MAT 121, Spring, 2012 **Test 1**

100 Points (5 pts) Name KEY

1. (5 pts) State whether the relation below represents a function (Yes/No). If not, why or why not? What is the domain and what is the range?

$$\{(1, 1), (-3,-1), (6,4), (4,-1), (1,2)\}$$

No. (1,1) & (1,2) mess it up.

$\mathcal{D} = \{1, -3, 6, 4\}$, $R = \{1, -1, 4, 2\}$

2. (5 pts) Determine whether the equation $x - y^2 = 11$ defines y as a function of x. If it does *not*, show/explain why not, either by a general argument, or by finding an x-value in the domain that corresponds to more than one y-value in the range.

$-y^2 = 11 - x$

$y^2 = x - 11$

$y = \pm\sqrt{x-11}$

No. Not a function.

Let $x = 12$, then $y = 1$, $y = -1$ are both assigned to it

$(12, 1)$, $(12, -1)$ are members

3. (5 pts) Find the domain of $g(x) = \dfrac{x^2 + 5x + 17}{\sqrt{2x - 7}}$.

$2x - 7 \geq 0$

$2x \geq 7$

$x \geq \frac{7}{2}$

$\mathcal{D} = [7/2, \infty) = \{x \mid x \geq \frac{7}{2}\}$

4. Let $f(x) = x^2 - 2$.

a. (5 pts) Simplify the difference quotient $\dfrac{f(x+h) - f(x)}{h}$. You may use the alternative version of this given by $\dfrac{f(x) - f(c)}{x - c}$.

$$\frac{(x+h)^2 - 2 - (x^2 - 2)}{h} = \frac{x^2 + 2xh + h^2 - 2 - x^2 + 2}{h}$$

$$= \frac{2xh + h^2}{h} = 2x + h$$

b. (5 pts) Find the average rate of change of f from $x = 1$ to $x = 3$.

$$\frac{f(3) - f(1)}{3 - 1} = \frac{3^2 - 2 - (1^2 - 2)}{} = \frac{9 - 2 - 1 + 2}{2} = \boxed{4}$$

5. Let $f(x)=x^2-3$ and $g(x)=\sqrt{2x+1}$.

a. Determine each of the following functions.

i. (5 pts) $(f+g)(x) = x^2-3+\sqrt{2x+1}$

ii. (5 pts) $(f\cdot g)(x) = \cancel{(x^2-3)(\sqrt{2x+1})}\ (x^2-3)\sqrt{2x+1}$

iii. (5 pts) $\left(\dfrac{g}{f}\right)(x) = \dfrac{\sqrt{2x+1}}{x^2-3}$

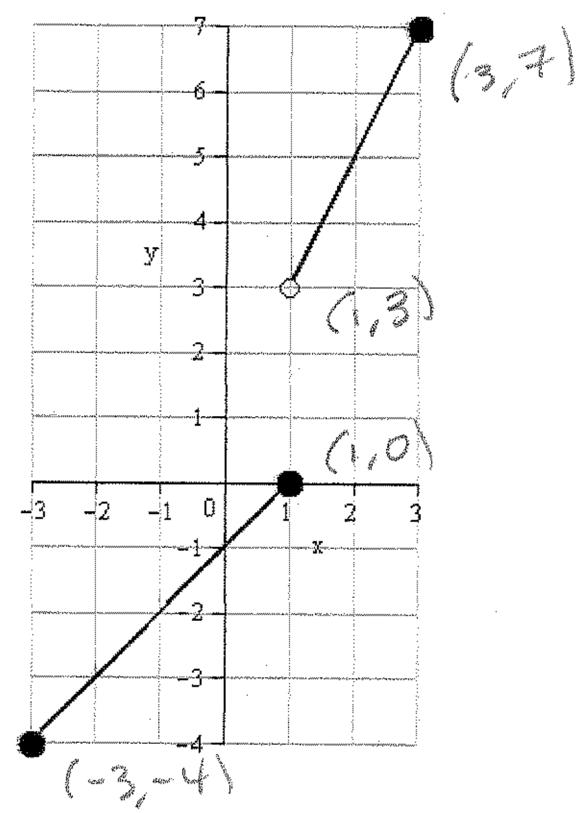
b. (5 pts) What is the domain of $\left(\dfrac{f}{g}\right)(x)$?

