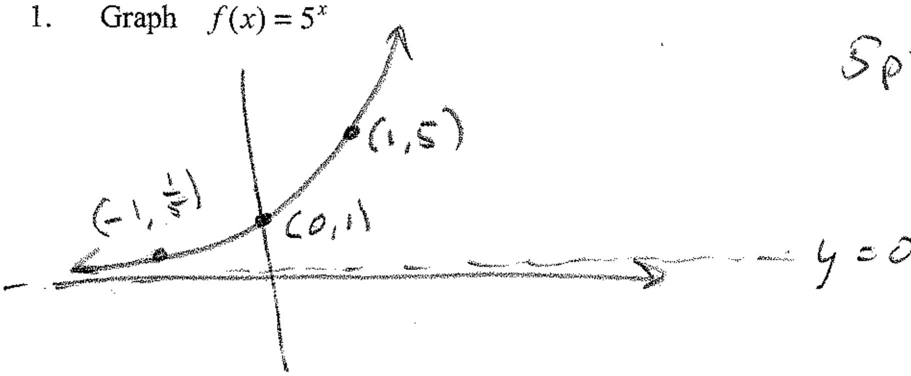
MAT 121 100 Points Fall, 2010

Test 4, Chapter 4 Name KEY

Work 10 of the following 12 problems. Omit two (2). If you omit a problem, write OMIT in the space provided. Otherwise, I'll grade the first 10 problems I come to, whether you work them or not.

1. Graph $f(x) = 5^x$

5pts



2. Graph $g(x) = -5^{1-x} + 7$ by transforming the basic function $f(x) = 5^x$

1. $f(x) = 5^x$ See above
2. $f(x+1) = 5^{x+1}$
3. $f(-x+1) = 5^{-x+1}$ $= 5^{1-x} = f(1-x)$
4. $-f(1-x) = -5^{1-x}$
5. $-f(1-x) + 7 = -5^{1-x} + 7$

Alternates:

5^x, 5^{-x}, $5^{-(x-1)}$

$-5^{-(x-1)}$, $-5^{-(x-1)} + 7$

Flip ↔

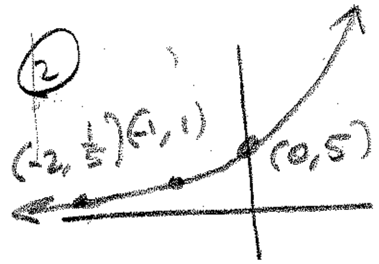
Right 1

Flip ↕

up 7

5^{-x+1}

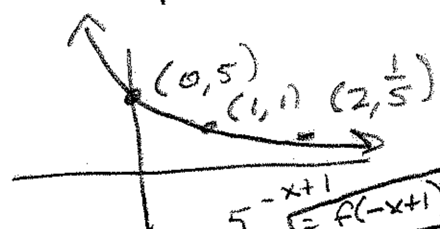
2



5^{x+1}

$\boxed{f(x+1)}$

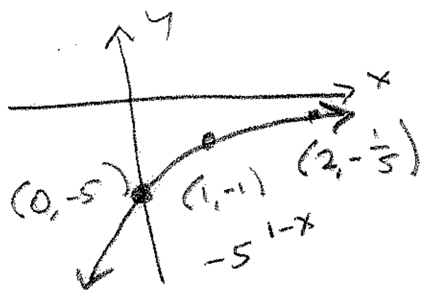
3



$5^{-x+1} \boxed{= f(-x+1)}$

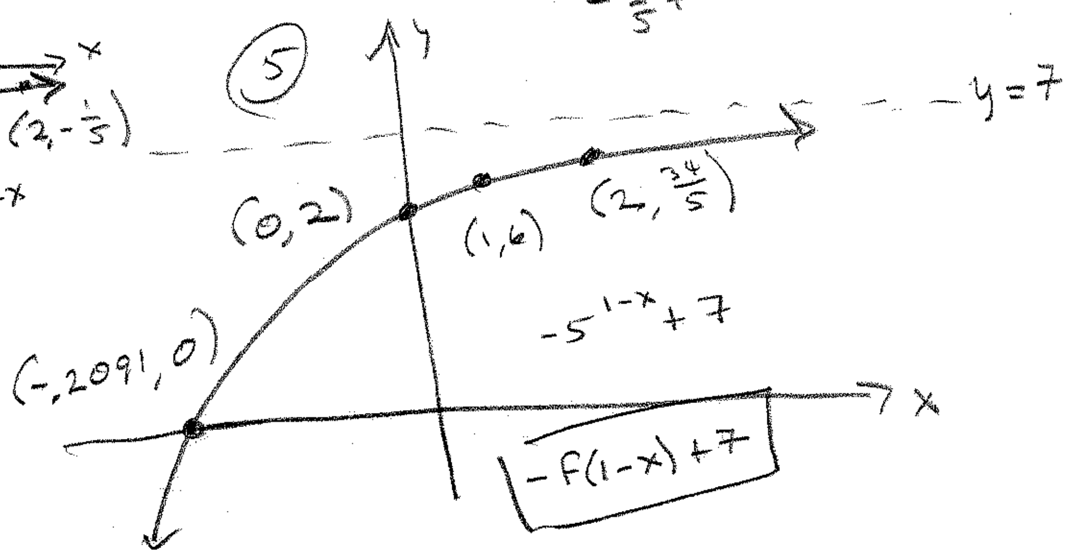
$-\frac{1}{5} + 7 = \frac{-1+35}{5} = \frac{34}{5}$

4



$\boxed{-f(-x+1)}$

5



$\boxed{-f(1-x) + 7}$

3. Find the inverse of the function $g(x) = -5^{1-x} + 7$

$x = -5^{1-y} + 7 = x$

$-5^{1-y} = x - 7$

$5^{1-y} = -x + 7$

$\log_5(5^{1-y}) = \log_5(-x+7)$

$1 - y = \log_5(7-x)$

$-y = \log_5(7-x) - 1$

$$\boxed{\begin{aligned} y &= -\log_5(7-x) + 1 \\ &= f^{-1}(x) \end{aligned}}$$

5pts

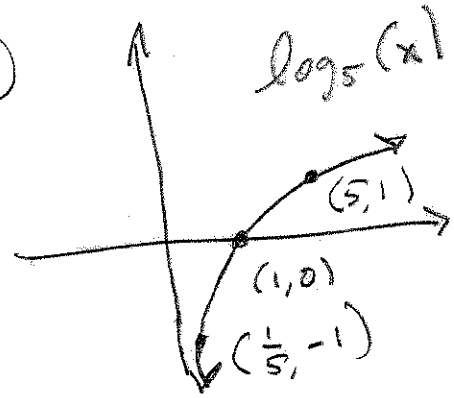
4. Graph $h(x) = -\log_5(x-3)$

① $\log_5(x)$

② $\log_5(x-3)$

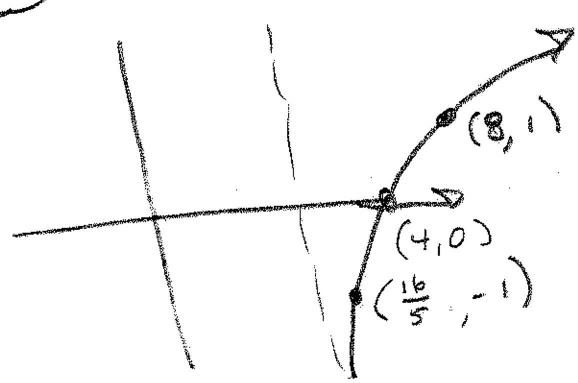
③ $-\log_5(x-3)$

①



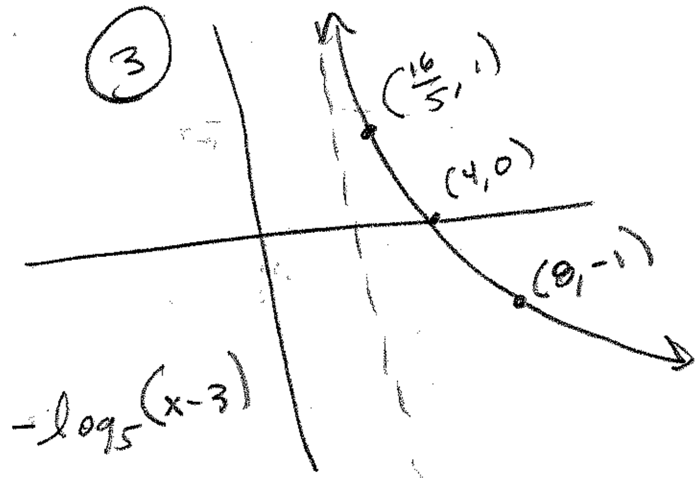
$\frac{1}{5} + 3 = \frac{1+15}{5} = \frac{16}{5}$

