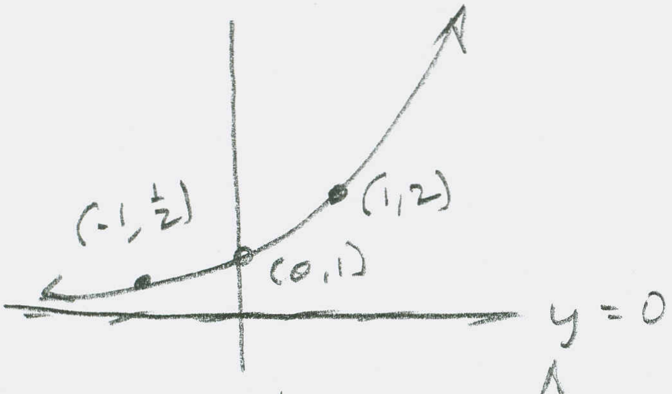
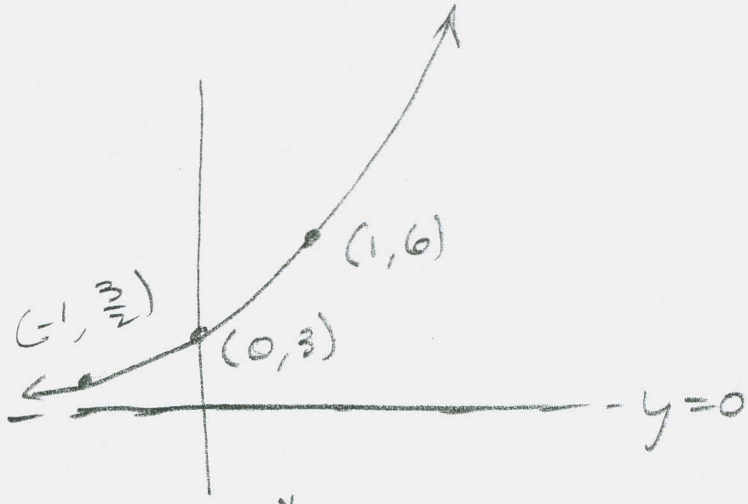
121 TEST 4 Spr' 2012

