Version 2

MAT 121 – G81 Spring, 2012

Test 3, Chapter 3 80 Points Name KEY

1.(5 pts) Form a polynomial in factored form with *real* coefficients with the given zeros and degree. Please do not expand the polynomial.

Zeros: $x = 2$, multiplicity 2; $x = -1$, multiplicity 2. Degree 4.

$(x-2)^2(x+1)^2$

2. (10 pts) Expand $(x-(2+i))(x-(2-i))$

$x^2 - (2-i)x - (2+i)x + 2^2 + 1^2$

$= x^2 - 2x + ix - 2x - ix + 5$

$= \boxed{x^2 - 4x + 5}$

3. (10 pts) Use synthetic division to find $P(3)$ if $P(x) = 3x^4 - 2x^3 + 2x^2 - x + 3$.

3	3	-2	2	-1	3
		9	21	69	204
	3	7	23	68	$\boxed{207 = P(3)}$

4. (5 pts) Divide $f(x) = 2x^4 - x^3 + 2x^2 - 5$ by $d(x) = x^2 + 3$. Then write the result in the form *Dividend = Divisor · Quotient + Remainder*.

$$\begin{array}{r|l}
 & 2x^2 - x - 4 \\
x^2+3 & 2x^4 - x^3 + 2x^2 + 0x - 5 \\
 & -(2x^4 \qquad + 6x^2) \\
 & \qquad -x^3 - 4x^2 + 0x - 5 \\
 & \qquad -(-x^3 \qquad - 3x) \\
 & \qquad\qquad -4x^2 + 3x - 5 \\
 & \qquad\qquad -(-4x^2 \qquad - 12) \\
 & \qquad\qquad\qquad 3x + 7
\end{array}$$

$\boxed{f(x) = (x^2+3)(2x^2 - x - 4) + 3x + 7}$

5. Let $f(x) = 3x^5 - 15x^4 + 21x^3 + 3x^2 - 24x + 12$, and suppose its factored form is given by $f(x) = 3(x-2)^2(x+1)(x-1)^2$

a. (10 pts) List each real zero and its multiplicity. Determine whether the graph of $f(x)$ touches or crosses the x-axis at the corresponding x-intercepts.

$x=2$ $m=2$ Touch

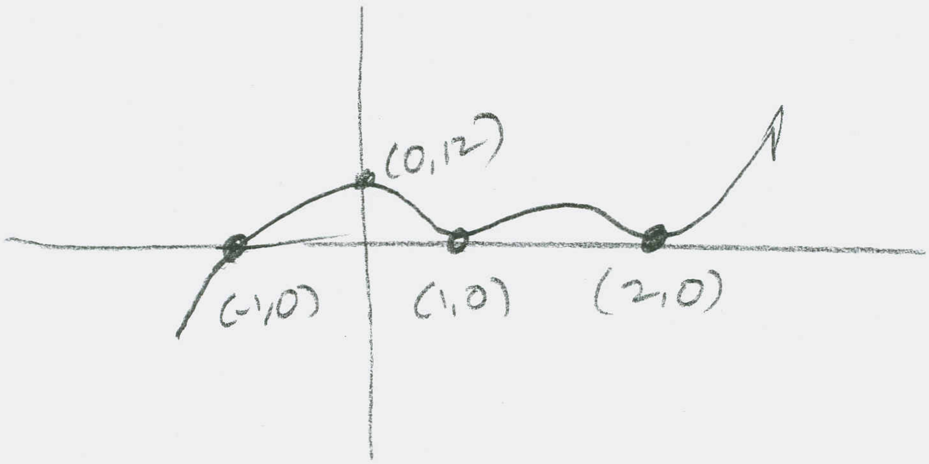
$x=-1$ $m=1$ cross

$x=1$ $m=2$ Touch

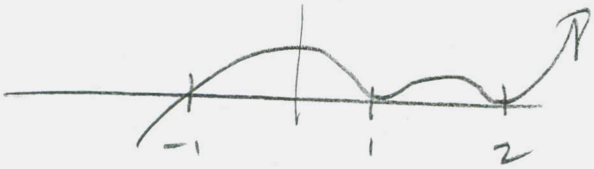
b. (5 pts) What power function does f resemble for large values of $|x|$? In other words, give the end behavior for f, along with a simple diagram.