② $\log_5(x-3)$



$x = 3$

③



$x = 3$

5. Solve $\log_5(x-4)+\log_5(x+2)=\log_5(7)$ for x.

$$\log_5(x^2-2x-8)=\log_5(7)$$

$$x^2-2x-8=7$$

$$x^2-2x-15=0$$

$$(x-5)(x+3)=0$$

$$x \in \{-3, 5\}$$

$$-3 \notin \mathcal{D}$$

$$\boxed{x \in \{5\}}$$

6. Solve for t: $A=P\left(1+\frac{r}{m}\right)^{mt}$.

$$P\left(1+\frac{r}{m}\right)^{mt}=A$$

$$\left(1+\frac{r}{m}\right)^{mt}=\frac{A}{P}$$

$$mt\ln\left(1+\frac{r}{m}\right)=\ln\left(\frac{A}{P}\right)$$

$$\boxed{t=\frac{\ln\left(\frac{A}{P}\right)}{m\ln\left(1+\frac{r}{m}\right)}}$$

7. Solve $-5^{1-x}+7=0$ for x. Give an exact answer and then round to 4 decimal places. If you use this to supply the x-intercept for the appropriate graph on Page 1, it's worth a couple bonus points.

$$-5^{1-x} = -7$$
$$5^{1-x} = 7$$
$$1-x = \log_5(7)$$
$$-x = \log_5(7) - 1$$
$$\boxed{x = -\log_5(7) + 1}$$
$$= -\frac{\ln(7)}{\ln(5)} + 1$$
$$\approx -.2090619551$$
$$\approx -.2091$$

8. Solve $5^{x-1}=3^x$ for x. Give an exact answer and then round your answer to 4 decimal places.

$$(x-1)\ln(5) = x\ln(3)$$
$$a = \ln(5),\ b = \ln(3)$$
$$a(x-1) = bx$$
$$ax - a = bx$$
$$ax - bx = a$$
$$(a-b)x = a$$
$$x = \frac{a}{a-b}$$
$$= \boxed{\frac{\ln(5)}{\ln(5)-\ln(3)} = x}$$
$$\approx 3.150660103$$
$$\approx \boxed{3.1507 \approx x}$$

9. Radioactive Wieligminium-12.5 has a half-life of 100 years. What's its decay rate?

$$Pe^{-(k)(100)} = \frac{1}{2}P$$

$$e^{-100k} = \frac{1}{2}$$

$$-100k = \ln\left(\frac{1}{2}\right)$$

$$k = \frac{\ln\left(\frac{1}{2}\right)}{-100} = \frac{\ln(2)}{100} \approx .006931478$$

Decay rate is approximately .6931478%

10. Using your work from the previous problem, a very old sample of radioactive Wieligminium decayed from 14 grams to 5 grams. To the nearest *day*, how old is the sample?

$$14e^{-kt} = 5$$

$$e^{-kt} = \frac{5}{14}$$

$$-kt = \ln\left(\frac{5}{14}\right)$$

$$t = \frac{\ln\left(\frac{5}{14}\right)}{-k} = \frac{\ln\left(\frac{14}{5}\right)}{k}$$

$$= \frac{\ln\left(\frac{14}{5}\right)}{\frac{\ln(2)}{100}} = \frac{100\ln\left(\frac{14}{5}\right)}{\ln(2)} \approx 148.5426827$$