$\left(\dfrac{f}{g}\right)(x) = \dfrac{x^2-3}{\sqrt{2x+1}}$ Need $2x+1 \geq 0$ AND $2x+1 \neq 0$

$\Rightarrow 2x > -1$

$\mathcal{D} = \{x \mid x > -\frac{1}{2}\} = (-\frac{1}{2}, \infty)$

6. (5 pts) The graph of a piecewise-defined function is given. Write its definition.



$m = \dfrac{7-3}{3-1} = \dfrac{4}{2} = 2$

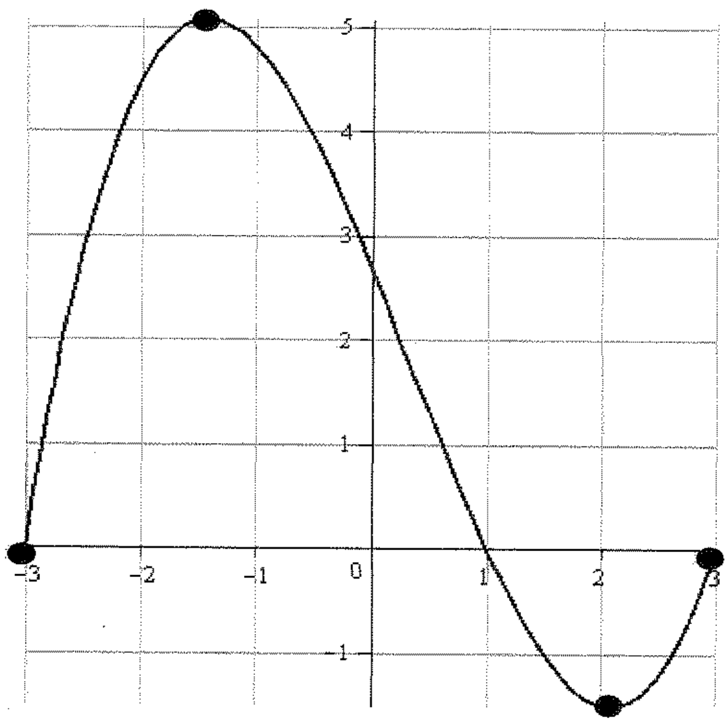
$m = \dfrac{0+4}{1+3} = \dfrac{4}{4} = 1$

$y = 2(x-3)+7 = 2x-6+7 = 2x+1$

$y = 1(x-1)+0 = x-1$

$$f(x) = \begin{cases} x-1, & -3 \leq x \leq 1 \\ 2x+1, & 1 < x \leq 3 \end{cases}$$

7. Use the graph of the function f, below, to answer the following questions. Some of your answers will be estimates, and that's OK:



a. The intercepts (Express answers as ordered pairs.)

i. (4 pts) x-intercept(s):

(-3,0), (1,0)

ii. (4 pts) y-intercept(s):

(0, 2.7) (about)

b. (5 pts) The domain and range:

$\mathcal{D} = [-3, 3]$, $\mathcal{R} = [-1.5, 5]$ approx

c. Intervals of increase/decrease:

i. (3 pts) f is increasing on (-3, -1.4) ∪ (2, 3).

ii. (3 pts) f is decreasing on (-1.4, 2) approx.

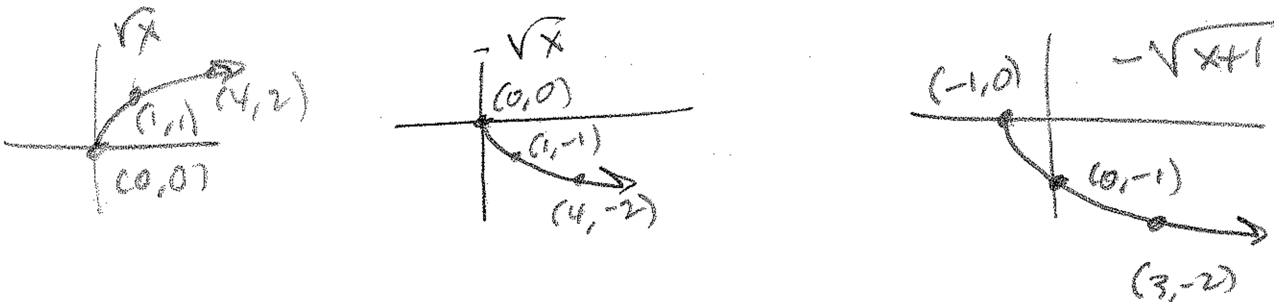
d. Extrema:

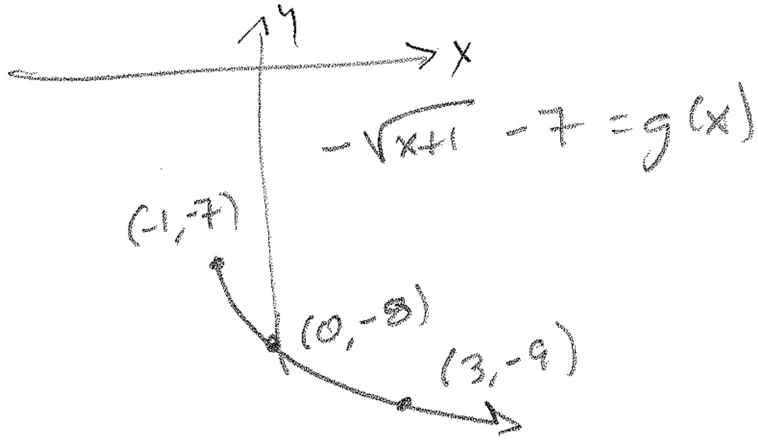
i. (3 pts) f has local minimum of -1.4 at 2.

ii. (3 pts) f has a local maximum of 5 at -1.4.

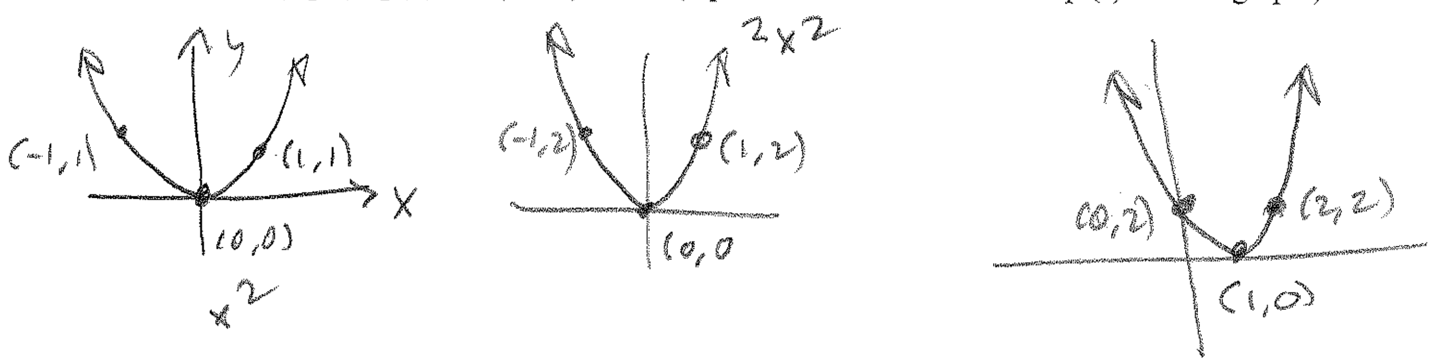
8. Graph each of the following functions using the techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function and show all stages in separate sketches. Track 3 key points through the transformations, and show the y-intercept in the final sketch.

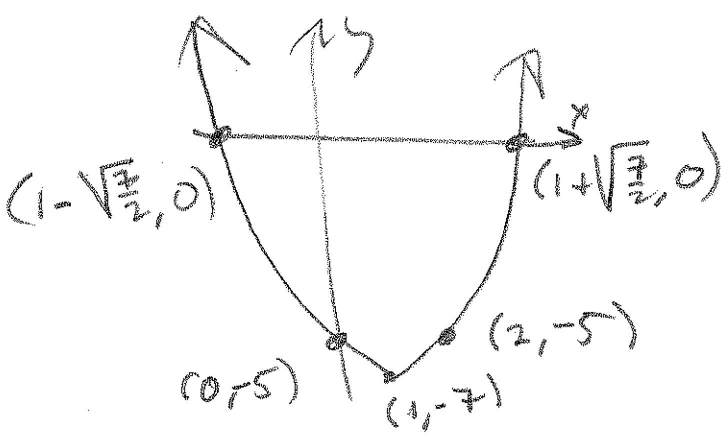
a. (7 pts) $g(x) = -\sqrt{x+1} - 7$. (**2 pts bonus** – Show x-intercepts in final graph.)





b. (7 pts) $g(x) = 2(x-1)^2 - 7$ (**2 pts bonus** – Show x-intercept(s) in final graph.)





