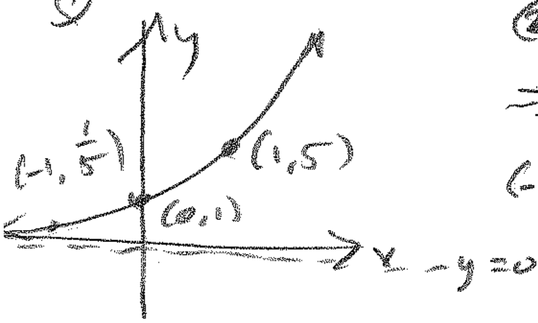
121 - ONLINE TEST 4, FALL, 2012 SOLNS

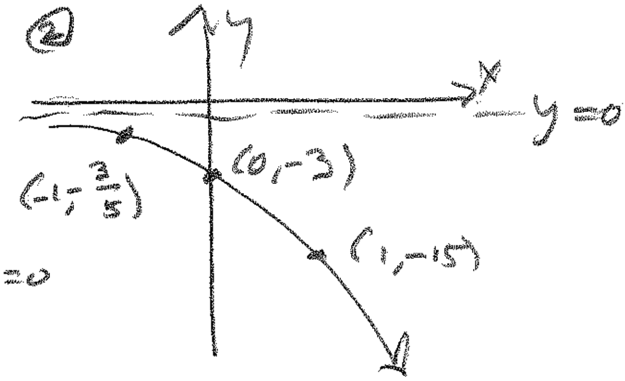
1) $g(x) = -3 \cdot 5^{x+1} - 2$

1)



$f(x) = 5^x$

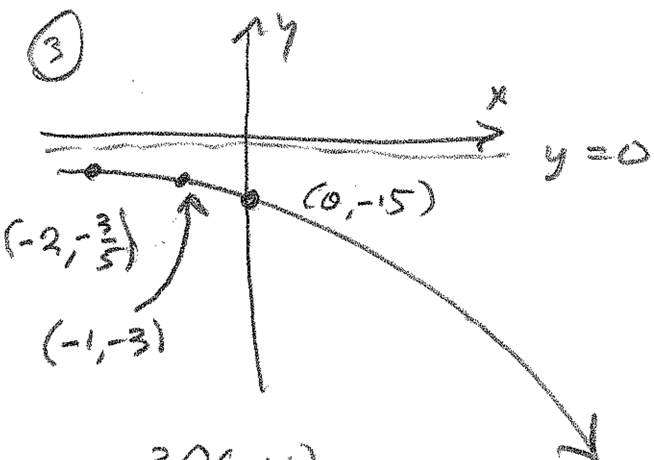
2)



$-3f(x)$

$= -3 \cdot 5^x$

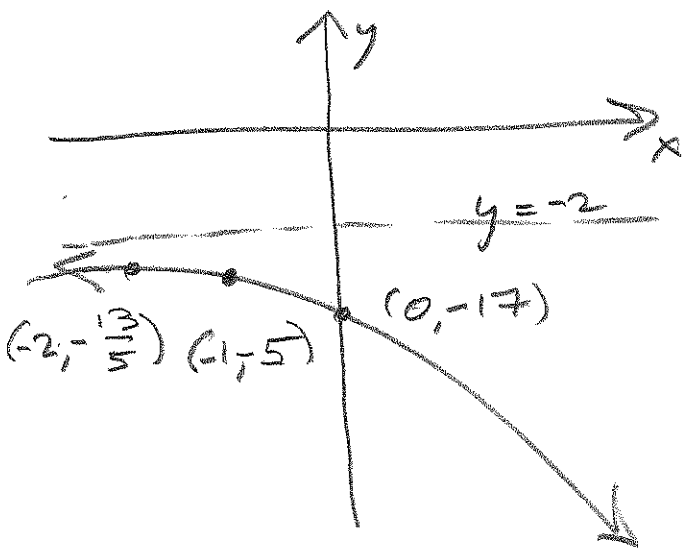
3)