$$(.5426827\text{ yrs})\left(\frac{365\text{ days}}{1\text{ yr}}\right)$$

$$\approx 198.0791917$$

$$\boxed{148\text{ yrs}, 198\text{ days}}$$

54218 days

11. Solve $(\log(x))^2 = \log(x^2)$ for x.

$(\log(x))^2 = 2\log(x)$

$u^2 = 2u$

$u^2 - 2u = 0$

$u(u-2) = 0$

$u = 0$ OR $u = 2$

$\log x = 0$ $\log x = 2$

$x = 1$ $x = 100$

$\boxed{x \in \{1, 100\}}$

12. What's the future value of \$5,000 invested at 4% APR, if interest is compounded... 5 yrs

a. ... monthly? 6104.98297

$5000\left(1 + \frac{.04}{12}\right)^{12(5)} \approx \$6104.98$

b. ... daily? 6106.94687

$5000\left(1 + \frac{.04}{365}\right)^{(365)(5)} \approx \$6106.95$

c. ... continuously? 6107.013791

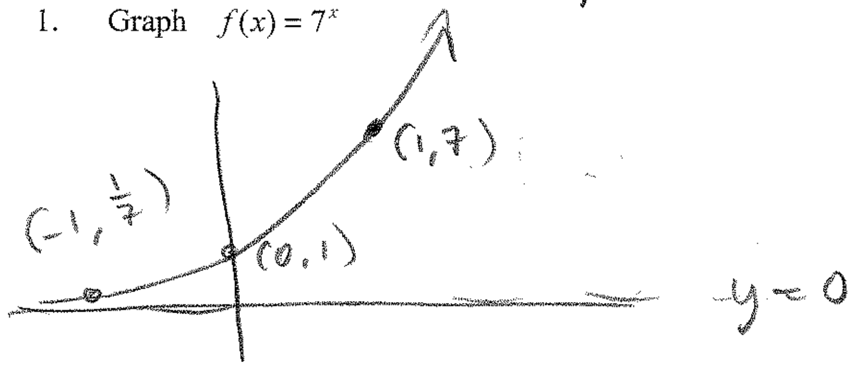
$5000e^{.04(5)} \approx \$6107.01$

MAT 121 100 Points Fall, 2012

Test 4, Chapter 4 #8: Millsium Name KEY

#10: 7 years.

1. Graph $f(x) = 7^x$

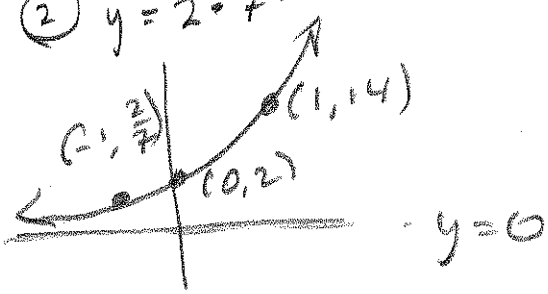


10 pts

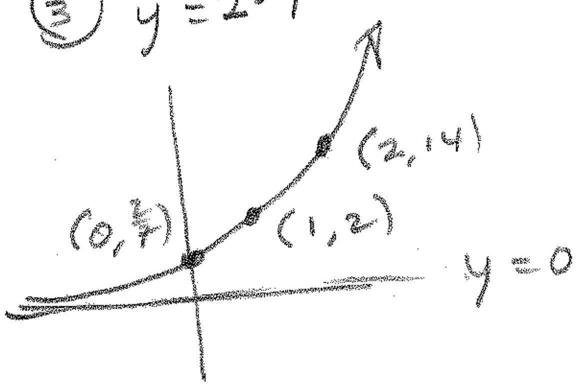
2. Graph $g(x) = 2\cdot 7^{x-1} - 3$ by transforming the basic function $f(x) = 7^x$

① $y = 7^x$ See #1

② $y = 2\cdot 7^x$



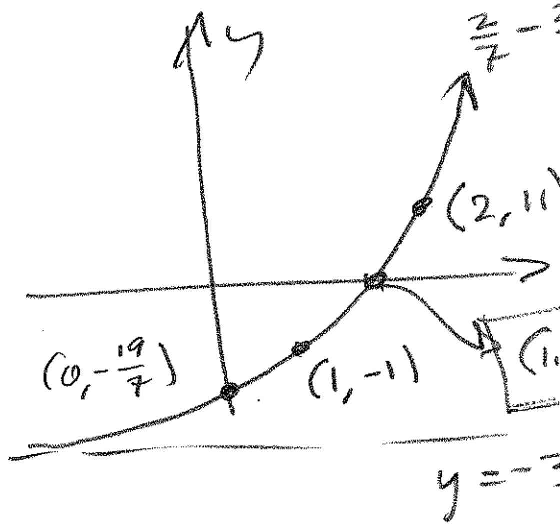
③ $y = 2\cdot 7^{x-1}$



5 pts

④ $g(x) = 2\cdot 7^{x-1} - 3$

$\frac{2}{7} - \frac{21}{7} = -\frac{19}{7}$



$\boxed{(1.2084, 0)}$

Bonus

.5

4. Solve $\log_5(x-4)+\log_5(x+2)=\log_5(7)$ for x.

10 pts

$\log_5((x-4)(x+2)) = \log_5(7)$

$x^2-2x-8=7$

$x^2-2x-15=0$

$(x-5)(x+3)=0$

$x=5$ OR $x=-3$ $\Rightarrow \notin \mathcal{D}$ (problem)

