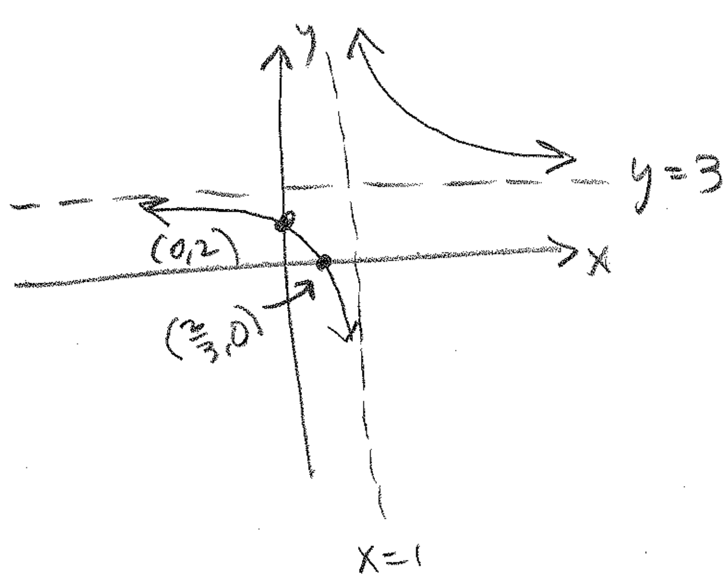
$D=\{x \mid x\neq 1\}$

$\boxed{\text{V.A.: } x=1}$

$\dfrac{3(0)-2}{0-1}=2 \rightsquigarrow \boxed{(0,2) \text{ is } y\text{-int}}$

$\dfrac{3x-2}{x-1} \xrightarrow{x\to\infty} \dfrac{3x}{x}=3$

$\boxed{y=3 \text{ is H.A.}}$



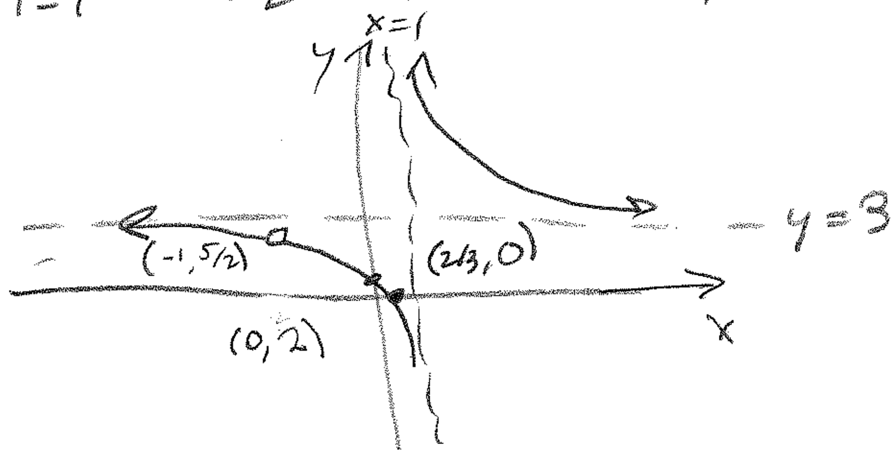
$3x-2=0$

$3x=2$

$x=\frac{2}{3} \rightsquigarrow \boxed{(\tfrac{2}{3},0) \text{ is } x\text{-int}}$

5. (5 pts) Based on your work on the previous problem, give a sketch of $\left(\frac{3x-2}{x-1}\right)\left(\frac{x+1}{x+1}\right)$

$\frac{3(-1)-2}{-1-1} = \frac{-3-2}{-2} = \frac{-5}{-2} = \frac{5}{2} \rightarrow (-1, \frac{5}{2})$ is hole



6. (5 pts) Solve the inequality $x^2 - 7x - 11 > 0$. Give answer in set-builder and interval notation.

$x^2 - 7x = 11$

$x^2 - 7x + \left(\frac{7}{2}\right)^2 = 11 + \frac{49}{4}$

$\left(x - \frac{7}{2}\right)^2 = \frac{44+49}{4} = \frac{93}{4}$

$x - \frac{7}{2} = \pm\frac{\sqrt{93}}{2}$

$x = \frac{7 \pm \sqrt{93}}{2}$

$3\,\overline{)93}$
31

$+ \quad \frac{7-\sqrt{93}}{2} \quad - \quad \frac{7+\sqrt{93}}{2} \quad +$

$x \in \left(-\infty, \frac{7-\sqrt{93}}{2}\right) \cup \left(\frac{7+\sqrt{93}}{2}\right)$

$\approx (-\infty, -1.321825380)$
$\cup (8.321825380, \infty)$

7. (10 pts) Let $f(x) = \frac{x-3}{x-5}$ and $g(x) = \sqrt{x-7}$. Form the composite function $(f \circ g)(x)$. Do not simplify.

What is the domain of $f \circ g$?