$-3f(x+1)$

$= -3 \cdot 5^{x+1}$

$-\frac{3}{5} - \frac{10}{5} = -\frac{13}{5}$



121-ONLINE FALL, 2012 TEST 4

3) $f(x)=\sqrt{x+6}$, $g(x)=x^2-2x+2$

a) $\boxed{f\circ g=\sqrt{(x^2-2x+2)+6}}=\sqrt{x^2-2x+8}$

In later semesters, I said "Don't simplify."

b) $\mathcal{D}(f\circ g)=\{x \mid \underline{x\in\mathcal{D}(g)} \text{ and } \underline{g(x)\in\mathcal{D}(f)}\}$

$\mathcal{D}(g)=\mathbb{R}$
$=(-\infty,\infty)$

Need
$x^2-2x+8\geq 0$

Nope. Doesn't factor this way, Steve. See last page for corrections.

$x^2-2x+8=0$

$(x-4)(x+2)=0$

$x=-2$, $x=4$ are where it can change signs

$x=-2$, $m=1$, odd, sign changes
$x=4$, $m=1$, " " "

Yes + yes NO − Yes yes +

-2 $=0$ 4 $=0$

Need ≥ 0

Check $x=5$:
$5^2-2(5)+8=25-10+8$,
which is positive.
That's a "+"

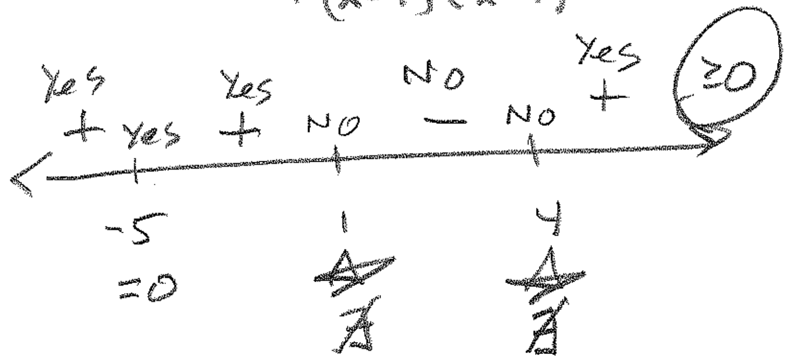
$\boxed{(-\infty,-2]\cup[4,\infty)=\mathcal{D}(f\circ g)}$

3) $g(x)=\ln(x+6)\Rightarrow\mathcal{D}(g)=\{x \mid x+6>0\}$

$=\{x \mid x>-6\}=\boxed{(-6,\infty)=\mathcal{D}(g)}$

(4) $f(x)=\sqrt{\dfrac{(x+5)^2}{(x-4)(x-1)^3}} \longrightarrow$

$\mathcal{D}(f)=\left\{x \;\middle|\; \dfrac{(x+5)^2}{(x-4)(x-1)^3}\geq 0 \text{ and } x\neq 1,4\right\}$



$\boxed{\mathcal{D}=(-\infty,1)\cup(4,\infty)}$

(5) $H(x)=\log_5(x^2-4)$. Let $f(x)=\log_5(x)$ and $g(x)=x^2-4$. Then $H(x)=(f\circ g)(x)$.

OTER ANSWERS:

$f(x)=\log_5(x-4),\ g(x)=x^2$

$f(x)=\log_5(x^2-4),\ g(x)=x$ Bleah!

(6) $f(x)=y=\log_5(2x+7)-4$

$\log_5(2y+7)-4=x$

$\log_5(2y+7)=x+4$

$2y+7=5^{x+4}$

$2y=5^{x+4}-7$

$$\boxed{\begin{aligned} y&=\tfrac{1}{2}\left(5^{x+4}-7\right)=f^{-1}(x)\\ \text{OR }&=\tfrac{1}{2}\cdot 5^{x+4}-\tfrac{7}{2}\end{aligned}}$$