$\log_5(5-4)+\log_5(7)$

$=\log_5(1)+\log_5(7)$

$=\log_5(7)$ ✓

Bonus Solve for t: $A = P\left(1+\frac{r}{m}\right)^{mt}$.

$P\left(1+\frac{r}{m}\right)^{mt} = A$

$\left(1+\frac{r}{m}\right)^{mt} = \frac{A}{P}$

$\ln\left(\left(1+\frac{r}{m}\right)^{mt}\right) = \ln\left(\frac{A}{P}\right)$

$\left(\ln\left(1+\frac{r}{m}\right)\right) mt = \ln(A/P)$

$$\boxed{t = \frac{\ln(A/P)}{m\ln\left(1+\frac{r}{m}\right)}}$$

7. Millsium has a half-life of 50 years, if I'm lucky. What's its decay rate?

$$Pe^{50k} = \frac{1}{2}P$$

$$e^{50k} = \frac{1}{2}$$

$$50k = \ln\left(\frac{1}{2}\right) = -\ln(2)$$

$$k = -\frac{\ln(2)}{50} \approx -.0138629436$$

10 pts

8. Using your work from the previous problem, a very old sample of radioactive ~~Wielgminium~~ Millsium decayed from 20 grams to 3 grams. To the nearest *day*, how old is the sample?

$$20e^{kt} = 3$$

$$e^{kt} = \frac{3}{20}$$

$$kt = \ln\left(\frac{3}{20}\right)$$

$$t = \frac{\ln\left(\frac{3}{20}\right)}{k} =$$

$$= \frac{\ln\left(\frac{3}{20}\right)}{-\frac{\ln(2)}{50}} = \frac{-50\ln(3/20)}{\ln(2)} \approx 136.8482797$$

$$(.8482797\ \text{yr})\left(\frac{365\ \text{days}}{1\ \text{yr}}\right)$$

$$\approx 309.6220935$$

$$\approx 310\ \text{days}$$

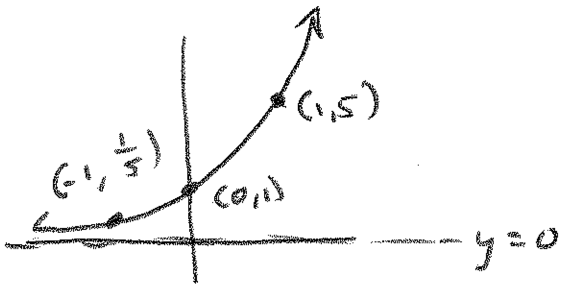
$\boxed{136\ \text{yrs},\ 310\ \text{days}}$