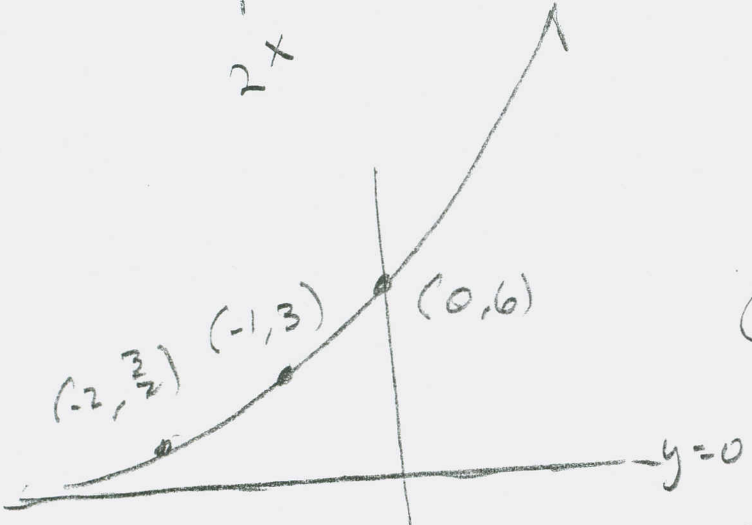
$$g(x) = 3 \cdot 2^{x+1} - 5$$



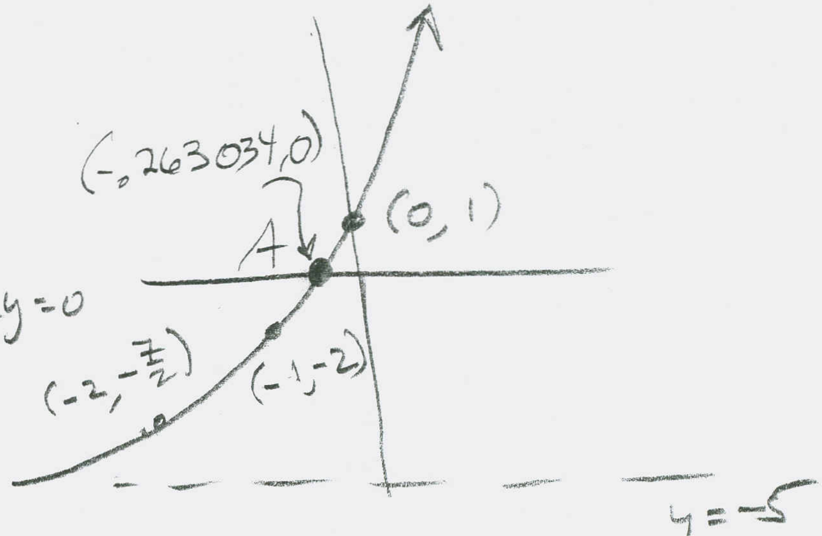
2^x



$3 \cdot 2^x$



$3 \cdot 2^{x+1}$



$g(x) = 3 \cdot 2^{x+1} - 5$

$\frac{3}{2} - \frac{10}{2} = -\frac{7}{2}$

x-intercept:

$$3 \cdot 2^{x+1} - 5 = 0$$

$$3 \cdot 2^{x+1} = 5$$

$$2^{x+1} = \frac{5}{3}$$

$$x+1 = \log_2\left(\frac{5}{3}\right)$$

$$x = -1 + \log_2\left(\frac{5}{3}\right)$$

$$= -1 + \frac{\ln(5/3)}{\ln(2)} \approx -.2630344058$$

$$\Longrightarrow A \approx (-.263034, 0)$$

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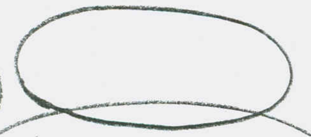
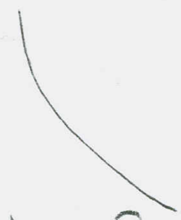
2

10 pts



Not func.



Yes func
Yes 1-to-1

2

Yes func
Yes 1-to-1

3 $f(x) = x^2 - 1$, $g(x) = \sqrt{2x-6}$ $\longrightarrow$

a $(f \circ g)(x) = f(g(x)) = \sqrt{2x-6}^{\,2} - 1 = 2x-6-1 = \boxed{2x-7}$

$\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

$= \{x \mid 2x-6 \geq 0 \text{ and } \underbrace{g(x) \in \mathbb{R}}_{\text{always true when } x \in \mathcal{D}(g)}\}$

$= \boxed{\{x \mid x \geq 3\}}$

b $(g \circ f)(x) = g(f(x))$

$= \sqrt{2(x^2-1)-6} = \sqrt{2x^2 - 2 - 6} = \boxed{\sqrt{2x^2 - 8}}$

$\mathcal{D}(g \circ f) = \{x \mid x \in \mathcal{D}(f) \text{ and } f(x) \in \mathcal{D}(g)\}$