$2(x-1)^2 = 7$

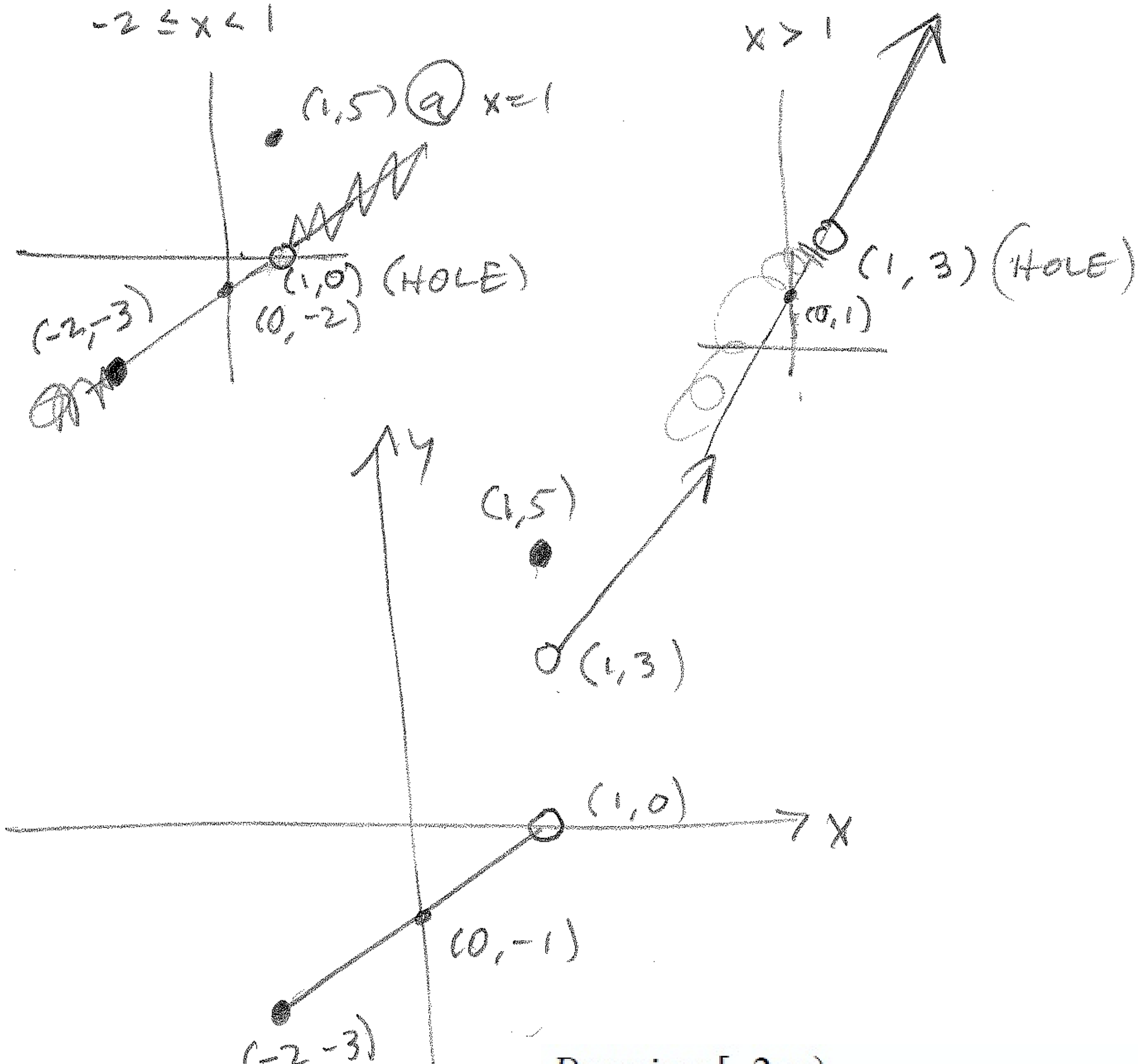
$(x-1)^2 = \frac{7}{2}$

$x-1 = \pm\sqrt{\frac{7}{2}}$

$x = 1 \pm \sqrt{\frac{7}{2}}$

9. (6 pts) Sketch the graph of $f(x)=\begin{cases} x-1 & if\ -2\le x<1 \\ 5 & if\ x=1 \\ 2x+1 & if\ x>1 \end{cases}$. Include all intercepts.

State the domain and range.



$$Domain = [-2,\infty)$$

$$Range = [-3,1)\cup\{5\}\cup(3,\infty)$$

$$= [-3,1)\cup(3,\infty),$$

since 5 is already in that last interval.