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MAT 121
100 Points

Covers Chapter 1

Test 1 - Fall, 2013

(5 pts) Name Key

Find all real or imaginary solutions in #s 1 - 5..

1. (5 pts) $3x + 2 = -x - 5$

$$4x = -7$$

$$x = -\frac{7}{4}$$

$$x \in \left\{ -\frac{7}{4} \right\}$$

2. (5 pts) $\frac{2}{3}x - \frac{1}{4} = \frac{5}{6}$ LCD = 12

$$\frac{2x}{3} \cdot \frac{4}{4} - \frac{1}{4} \cdot \frac{3}{3} = \frac{5}{6} \cdot \frac{2}{2}$$

$$\frac{8x - 3}{\text{LCD}} = \frac{10}{\text{LCD}}$$

$$8x - 3 = 10$$

$$8x = 13$$

$$x = \frac{13}{8}$$

$$x \in \left\{ \frac{13}{8} \right\}$$

3. (5 pts) $3x^2 = 5$

$$x^2 = \frac{5}{3}$$

$$x = \pm \sqrt{\frac{5}{3}} = \pm \frac{\sqrt{5}}{\sqrt{3}} = \pm \frac{\sqrt{5} \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \pm \frac{\sqrt{15}}{3} \Rightarrow x \in \left\{ \pm \frac{\sqrt{15}}{3} \right\}$$

4. (5 pts) $3x^2 + 6x + 13 = 0$

$$a = 3, b = 6, c = +13$$

$$b^2 - 4ac = 6^2 - 4(3)(13)$$

$$= 36 - 156 = -120$$

$$\sqrt{-120} = 2\sqrt{30}$$

2 | 20
2 | 60
2 | 30
3 | 15
5

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-6 \pm 2i\sqrt{30}}{2(3)}$$

$$= \frac{-3 \pm i\sqrt{30}}{3}$$

$$\Rightarrow x \in \left\{ \frac{-3 \pm i\sqrt{30}}{3} \right\}$$

5. (10 pts) Compute the discriminant for each of the following equations and tell me what it tells you about the solutions of the equations, *without having to solve them*, i.e., don't solve.

a. $x^2 - 6x - 5 = 0$

$$a=1, b=-6, c=-5$$

$$b^2 - 4ac = (-6)^2 - 4(1)(-5)$$

$$= 36 + 20 = 56$$

~~$$56 = 8 \cdot 7 = 2 \cdot 2 \cdot 2 \cdot 7$$~~

~~$$\sqrt{56} = 2\sqrt{14}$$~~

2 real solms

$$(56 > 0)$$

b. $x^2 + 6x + 17 = 0$

$$a=1, b=6, c=17$$

$$b^2 - 4ac = 6^2 - 4(1)(17)$$

$$= 36 - 68 = -32$$

$$-32 < 0$$

2 nonreal solms

6. (10 pts) Solve $x^2 + 6x - 17 = 0$ by completing the square.

$$x^2 + 6x = 17$$

$$x^2 + 6x + 3^2 = 17 + 9$$

$$(x+3)^2 = 26$$

$$x+3 = \pm \sqrt{26}$$

$$x = -3 \pm \sqrt{26}$$

$$x \in \left\{ -3 \pm \sqrt{26} \right\}$$

7. (5 pts) Find an equation of the line through $(-3,1)$ and $(2,7)$. Point-slope is preferred, but not required.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 1}{2 - (-3)} = \frac{6}{5}$$

$$\boxed{y = m(x - x_1) + y_1}$$

$$\boxed{y = \frac{6}{5}(x + 3) + 1} = \frac{6}{5}x + \frac{23}{5}$$

8. (5 pts) Find an equation of the line thru $(3,5)$ that is perpendicular to the line $y = \frac{4}{7}x - 11$.

$$m = \frac{4}{7} \implies m_{\perp} = -\frac{7}{4}$$

$$y = m(x - x_1) + y_1 \implies$$

$$\boxed{y = -\frac{7}{4}(x - 3) + 5} = -\frac{7}{4}x + \frac{21 + 20}{4} = -\frac{7}{4}x + \frac{41}{4}$$

9. (5 pts) Sketch the graph of the line $y = \frac{4}{7}x - 11$.