$= \{x \mid x \in \mathbb{R} \text{ and } x^2 - 1 \geq 3\}$

$= \{x \mid x^2 - 4 \geq 0\}$

$= \boxed{\{x \mid x \leq -2 \text{ OR } x \geq 2\}}$

Need $x^2 - 4 \geq 0$
$\Longrightarrow$
$x \in (-\infty, -2] \cup [2, \infty)$

TEST 4

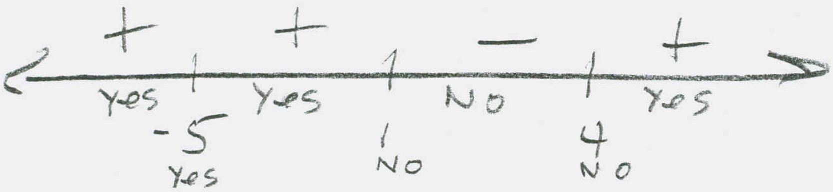
4 $g(x) = \ln(2x-6) \longrightarrow$

$\mathcal{D}(g) = \{x \mid 2x-6 > 0\}$

$= \{x \mid x > 3\} = \boxed{(3, \infty)}$

5 $f(x) = \sqrt{\dfrac{(x+5)^2}{(x-4)(x-1)^3}} \longrightarrow$

$\mathcal{D}(f) = \left\{x \;\middle|\; \dfrac{(x+5)^2}{(x-4)(x-1)^3} \geq 0 \text{ and } (x-4)(x-1)^3 \neq 0\right\}$



$\Longrightarrow \mathcal{D}(f) = (-\infty, 1) \cup (4, \infty)$

6 $\boxed{f(x) = x^5 \text{ and } g(x) = 2x-7} \longrightarrow$

$H(x) = (2x-7)^5 = (f \circ g)(x)$

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(4)

(7) $f(x) = 2e^{x-1} + 5$ Let

$x = 2e^{y-1} + 5 \longrightarrow$

$2e^{y-1} + 5 = x \longrightarrow$

$2e^{y-1} = x - 5 \longrightarrow$

$e^{y-1} = \frac{x-5}{2} \longrightarrow$

$y - 1 = \ln\left(\frac{x-5}{2}\right) \longrightarrow$

$$\boxed{y = 1 + \ln\left(\frac{x-5}{2}\right) = f^{-1}(x)}$$

OR

$x - 7 = \log_3(5^{x-4})$

$x - 7 = (\log_3 5)(x-4)$

$x - 7 = (\log_3 5)x - 4\log_3 5$

$x - (\log_3 5)x = 7 - 4\log_3 5$

$(1 - \log_3 5)x = 7 - 4\log_3 5$

$$\boxed{x = \frac{7 - 4\log_3 5}{1 - \log_3 5}}$$

(8) $3^{x-7} = 5^{x-4}$

$\ln(3^{x-7}) = \ln(5^{x-4})$

$\ln(3)(x-7) = (\ln 5)(x-4)$

$(\ln 3)x - 7\ln 3 = (\ln 5)x - 4\ln 5$

$(\ln 3)x - (\ln 5)x = 7\ln 3 - 4\ln 5$

$(\ln 3 - \ln 5)x = 7\ln 3 - 4\ln 5$

$$\boxed{x = \frac{7\ln 3 - 4\ln 5}{\ln 3 - \ln 5}}$$

$\log_5(3^{x-7}) = x - 4$

$(\log_5 3)(x-7) = x - 4$

$(\log_5 3)x - 7\log_5 3$
$= x - 4$

$(\log_5 3)x - x$
$= 7\log_5 3 - 4$

$(\log_5 3 - 1)x$
$= 7\log_5 3 - 4$

$$\boxed{x = \frac{7\log_5 3 - 4}{\log_5 3 - 1}}$$

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5

9 a $7 - \frac{7}{3} + \frac{7}{9} - \frac{7}{27} + \cdots + \frac{7}{243} \rightarrow 7\cdot\left(-\frac{1}{3}\right)^5$

1 2 . . .

