MAT 121 Final Test
200 Points Name (5 pts) _____

1. Find the domain of each of the following:

   a. (10 pts) $f(x) = \dfrac{x^2 + 5}{x^2 + 3x - 10}$

   b. (10 pts) $f(x) = \sqrt{x^2 + 3x - 10}$

   c. (10 pts) $f(x) = \ln\left(x^2 + 3x - 10\right)$

2. (10 pts) Graph $g(x) = -2(x-4)^2 + 18$ using the techniques of shifting and reflecting. Start with the graph of the basic function and show all stages. In the final graph, indicate (label as ordered pairs) the *x*- and *y*-intercepts.

3. Let $f(x) = x^2 - 2x - 8$.

   a. (10 pts) Find the zeros of f by factoring.

   b. (10 pts) Find the zeros of f by quadratic formula.

   c. (10 pts) Find the zeros of f by completing the square.

4. Solve $(x+2)^2(x-3)(x-1)^3 \geq 0$.

5. (10 pts) Form a polynomial with real coefficients that has the given zeros and has degree 6. Please do not expand the polynomial.

Zeros: -4, multiplicity 1; 2, multiplicity 3; $3 + 2i$, multiplicity 1.

6. (10 pts) Let $f(x) = x^4 - 11x^3 + 42x^2 - 14x - 68$. Use synthetic division to determine $f(3)$.

7. (10 pts) Sketch the graph of $g(x) = \log_3(x+2) - 3$ by transforming $f(x) = \log_3(x)$ *x*-intercept is bonus.

8. (10 pts) Suppose the half-life of carbon-14 is 4700 years. (I think it's 5600 years in the textbook, but let's roll with 4700.) Use this half-life to obtain an exponential decay function

$$A(t) = A_0 e^{-kt}.$$

Find k symbolically (in terms of natural logarithm). This answer will be exact. Then estimate k to the 6^{th} decimal place.

Exact: $k =$ Approximate: $k \approx$

Radioactive decay model (using approximate k): $A(t) \approx$

9. Find the geometric sums:

a. (5 pts) $\displaystyle\sum_{k=1}^{20} 3 \cdot 2^{k-1}$

b. (5 pts) $\displaystyle\sum_{k=1}^{\infty} 3\left(\frac{2}{3}\right)^{k-1}$

10. (10 pts) Solve the system $\begin{aligned} x + y &= 7 \\ 3x - 2y &= 8 \end{aligned}$ by substitution.

11. (10 pts) Solve the system $\begin{matrix} x + y = 7 \\ 3x - 2y = 8 \end{matrix}$ by elimination. (Matrices are fine.)

12. Evaluate the following:

a. (5 pts) $P(5,3)$

b. (5 pts) $C(5,3) = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$

13. (10 pts) Expand $(3x - 2y)^5$ using the Binomial Theorem. Final answer less important than initial setup and use of the theorem.