$8 \div 4\,3$

MAT 121 Test 1 - Fall, 2013 (5 pts) Name______________________________
100 Points Covers Chapter 1

Find all real or imaginary solutions in #s 1 – 5..

1. (5 pts) $3x+2=-x-5$

$4x=-7$

$x=-\frac{7}{4}$

$\boxed{x \in \left\{-\frac{7}{4}\right\}}$

2. (5 pts) $\frac{2}{3}x-\frac{1}{4}=\frac{5}{6}$ LCD $=12$

$\frac{2x}{3}\cdot\frac{4}{4}-\frac{1}{4}\cdot\frac{3}{3}=\frac{5}{6}\cdot\frac{2}{2}$

$\frac{2x-3}{LCD}=\frac{10}{LCD}$

$2x-3=10$

$2x=13$

$x=\frac{13}{2}$

$\boxed{x \in \left\{\frac{13}{2}\right\}}$

3. (5 pts) $3x^2=5$

$x^2=\frac{5}{3}$

$x=\pm\sqrt{\frac{5}{3}}=\pm\frac{\sqrt{5}}{\sqrt{3}}=\pm\frac{\sqrt{5}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\pm\frac{\sqrt{15}}{3} \rightarrow \boxed{x \in \left\{\pm\frac{\sqrt{15}}{3}\right\}}$

4. (5 pts) $3x^2+6x+13=0$

$a=3, b=6, c=+13$

$b^2-4ac=6^2-4(3)(+13)$

$=36-156=-120$

$\sqrt{-120}=2i\sqrt{30}$

$2|120$
$2|60$
$2|30$
$3|15$
5

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{-6\pm2i\sqrt{30}}{2(3)}$

$=\frac{-3\pm i\sqrt{30}}{3}$

$\rightarrow x \in \left\{\frac{-3\pm i\sqrt{30}}{3}\right\}$

5. (10 pts) Compute the discriminant for each of the following equations and tell me what it tells you about the solutions of the equations, *without having to solve them*, i.e., don't solve.

a. $x^2 - 6x - 5 = 0$

$a=1, b=-6, c=-5$

$b^2-4ac = (-6)^2-4(1)(-5)$

$= 36+20 = 56$

$\cancel{56 = 8 \cdot 7 = 2 \cdot 2 \cdot 2 \cdot 7}$

$\cancel{\sqrt{56} = 2\sqrt{14}}$

2 real solns

$(56 > 0)$

b. $x^2 + 6x + 17 = 0$

$a=1, b=6, C=17$

$b^2-4ac = 6^2-4(1)(17)$

$= 36-68 = -32$

$-32 < 0$

2 nonreal sol'ns

6. (10 pts) Solve $x^2 + 6x - 17 = 0$ by completing the square.

$x^2+6x = 17$

$x^2+6x+3^2 = 17+9$

$(x+3)^2 = 26$

$x+3 = \pm\sqrt{26}$

$x = -3 \pm \sqrt{26}$

$\boxed{x \in \{-3 \pm \sqrt{26}\}}$

7. (5 pts) Find an equation of the line through $(-3,1)$ and $(2,7)$. Point-slope is preferred, but not required.

$m = \frac{y_2-y_1}{x_2-x_1} = \frac{7-1}{2-(-3)} = \frac{6}{5}$

$y = m(x-x_1)+y_1$

$\boxed{y = \frac{6}{5}(x+3)+1}$

8. (5 pts) Find an equation of the line thru $(3,5)$ that is perpendicular to the line $y = \frac{4}{7}x - 11$.

$m = \frac{4}{7} \Longrightarrow m_{\perp} = -\frac{7}{4}$

$y = m(x - x_1) + y_1 \Longrightarrow$

$\boxed{y = -\frac{7}{4}(x-3) + 5}$

9. (5 pts) Sketch the graph of the line $y = \frac{4}{7}x - 11$.

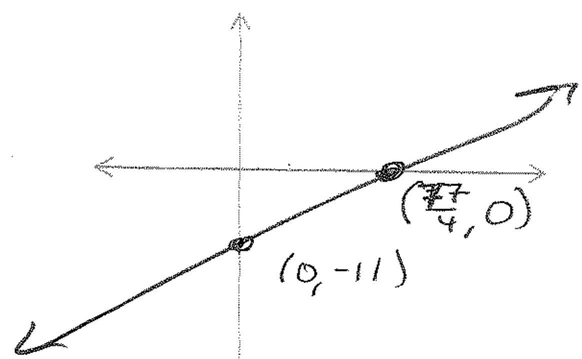
x	y
0	-11
$\frac{77}{4}$	0

$y = 0$:

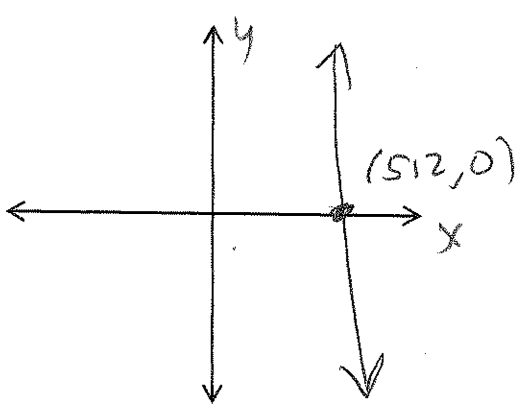
$\frac{4}{7}x - 11 = 0$

$\frac{4}{7}x = 11$

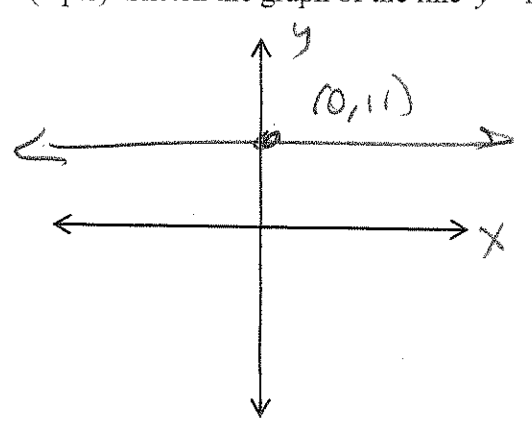
$x = \frac{11}{1} \cdot \frac{7}{4} = \frac{77}{4}$



10. (5 pts) Sketch the graph of the line $x = 512$



11. (5 pts) Sketch the graph of the line $y = 11$



Solve the inequalities.

12. (5 pts) $-3x-5\geq 4$

$-3x \geq 9$

$x \leq -3$

$\boxed{\{x \mid x \leq -3\} \text{ OR } (-\infty, -3]}$

13. (5 pts) $|2x-3|\geq 7$

$2x-3 \geq 7$ OR $2x-3 \leq -7$

$2x \geq 10$ OR $2x \leq -4$

$x \geq 5$ OR $x \leq -2$

$\boxed{x \in \{x \mid x \leq -2 \text{ OR } x \geq 5\} = (-\infty,-2] \cup [5,\infty)}$

14. (5 pts) $|2x-3|<7$

$2x-3 < 7$ AND $2x-3 > -7$

$2x < 10$ AND $2x > -4$

$x < 5$ AND $x > -2$

$\boxed{\{x \mid x < 5 \text{ and } x > -2\} = (-2,5)}$

15. (5 pts) $|2x-3|<-7$

Nevah!

$\boxed{\emptyset}$

16. (5 pts) $|2x-3|\geq -7$

Always $\boxed{\mathbb{R}}$

17. (5 pts) Suppose population growth in a small town is linear (a straight line). Also suppose the population was 10,000 in 1998 and 12,000 in 2011. Model the town's population (in thousands) as a function of time (in years after 1998). Then use your model to predict the population in 2014.

$x = \#$ of years after 1998
$y =$ Pop, in thousands

$(x_1, y_1) = (0, 10)$

$(x_2, y_2) = (13, 12)$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{12-10}{13-0} = \frac{2}{13}$

$y = m(x - x_1) + y_1$

$y = \frac{2}{13}(x - 0) + 10$

$\boxed{y = \frac{2}{13}x + 10}$

In 2014, $x = 2014 - 1998 = 16$

$\Rightarrow y = f(16) = \frac{2}{13}(16) + 10 = \frac{32}{13} + \frac{130}{13} = \frac{162}{13}$

$\approx 12.4615 \rightarrow$ $\boxed{12,462 \text{ people}}$

18. (5 pts) How many liters of 15% alcohol must be added to 90 liters of 47% alcohol to obtain a mixture of 35% alcohol?

Let $x =$ amount of 15% alcohol (in liters)

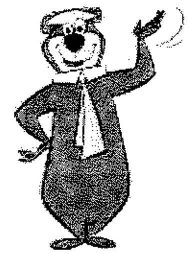
	conc.	vol.	Alcohol
15%	.15	x	$.15x$
47%	.47	90	$(.47)(90)$
35%	.35	$x+90$	$.35(x+90)$

$.15x + .47(90) = .35x + .35(90)$

$-.2x = .35(90) - .47(90) = -.12(90) = -10.8$

$\boxed{x = \frac{-10.8}{-.2} = 54}$ liters of 15% alcohol

BONUS Page. Work *one* of the following. Expect one or all three types on the next test.



BONUS (10 pts) Suppose I take 5 hours to do a job that Kelli can do in 4 hours. Then on top of that, I start work one hour late! How many hours does Kelli end up spending on the job, until it's finished? Hint: If you take the average of our times, you're doing it wrong.

BONUS (10 pts) Re-write the function $f(x)=x^2+6x+17$ in the form $f(x)=a(x-h)^2+k$. State the vertex of this parabola.

BONUS (10 pts) Re-write the function $g(x)=3x^2+6x-13$ in the form $g(x)=a(x-h)^2+k$. State the vertex of this parabola.

1. x = # of Hours Kelli spent

$\Rightarrow$ x − 1 = # of hours I spent

$$\frac{1}{4}x+\frac{1}{5}(x-1)=1$$

$$\left[\frac{1}{4}x+\frac{1}{5}(x-1)=1\right](20)$$

$$5x+4(x-1)=20$$

$$5x+4x-4=20$$

$$9x=24$$

$$\boxed{x=\frac{24}{9}\text{ hr}}$$

2. $x^2+6x+17$

$$=x^2+6x+3^2-9+17$$

$$=\boxed{(x+3)^2+8}$$

$$\boxed{(h,k)=(-3,8)}$$

3. $3x^2+6x-13$

$$=3(x^2+2x\quad)-13$$

$$=3(x^2+2x+1-1)$$

$$=3(x^2+2x+1)-3-13$$

$$=\boxed{3(x+1)^2-16}$$

$$\boxed{(h,k)=(-1,-16)}$$