Version 1

MAT 121 – G81 Spring, 2012

Test 3, Chapter 3 80 Points Name KEY

1.(5 pts) Form a polynomial in factored form with *real* coefficients with the given zeros and degree. Please do not expand the polynomial.

Zeros: $x = 2$, multiplicity 2; $x = -1$, multiplicity 2. Degree 4.

$$(x-2)^2(x+1)^2$$

2. (10 pts) Expand $(x-(2+i))(x-(2-i))$

$$x^2 - (2-i)x - (2+i)x + (2+i)(2-i)$$

$$= x^2 - 2x + ix - 2x - ix + 2^2 + 1^2$$

$$= \boxed{x^2 - 4x + 5}$$

3. (10 pts) Use synthetic division to find $P(3)$ if $P(x) = 2x^4 + x^3 + 3x^2 - 2x + 5$.

$$\begin{array}{r|rrrrr} 3 & 2 & 1 & 3 & -2 & 5 \\ & & 6 & 21 & 72 & 210 \\ \hline & 2 & 7 & 24 & 70 & \boxed{215 = P(3)} \end{array}$$

4. (5 pts) Divide $f(x) = x^4 - 3x^3 + 2x^2 + 5$ by $d(x) = x^2 + 3$. Then write the result in the form *Dividend = Divisor · Quotient + Remainder*.

$$\begin{array}{r} x^2 - 3x - 1 \\ x^2+3 \overline{\big)\, x^4 - 3x^3 + 2x^2 + 0x + 5} \\ -(x^4 \qquad\quad + 3x^2) \\ \hline -3x^3 - x^2 + 0x + 5 \\ -(-3x^3 \qquad - 9x) \\ \hline -x^2 + 9x + 5 \\ -(-x^2 \qquad\quad - 3) \\ \hline 9x + 8 \end{array}$$

$$\boxed{f(x) = (x^2+3)(x^2-3x-1) + 9x + 8}$$

5. Let $f(x) = 3x^5 - 15x^4 + 21x^3 + 3x^2 - 24x + 12$, and suppose its factored form is given by $f(x) = 3(x-2)^2(x+1)(x-1)^2$

a. (10 pts) List each real zero and its multiplicity. Determine whether the graph of $f(x)$ touches or crosses the $x$-axis at the corresponding $x$-intercepts.

$x=2$ $m=2$ Touch

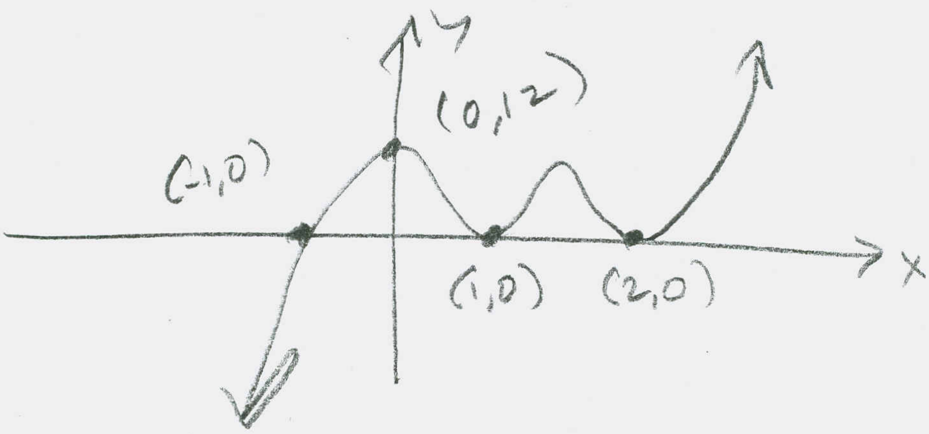
$x=-1$ $m=1$ cross

$x=1$ $m=2$ touch

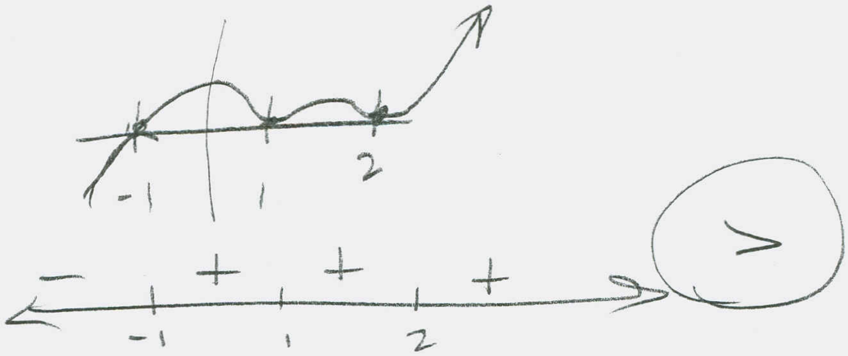
b. (5 pts) What power function does $f$ resemble for large values of $|x|$? In other words, give the end behavior for $f$, along with a simple diagram.

$3x^5$

c. (5 pts) Use your work, above, to help you sketch the graph of $f(x)$, showing all intercepts (including the $y$-intercept).