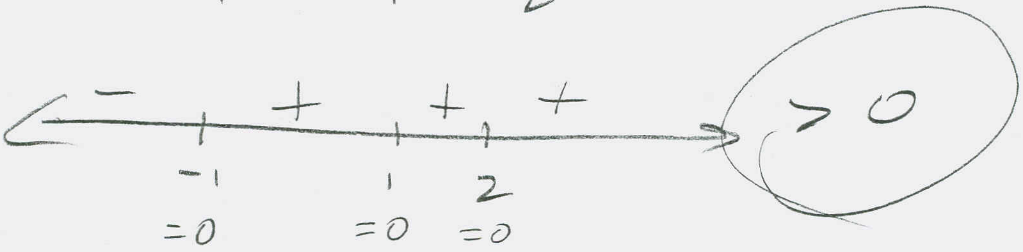
$3x^5$

c. (5 pts) Use your work, above, to help you sketch the graph of $f(x)$, showing all intercepts (including the y-intercept).



6. Use your sketch from the previous problem to help you solve the following inequalities. You might want to re-sketch it, below, just to have it on the same page.

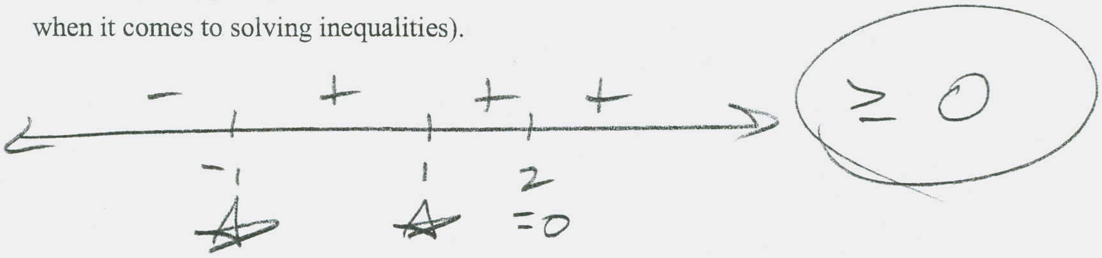




a. (5 pts) $3(x-2)^2(x+1)(x-1)^2 > 0$

$x \in (-1,1) \cup (1,2) \cup (2,\infty)$

b. (5 pts) $\dfrac{3(x-2)^2}{(x+1)(x-1)^2} \geq 0$ (A very different-looking function, but not so very different, when it comes to solving inequalities).



$x \in (-1,1) \cup (1,2] \cup [2,\infty)$

$= \boxed{(-1,1) \cup (1,\infty)}$

7. Let $f(x) = x^5 - 5x^4 + x^3 + 39x^2 - 88x + 60$

   a. (10 pts) Find the *real* zeros of $f(x)$. Factor f over the set of real numbers. Use scratch paper (the back of page 5) to make your guesses, and then use the *correct* guesses to break f down in the space, below.

$$
\begin{array}{r|rrrrrr}
2 & 1 & -5 & 1 & 39 & -88 & 60 \\
 & & 2 & -6 & -10 & 58 & -60 \\
\hline
2 & 1 & -3 & -5 & 29 & -30 & 0 \\
 & & 2 & -2 & -14 & 30 & \\
\hline
-3 & 1 & -1 & -7 & 15 & 0 & \\
 & & -3 & 12 & -15 & & \\
\hline
 & 1 & -4 & 5 & 0 & &
\end{array}
$$

$x^2 - 4x + 5$

$a = 1, b = -4, c = 5$

$b^2 - 4ac = (-4)^2 - 4(1)(5)$

$= 16 - 20$

$= -4$ No real zero, so

$f(x) = (x-2)^2(x+3)(x^2-4x+5)$ is factored over $\mathbb{R}$, with zeros $x = 2$, $x = -3$
$m = 2$ $\quad m = 1$

   b. (5 pts) Find the remaining (nonreal) zeros of f and factor f over the set of *complex* numbers.

$x = \dfrac{4 \pm \sqrt{-4}}{2} = \dfrac{4 \pm 2i}{2} = \boxed{2 \pm i \text{ are the nonreal zeros}}$

So $\boxed{f(x) = (x-2)^2(x+3)(x-(2+i))(x-(2-i))}$

8. (10 pts) Suppose $R(x) = \dfrac{2x^3 - 16x^2 + 2x + 84}{x^3 - x^2 - 10x - 8}$ can be factored into $\dfrac{2(x-3)\cancel{(x+2)}(x-7)}{\cancel{(x+2)}(x-4)(x+1)}$.

(It can.) Sketch the graph of R showing all intercepts, asymptotes and holes (if any).

HA: $y = 2$

VA: $x = 4, x = -1$

HOLE: $x = -2$: $\dfrac{2(-2-3)(-2-7)}{(-2-4)(-2+1)} = \dfrac{\cancel{2}(-5)(-9)}{(\cancel{-6})(-1)} = \dfrac{45}{3} = 15$

-3

zeros: $x = 3, 7$

y-int: $(0, -\frac{84}{8})$

$= (0, -\frac{21}{2})$