(7) $2^{x+3} = 5^{x-4}$

$\ln(2^{x+3}) = \ln(5^{x-4})$

$(\ln(2))(x+3) = (\ln(5))(x-4)$

$A(x+3) = B(x-4)$, where $\boxed{A = \ln(2),\ B = \ln(5)}$

$Ax + 3A = Bx - 4B$

$Ax - Bx = -4B - 3A$

$(A-B)x = -4B - 3A$

$x = \dfrac{-4B-3A}{A-B} = \dfrac{3A+4B}{B-A}$ ← with A & B defined, THIS is a final answer

$= \boxed{\dfrac{3\ln(2) + 4\ln(5)}{\ln(5) - \ln(2)} = x}$

(8) (a) $10 - 2 + \frac{2}{5} - \frac{2}{25} + \cdots + \frac{2}{78125}$

$\boxed{a = 10,\ r = \frac{1}{5}}$

$ar^{n-1} = 2\left(\frac{1}{5}\right)^{n-1}$ Need to find n.

$$\begin{array}{r|l} 5 & 78125 \\ 5 & 15625 \\ 5 & 3125 \\ 5 & 625 \\ 5 & 125 \\ 5 & 25 \\ & 5 \end{array}$$

$2\left(\frac{1}{5}\right)^7 = 2\left(\frac{1}{5}\right)^{n-1}$

$7 = n-1$

$n = 8$

$a_k = ar^{k-1}$

$$\sum_{k=1}^{n} ar^{k-1} = \frac{a(1-r^n)}{1-r}$$

$$= \frac{2\left(1-\left(\frac{1}{5}\right)^8\right)}{1-\frac{1}{5}}$$

$$= \boxed{\frac{195312}{78125} \approx 2.499993600}$$

(8) (b) $\sum_{k=1}^{\infty} 10\left(\frac{1}{5}\right)^{k-1}$ $a=10,\ r=\frac{1}{5} \longrightarrow$

$$S = \frac{a}{1-r} = \frac{10}{1-\frac{1}{5}} = \frac{10}{\frac{4}{5}} = \frac{50}{4} = \boxed{\frac{25}{2}}$$

(9) $\log_5(x-4) + \log_5(x+2) = \log_5(7)$ $D = \{x \mid x > 2\}$

$$\log_5((x-4)(x+2)) = \log_5(7)$$

$$(x-4)(x+2) = 7$$

$$x^2 - 2x - 8 = 7$$

$$x^2 - 2x - 15 = 0$$

$$(x-5)(x+3) = 0$$

$x \in \{-3, 5\}$, but $x = -3 \notin D$ so $\boxed{x \in \{5\}}$

Check $x=5$: $\underbrace{\log_5(5-4)}_{\to 0} + \log_5(7) = 7$

(10) Half life is 250 yrs

$$A(t) = A_0 e^{kt}$$

$$A(250) = A_0 e^{250k} = \frac{1}{2} A_0$$

$$e^{250k} = \frac{1}{2}$$

$$250k = \ln(1/2) = -\ln(2)$$

$$k = -\frac{\ln 2}{250}$$

$$A(t) = A_0 e^{-\frac{\ln 2}{250} t}$$

$$\approx A_0 e^{-.00277258872 t}$$

(b) $A_0 e^{kt} = \frac{1}{3} A_0$ <u>$\frac{1}{3}$ Remains</u>

$$e^{kt} = \frac{1}{3}$$

$$kt = \ln\left(\frac{1}{3}\right) = -\ln(3)$$

$$t = \frac{-\ln 3}{k} = \frac{-\ln 3}{-\ln 2} \cdot 250$$

$$\approx 396.2406252 \approx 396 \text{ years}$$

121-ONLINE TEST 4, FALL 2012 ERRATA.

Let's fix the $\mathcal{D}(f\circ g)$ from Page 2

Need $x^2-2x+8 \geq 0$, basically. The way I factored was an optical illusion.

$x^2-2x+8=0$ Find zeros

$x^2-2x \quad = -8$

$x^2-2x+1^2 = -8+1^2$

$(x-1)^2=-7$ No real solutions. No sign changes.

Either always positive or always negative, so always positive, since $0^2-2(0)+8$ is positive.

$\therefore$ $\mathcal{D}(f\circ g) = (-\infty,\infty)$ and the question wasn't very well devised.