10 pts

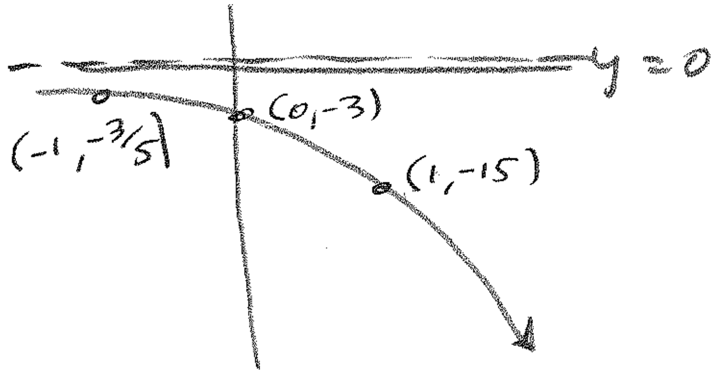
20

121-G01 E4 C4, C7 KEY

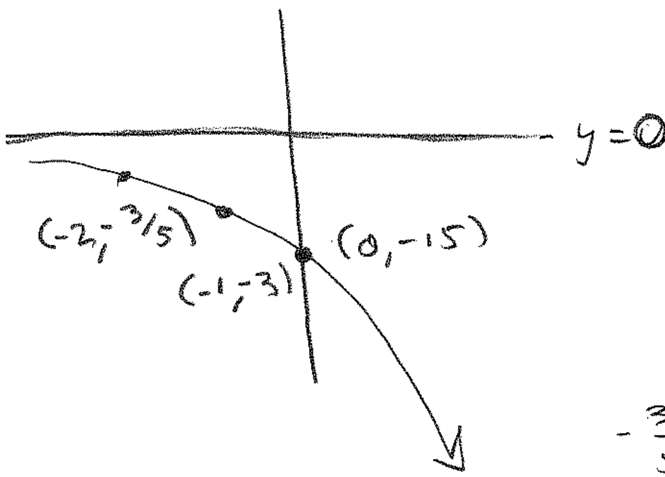
① $f(x) = 5^{x}$



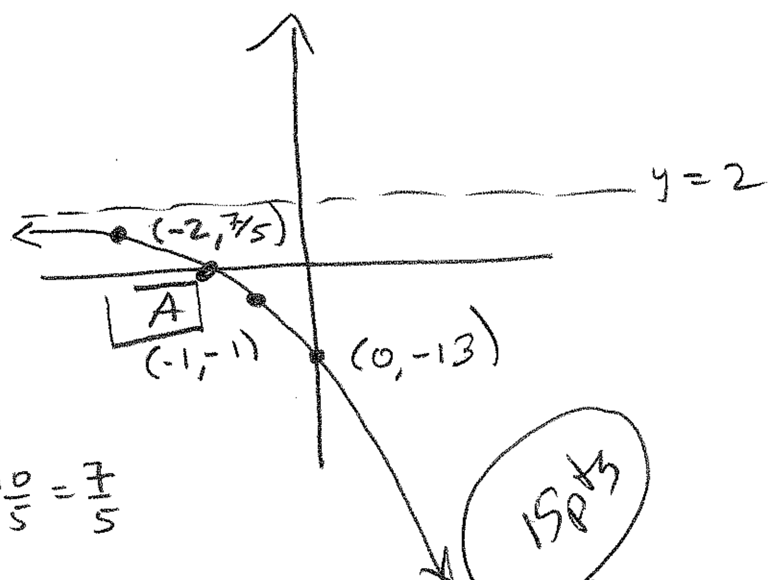
② $-3f(x) = -3 \cdot 5^{x}$



③ $-3f(x+1) = -3 \cdot 5^{x+1}$



④ $-3f(x+1)+2 = -3 \cdot 5^{x+1} + 2$



$-\frac{3}{5}+2$

$= -\frac{3}{5}+\frac{10}{5} = \frac{7}{5}$

15 pts

$-3 \cdot 5^{x+1} + 2 = 0$

$-3 \cdot 5^{x+1} = -2$

$5^{x+1} = \frac{2}{3}$

$x+1 = \log_5\left(\frac{2}{3}\right)$

$x = \log_5\left(\frac{2}{3}\right) - 1 = \frac{\ln(2/3)}{\ln(5)} - 1 \approx -1.251929636$

$\boxed{A = \left(\log_5\left(\tfrac{2}{3}\right) - 1, 0\right)} \approx (-1.251929636, 0) \approx A$

5B

121 - G81 E4

(2) $f(x) = \sqrt{x+6}$ & $g(x) = x^2 - 2x + 2 \longrightarrow$

(a) $\overline{(f \circ g)(x)} = \sqrt{(x^2-2x+2)+6} = f(g(x))$

(b) $= \sqrt{x^2-2x+8}$ so its domain is

$\{x \mid x \in \mathcal{D}(g)$ and $g(x) \in \mathcal{D}(f)\}$

$= \{x \mid x \in \mathbb{R}$ and $x^2-2x+2 \geq -6\}$, since

$\mathcal{D}(f) = \{x \mid x \geq -6\}$. We solve

$x^2-2x+2 \geq -6$

$\Longrightarrow x^2-2x+8 \geq 0$.