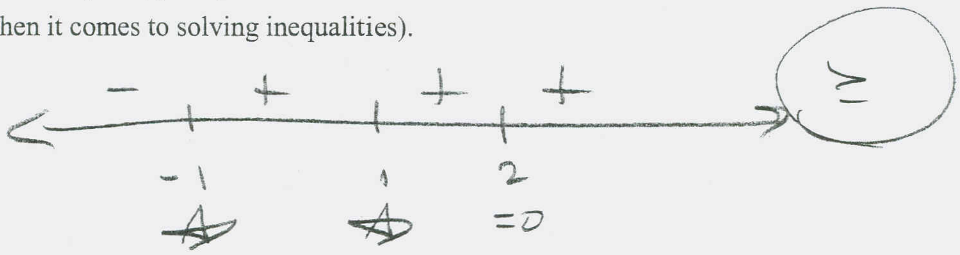
6. Use your sketch from the previous problem to help you solve the following inequalities. You might want to re-sketch it, below, just to have it on the same page.




a. (5 pts) $3(x-2)^2(x+1)(x-1)^2 > 0$

$x \in (-1,1) \cup (1,2) \cup (2,\infty)$

b. (5 pts) $\dfrac{3(x-2)^2}{(x+1)(x-1)^2} \geq 0$ (A very different-looking function, but not so very different, when it comes to solving inequalities).




$x \in (-1,1) \cup (1,2] \cup [2,\infty)$

$= \boxed{(-1,1) \cup (1,\infty)}$

Teacher screw-up ! ?

7. Let $f(x)=x^5-4x^4+7x^3-10x^2-62x-40$

a. (10 pts) Find the *real* zeros of $f(x)=x^4-5x^3+15x^2-5x-26$. Factor $f$ over the set of real numbers. Use scratch paper (the back of page 5) to make your guesses, and then use the *correct* guesses to break $f$ down in the space, below.

$x^5-4x^4+7x^3-10x^2-62x-40$

$$\begin{array}{r|rrrrrr} -1 & 1 & -4 & 7 & -10 & -62 & -40 \\ & & -1 & 5 & -12 & 22 & 40 \\ \hline -1 & 1 & -5 & 12 & -22 & -40 & 0 \\ & & -1 & 6 & -18 & 40 & \\ \hline 4 & 1 & -6 & 18 & -40 & 0 & \\ & & 4 & -8 & 40 & & \\ \hline & 1 & -2 & 10 & 0 & & \\ & a & b & c & & & \end{array}$$

$b^2-4ac=(-2)^2-4(1)(10)=4-40=-36$

No real zero of the depressed polynomial $x^2-2x+10$, so

$\boxed{f(x)=(x+1)^2(x-4)(x^2-2x+10),}$

if you factor over the field of real numbers

$\boxed{x=-1, 4 \text{ are the real zeros}}$

$x^4-5x^3+15x^2-5x-26$

$$\begin{array}{r|rrrrr} -1 & 1 & -5 & 15 & -5 & -26 \\ & & -1 & 6 & -21 & 26 \\ \hline 2 & 1 & -6 & 21 & -26 & 0 \\ & & 2 & -8 & 26 & \\ \hline & 1 & -4 & 13 & 0 & \\ & a & b & c & & \end{array}$$

$b^2-4ac=(-4)^2-4(1)(13)=$
$=16-52=-36$

No real zero, so

$f(x)=(x+1)(x-2)(x^2-4x+13)$

is factored over $\mathbb{R}$, and

$\boxed{\text{real zeros are } x=-1, 2}$

$x=\dfrac{4\pm 6i}{2}=2\pm 3i$ are the $\boxed{\text{nonreal zeros}}$

b. (5 pts) Find the remaining (nonreal) zeros of $f$ and factor $f$ over the set of *complex* numbers.

$b^2-4ac=-36 \Longrightarrow x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{2\pm\sqrt{-36}}{2(1)}=\dfrac{2\pm 6i}{2}$

$=1\pm 3i$ are the zeros of $x^2-2x+10$, so

$f(x)=(x+1)^2(x-4)(x-(1+3i))(x-(1-3i))$ (LEFT)

$f(x)=(x+1)(x-2)(x-(2+3i))(x-(2-3i))$ (RIGHT)

8. (10 pts) Suppose $R(x) = \dfrac{2x^3 - 16x^2 + 2x + 84}{x^3 - x^2 - 10x - 8}$ can be factored into $\dfrac{2(x-3)\cancel{(x+2)}(x-7)}{\cancel{(x+2)}(x-4)(x+1)}$.

(It can.) Sketch the graph of $R$ showing all intercepts, asymptotes and holes (if any).

H.A.: $y = 2$

V.A: $x = -1$, $x = 4$

HOLE: $x = -2$: $\dfrac{2(-2-3)(-2-7)}{(-2-4)(-2+1)} = \dfrac{\overset{1}{\cancel{2}}(-5)(-9)}{(-\underset{3}{\cancel{6}})(-1)} = \dfrac{45}{3} = 15$

(HOLE @ $(-2, 15)$)

zeros: x-int $x = 3, 7$

y-int: $\left(0, -\frac{84}{8}\right)$ $= \left(0, -\frac{21}{2}\right)$