$(f \circ g)(x) = \frac{\sqrt{x-7} - 3}{\sqrt{x-7} - 5}$

$\sqrt{x-7} = 5$
$x - 7 = 25$
$x = 32$

$\mathcal{D}(f) = \mathbb{R} \setminus \{5\}$

$\mathcal{D}(g) = [7, \infty) = \{x \mid x \geq 7\}$

$\mathcal{D}(f \circ g) = \{x \mid x \geq 7 \text{ and } \sqrt{x-7} \neq 5\}$

$\mathcal{D} = \{x \mid x \geq 7 \text{ and } x \neq 32\}$
$= [7, 32) \cup (32, \infty)$

8. Let $f(x)=4x^5-16x^4+49x^3+11x^2-35x-13$

a. (10 pts) Find all possible rational zeros.

$\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 13, \pm \frac{13}{2}, \pm \frac{13}{4}$

b. (10 pts) Use Descartes' Rule to find the possible number of positive and negative zeros of f.

3 or 1 positive zeros

$f(-x) = -4x^5 - 16x^4 - 49x^3 + 11x^2 + 35x - 13$

2 or 0 neg.

c. (10 pts) Use synthetic division to show that $x=4$ is an upper bound on real zeros for f.

$$\begin{array}{r|rrrrrr} 4 & 4 & -16 & 49 & 11 & -55 & -13 \\ & & 16 & 0 & 196 & \text{BIGG} & \text{HUGE} \\ \hline & 4 & 0 & 49 & 207 & \text{BIGG} & \text{HUGE} \end{array}$$

All nonnegative.

$$\begin{array}{r} 49 \\ \times 4 \\ \hline 196 \\ 11 \\ \hline 207 \end{array}$$

d. (10 pts) Find all real zeros of f. Then factor f over the field of real numbers.

$$\begin{array}{r|rrrrrr} -\frac{1}{2} & 4 & -16 & 49 & 11 & -35 & -13 \\ & & -2 & 9 & -29 & 9 & 13 \\ \hline -\frac{1}{2} & 4 & -18 & 58 & -18 & -26 & 0 \\ & & -2 & 10 & -34 & 26 & \\ \hline 4 & 4 & -20 & 68 & -52 & 0 & \\ & & 4 & -16 & 52 & & \\ \hline & 4 & -16 & 52 & & & \end{array}$$

$$\boxed{\begin{array}{c} x = -\frac{1}{2},\ m=2;\ x=4,\ m=1 \\ \left(x+\frac{1}{2}\right)^2(x-4)(4x^2-16x+52) \end{array}}$$

e. (10 pts) Find the remaining nonreal zeros of f and factor f over the field of complex numbers.

$4x^2-16x+52=0$

$x^2-4x+13=0$

$x^2-4x=-13$

$x^2-4x+2^2=-13+4$

$(x-2)^2=-9$

$x-2=\pm 3i$ → $2+3i$, $2-3i$

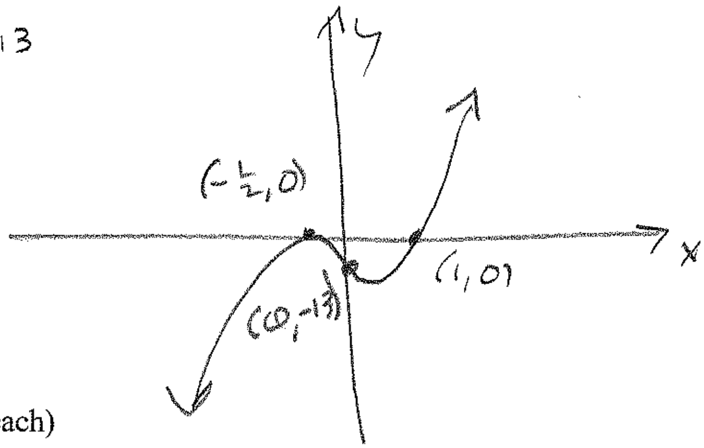
$x \in \{2 \pm 3i\}$

$f(x)=4(x+\frac{1}{2})^2(x-1)(x-(2+3i))(x-(2-3i))$

9. (10 pts) Based on your work in the previous problem, provide a rough sketch of f.

$f(0)=-13$

$(0,-13)$



Bonus (5 pts each)

a. Solve the inequality: $(x-2)(x+5)^2(x-\sqrt{30})(x+\sqrt{7}) \le 0$

b. Solve the inequality: $\dfrac{(x-\sqrt{30})(x+\sqrt{7})}{(x-2)(x+5)^2} \le 0$

(a)

$- \quad - \quad + \quad - \quad +$

$-5 \quad -\sqrt{7} \quad 2 \quad \sqrt{30}$

$\boxed{x \in (-\infty, -\sqrt{7}] \cup [2, \sqrt{30}]}$

(b) $x \in \boxed{(-\infty,-5) \cup (-5,-\sqrt{7}] \cup (2, \sqrt{30}]}$