$a=1, b=-2, c=8 \longrightarrow$

$b^2-4ac = (-2)^2-4(1)(8)$

$= 4-32$

$= -28$, so it's

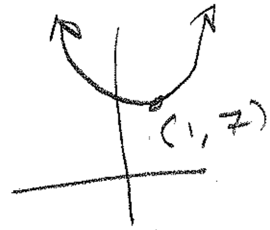
never equal zero.

What does

x^2-2x+8 look like?

$(x^2-2x+1)-1+8$

$= (x-1)^2+7$



This is ALWAYS greater

than zero, so

$\boxed{\mathcal{D}(f \circ g) = \mathbb{R} = (-\infty, \infty)}$

(3) $g(x) = \ln(x+6) \longrightarrow$

$\mathcal{D}(g) = \{x \mid x+6 > 0\} = \boxed{\{x \mid x > -6\} = (-6, \infty)}$

121 – G81 E4

(4) $f(x) = \sqrt{\dfrac{(x+5)^2}{(x-4)(x-1)^3}} \Longrightarrow \mathcal{D}(f) = \left\{ x \,\middle|\, \dfrac{(x+5)^2}{(x-4)(x-1)^3} \geq 0 \right\}$

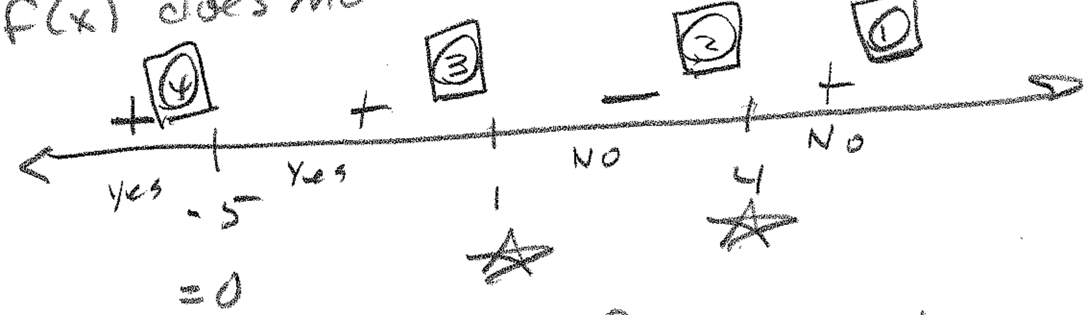
We solve $\dfrac{(x+5)^2}{(x-4)(x-1)^3} \geq 0$, keeping in mind

$x = 1, 4$ are not allowed.

$f(x) = 0 \Longrightarrow x = -5$

$f(x)$ does not exist @ $x = 1, 4$

places where f can change sign



$\boxed{1}$ $f(5) = \dfrac{(5+5)^2}{(5-4)(5-1)^3} = \dfrac{10^2}{(1)(4)^3} > 0$ $+$

$\boxed{2}$ $x-4$ controls. $(x-4)^1$ & 1 is odd
Sign changes

$\boxed{3}$ $(x-1)^3$ controls. 3 is odd. sign changes.

$\boxed{4}$ $(x+5)^2$ controls. 2 is even sign DOES NOT change.

Take the $+$ & the $=0$:

$(-\infty, 1) \cup (4, \infty)$ Remember $x = 1, 4$ not included.

(5) $\boxed{f(x) = \log_5(x),\ g(x) = x^2 - 4} \Longrightarrow$

$(f \circ g)(x) = \log_5(x^2 - 4) = H(x)$

121 E4

(6) $f(x) = \log_5(2x+7) - 4 \longrightarrow f^{-1}$ is found by swapping x & y & solving for y.

$$\log_5(2y+7) - 4 = x$$

$$\log_5(2y+7) = x+4$$

$$2y+7 = 5^{x+4}$$

$$2y = 5^{x+4} - 7$$

$$y = \frac{1}{2}\left(5^{x+4} - 7\right) \text{ OR } \boxed{\frac{5^{x+4} - 7}{2} = f^{-1}(x)}$$

(7) $2^{x+3} = 5^{x-4}$

$$\log_2(2^{x+3}) = \log_2(5^{x-4})$$

$$x+3 = (x-4)\log_2(5)$$

Let $a = \log_2(5)$

$$x+3 = a(x-4) = ax - 4a$$

$$x - ax = -4a - 3$$

$$x(1-a) = -4a - 3$$

$$x = \frac{-4a-3}{1-a} \text{ OR } \frac{4a+3}{a-1} = \boxed{\frac{4\log_2(5) + 3}{\log_2(5) - 1} = x}$$

$$= \frac{\frac{4\ln 5}{\ln 2} + \frac{3\ln 2}{\ln 2}}{\frac{\ln 5}{\ln 2} - \frac{\ln 2}{\ln 2}} = \frac{4\ln 5 + 3\ln 2}{\ln 5 - \ln 2}$$