$a = 7$

$r = -\frac{1}{3}$

$n = 6$

$S_6 = 7\,\frac{\left(1-\left(-\frac{1}{3}\right)^{6}\right)}{1-\left(-\frac{1}{3}\right)}$

$3\,\underline{|243}$
$3\,\underline{|81}$
$3\,\underline{|27}$
$3\,\underline{|9}$
3

$n-1=5$

$a\left(\frac{1-r^n}{1-r}\right) \quad = 7\,\frac{\left(1-\frac{1}{729}\right)}{1-\left(-\frac{1}{3}\right)}$

$243 \times 3 = 729 \rightarrow \left(\frac{1}{3}\right)^6$

$= 7\,\frac{\left(\frac{728}{729}\right)}{\frac{4}{3}} = 7\left(\frac{728}{729}\right)\left(\frac{3}{4}\right)$

$= 7\left(\frac{182}{243}\right) = \frac{1274}{243} \approx \boxed{5.242798}$

your teacher got the sign wrong on the last term. $\left(-\frac{1}{3}\right)^5 = -\frac{1}{243}$.
So the last term doesn't fit the pattern
5 pts Bonus to anyone who picks up on this. *sigh*

b $\sum_{k=1}^{\infty} 7\left(\frac{1}{3}\right)^{k-1} = 7\left(\frac{1}{1-\frac{1}{3}}\right) = \frac{7}{\frac{2}{3}} = \boxed{\frac{21}{2}}$

$a\left(\frac{1}{1-r}\right)$

10 $\ln(x-4) + \ln(x+1) = \ln(6) \Rightarrow$

$\ln((x-4)(x+1)) = \ln 6 \Rightarrow$

$x^2 - 3x - 4 = 6 \Rightarrow$

$x^2 - 3x - 10 = 0 \Rightarrow$

$(x-5)(x+2) = 0 \Rightarrow x \in \{-2, 5\}$

Not in the domain

So, $x = 5$ is the only solution.

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(11) Doubling time is 15 yrs

t = time from beginning of study, in years

(a) $A(t) = A_0 e^{kt} \longrightarrow$

$A(15) = A_0 e^{15k} = 2A_0$

$e^{15k} = 2$

$15k = \ln 2$

$k = \frac{\ln 2}{15} \rightarrow \boxed{A(t) = A_0 e^{\frac{\ln 2}{15} t}}$

(b) $A_0 = 1000$, $A(t) = 6000$ in 1998

where t is unknown.

$A(t) = 1000 e^{\frac{\ln 2}{15} t} \overset{\text{SET}}{=} 6000 \longrightarrow$

$e^{\frac{\ln 2}{15} t} = 6 \longrightarrow$

$\frac{\ln 2}{15} t = \ln 6$

$t = \frac{15 \ln 6}{\ln 2} \approx 38.77 \approx 39$ years.

$1998 - 39 =$ $\boxed{1959 \text{ is when the study began}}$

Test 4 What Students don't get

Polynomial & Rational Inequalities

Comes up on pages 2 & 3

3a $(f \circ g)(x) = 2x-7$

$\mathcal{D} = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

$= \{x \mid x \geq 3 \text{ and } g(x) \text{ is real}\} = [3, \infty)$

3b $(g \circ f)(x) = \sqrt{2(x^2-1)-6} = \sqrt{2x^2-8}$

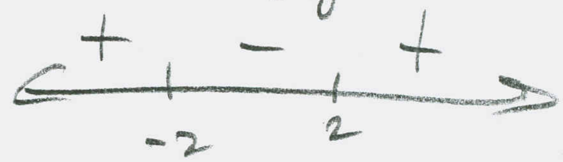
$\mathcal{D} = \{x \mid x \in \mathcal{D}(f) \text{ \& } f(x) \in \mathcal{D}(g)\}$

$= \{x \mid x \text{ is real \& } x^2-1 \geq 3\}$

Solve $x^2-1 \geq 3$, people! Sign Pattern!

$x^2-4 \geq 0$

$(x-2)(x+2) \geq 0$



$x = \pm 2$ is where x^2-4 changes sign.

So see what the sign is on each interval

want ≥ 0. want "+" and include

$x = \pm 2$. $(-\infty, -2] \cup [2, \infty)$

#5 Same deal. See Solutions. If you're not analyzing this the way I am, then you're not getting it. This represents close to a letter grade in MAT 121!