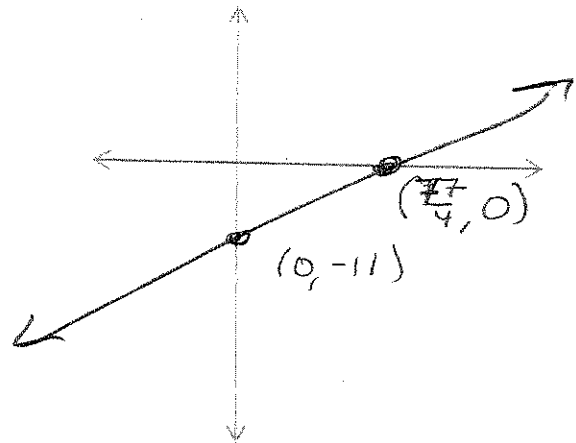
x	y
0	-11
$\frac{77}{4}$	0

$$y = 0:$$

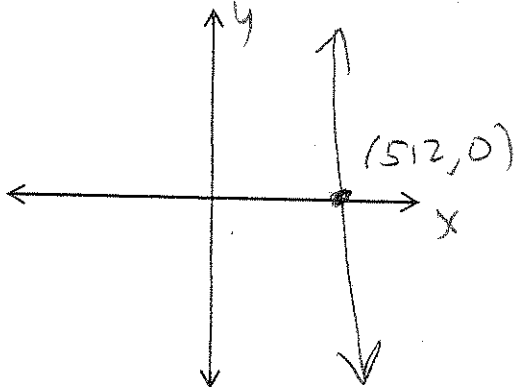
$$\frac{4}{7}x - 11 = 0$$

$$\frac{4}{7}x = 11$$

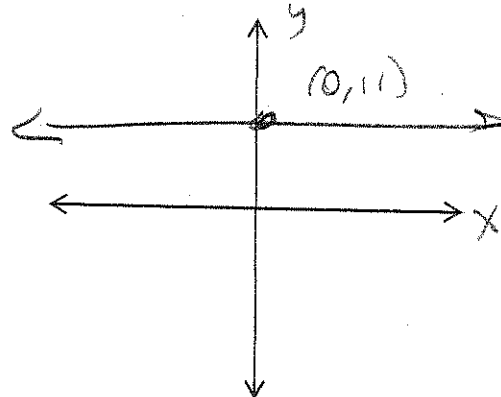
$$x = \frac{11 \cdot 7}{4} = \frac{77}{4}$$



10. (5 pts) Sketch the graph of the line $x = 512$



11. (5 pts) Sketch the graph of the line $y = 11$



Solve the inequalities.

12. (5 pts) $-3x - 5 \geq 4$

$$-3x \geq 9$$

$$x \leq -3$$

$$\{x \mid x \leq -3\} \text{ OR } (-\infty, -3]$$

13. (5 pts) $|2x - 3| \geq 7$

$$2x - 3 \geq 7 \text{ OR } 2x - 3 \leq -7$$

$$2x \geq 10 \text{ OR } 2x \leq -4$$

$$x \geq 5 \text{ OR } x \leq -2$$

$$x \in \{x \mid x \leq -2 \text{ OR } x \geq 5\}$$

$$= (-\infty, -2] \cup [5, \infty)$$

14. (5 pts) $|2x - 3| < 7$

$$2x - 3 < 7 \text{ AND } 2x - 3 > -7$$

$$2x < 10 \text{ AND } 2x > -4$$

$$x < 5 \text{ AND } x > -2$$

$$\{x \mid x < 5 \text{ and } x > -2\}$$

$$= (-2, 5)$$

15. (5 pts) $|2x - 3| < -7$

Never

 \emptyset

16. (5 pts) $|2x - 3| \geq -7$

Always

 \mathbb{R}

17. (5 pts) Suppose population growth in a small town is linear (a straight line). Also suppose the population was 10,000 in 1998 and 12,000 in 2011. Model the town's population (in thousands) as a function of time (in years after 1998). Then use your model to predict the population in 2014.

$$x = \# \text{ of years after 1998}$$

$$y = \text{Pop, in thousands}$$

$$\frac{\Delta \text{Dependent}}{\Delta \text{Independent}} = m$$

$$(x_1, y_1) = (0, 10)$$

$$(x_2, y_2) = (13, 12)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{12 - 10}{13 - 0} = \frac{2}{13}$$

$$y = m(x - x_1) + y_1$$

$$y = \frac{2}{13}(x - 0) + 10$$

$$y = \frac{2}{13}x + 10$$

In 2014, $x = 2014 - 1998 = 16$

$$\Rightarrow y = f(16) = \frac{2}{13}(16) + 10$$

$$= \frac{32}{13} + \frac{130}{13} = \frac{162}{13}$$

$$\approx 12.4615 \rightarrow 12,462 \text{ people}$$

18. (5 pts) How many liters of 15% alcohol must be added to 90 liters of 47% alcohol to obtain a mixture of

35% alcohol?	conc.	vol.	Alcohol
15%	.15	x	.15x
47%	.47	90	(.47)(90)
35%	.35	x+90	.35(x+90)

Let x = amount of 15% alcohol (in liters)

$$.15x + .47(90) = .35x + .35(90)$$

$$-.2x = .35(90) - .47(90) = -.12(90) = -10.8$$

$$x = \frac{-10.8}{-.2} = 54 \text{ liters of 15\% alcohol}$$

BONUS Page. Work *one* of the following. Expect one or all three types on the next test.

BONUS (10 pts) Suppose I take 5 hours to do a job that Kelli can do in 4 hours. Then on top of that, I start work one hour late! How many hours does Kelli end up spending on the job, until it's finished? Hint: If you take the average of our times, you're doing it wrong.



BONUS (10 pts) Re-write the function $f(x) = x^2 + 6x + 17$ in the form

$f(x) = a(x-h)^2 + k$. State the vertex of this parabola.

BONUS (10 pts) Re-write the function $g(x) = 3x^2 + 6x - 13$ in the form $g(x) = a(x-h)^2 + k$. State the vertex of this parabola.

① $x = \#$ of Hours Kelli spent
 $\rightarrow x-1 = \#$ of hours I spent

$$\frac{1}{4}x + \frac{1}{5}(x-1) = 1$$

$$\left[\frac{1}{4}x + \frac{1}{5}(x-1) = 1 \right] (20)$$

$$5x + 4(x-1) = 20$$

$$5x + 4x - 4 = 20$$

$$9x = 24$$

$$x = \frac{24}{9} \text{ hr}$$

$$\textcircled{3} 3x^2 + 6x - 13$$

$$= 3(x^2 + 2x) - 13$$

$$= 3(x^2 + 2x + 1 - 1)$$

$$= 3(x^2 + 2x + 1) - 3 - 13$$

$$= 3(x+1)^2 - 16$$

$$(h, k) = (-1, -16)$$

$$\textcircled{2} x^2 + 6x + 17$$

$$= x^2 + 6x + 3^2 - 9 + 17$$

$$= (x+3)^2 + 8$$

$$(h, k) = (-3, 8)$$