121 - G81 E4

8

a) $10 - 2 + \frac{2}{5} - \frac{2}{25} + \cdots + \frac{2}{78125}$

$$\boxed{\begin{array}{l} a = 10 \\ r = -\frac{1}{5} \end{array}} \qquad \frac{2}{78125} = \frac{2 \cdot 5}{78125 \cdot 5} = 10\left(\frac{1}{5 \cdot 78125}\right)$$

$$\begin{array}{r|l} 5 & 78125 \\ 5 & 15625 \\ 5 & 3125 \\ 5 & 625 \\ 5 & 125 \\ 5 & 25 \\ & 5 \end{array}$$

So, by work on right

$\frac{2}{78125}$ is $10\left(\frac{1}{5^8}\right) = 10\left(\frac{1}{5}\right)^8$

The last term is always n-1,

so $n - 1 = 8 \longrightarrow \boxed{n = 9}$

$$\therefore S = a\left(\frac{1-r^n}{1-r}\right) = 10\left(\frac{1-\left(-\frac{1}{5}\right)^9}{1-\left(-\frac{1}{5}\right)}\right)$$

$$= 10\left(\frac{1 + \frac{1}{1953125}}{1 + \frac{1}{5}}\right) = 10\,\frac{\left(\frac{1953126}{1953125}\right)}{\frac{6}{5}}$$

$$= 10\left(\frac{1953126}{1953125}\right)\left(\frac{5}{6}\right)$$

$$= 10\left(\frac{325521}{1953125}\right)(5) = 10\left(\frac{325521}{390625}\right) = 2\left(\frac{325521}{78125}\right)$$

$$= \boxed{\frac{651042}{78125} \approx 8.33333 7600}$$

$a = 10$, $r = -\frac{1}{5}$, $S = \frac{651042}{78125} \approx 8.333337600$

b) $\sum_{k=1}^{\infty} 10\left(\frac{1}{5}\right)^{k-1} = 10\left(\frac{1}{1-\frac{1}{5}}\right) = 10\left(\frac{1}{\frac{4}{5}}\right) = \frac{50}{4} = \boxed{\frac{25}{2} = 12.5}$

121 - GB1 E4

(9) $\log_5(x-4) + \log_5(x+2) = \log_5(7)$

$\log_5((x-4)(x+2)) = \log_5(7)$

$(x-4)(x+2) = 7$

$x^2 - 2x - 8 = 7$

$x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

$x = -3$ OR $x = 5$

Not in Domain: $\log_5(-3-4)$ $\nexists$

Does not exist

$\log_5(5-4) + \log_5(5+2)$

$= \log_5(1) + \log_5(7)$

$= 0 + \log_5(7)$ ✓

(10) $A(t) = A_0 e^{kt}$ Given $\frac{1}{2}$-life $= 250 \Rightarrow$

(a) $A_0 e^{250k} = \frac{1}{2} A_0$

$e^{250k} = \frac{1}{2}$

$250k = \ln(\tfrac{1}{2}) = -\ln(2)$

$\boxed{k = -\frac{\ln(2)}{250} \approx .0027725887}$

$A(t) = A_0 e^{-\frac{\ln 2}{250} t}$

$\approx A_0 e^{-.0027725887t}$

(b) $A(t) = \frac{1}{3} A_0$: $A_0 e^{kt} = \frac{1}{3} A_0$

$e^{kt} = \frac{1}{3}$

$kt = \ln(1/3) = -\ln(3)$

$t = -\frac{\ln 3}{k} = \frac{-\ln(3)(250)}{-\ln(2)} = 250 \frac{\ln 3}{\ln 2} \approx$ 396.2406252 years old

ROUGH CHECK

$\frac{1}{2} \rightsquigarrow 250$ yrs

$\frac{1}{3} \rightsquigarrow 396$ is between ✓

$\frac{1}{4} \rightsquigarrow 500$ yrs