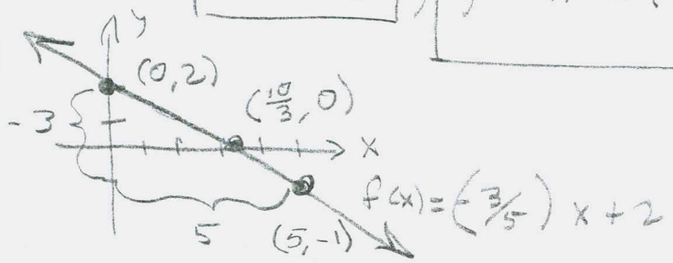
121-GBI TEST 2 SPRING, 2012

(1) $f(x) = -\frac{3}{5}x + 2 = mx + b$ (1)

(a) slope = $\boxed{m = -\frac{3}{5}}$, $\boxed{y\text{-int} = (0, 2)}$

(b)



(c) x-int: $f(x) = -\frac{3}{5}x + 2 \overset{\text{SET}}{=} 0$

$-\frac{3}{5}x = -2$

$x = (-2)(-\frac{5}{3}) = \frac{10}{3}$ → $\boxed{(\frac{10}{3}, 0)}$

(d) f(x) is decreasing

(2) (a) $h(x) = 5x^2 - 3x + 2 = ax^2 + bx + c \Rightarrow$

$a = 5, b = -3, c = 2 \Rightarrow$

$b^2 - 4ac = (-3)^2 - 4(5)(2) = 9 - 40 = \boxed{-31} < 0$

$\Rightarrow$ $\boxed{\text{Two, distinct, nonreal zeros}}$

(b) $h(x) = 3x^2 - 5x + 2$

$\Rightarrow a = 3, b = -5, c = 2$

$\Rightarrow b^2 - 4ac = (-5)^2 - 4(3)(2)$

$= 25 - 24$

$= \boxed{1} > 0$

$\Rightarrow$ $\boxed{\text{Two distinct, real zeros.}}$

Also, $1 = 1^2$ is a perfect square, so we know the zeros are rational.

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(2)

(3) $f(x) = 6x^2 - 13x + 6$

(a) Factor by ac method(s)

$ac = (6)(6) = 36$ WANT factors of 36 whose sum is -13.

From the factors direction:

$2 \mid 36$ $(-2)(-18) = 36$

$2 \mid 18$ $-2-18 = -20$

$3 \mid 9$ $(-4)(-9) = 36$

3 $-4-9 = -13$ ✓

From the "sum $= -13$" direction:

$-13 = -1-12$ $(-1)(-12) = 12$

$-13 = -2-11$ $(-2)(-11) = 22$

$-13 = -3-10$ $(-3)(-10) = 36$

$-13 = -4-9$ $(-4)(-9) = 36$ ✓

$6x^2 - 4x - 9x + 6$

$= 2x(3x-2) - 3(3x-2)$

$= (3x-2)(2x-3) \overset{\text{SET}}{=} 0$

$\Rightarrow 3x-2 = 0$ OR $2x-3 = 0$

$\Rightarrow 3x = 2$ OR $2x = 3$

$\Rightarrow x = \frac{2}{3}$ OR $x = \frac{3}{2}$

$\Rightarrow \boxed{x \in \left\{\frac{2}{3}, \frac{3}{2}\right\}}$

(b) $a = 6, b = -13, c = 6$

$b^2 - 4ac = (-13)^2 - 4(6)(6)$

$= 169 - 144 = 25$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{13 \pm \sqrt{25}}{2(6)} = \frac{13 \pm 5}{12}$

$\frac{13+5}{12} = \frac{18}{12} = \frac{3}{2}$

$\frac{13-5}{12} = \frac{8}{12} = \frac{2}{3}$

$\boxed{x \in \left\{\frac{2}{3}, \frac{3}{2}\right\}}$

(4) $f(x) = x^2 + 6x - 5 \overset{\text{SET}}{=} 0$

$\Rightarrow x^2 + 6x = 5$

$\Rightarrow x^2 + 6x + \left(\frac{6}{2}\right)^2 = 5 + 3^2$

$\Rightarrow (x+3)^2 = 5 + 9$

$\Rightarrow (x+3)^2 = 14$

$\Rightarrow \sqrt{(x+3)^2} = \sqrt{14}$

$\Rightarrow |x+3| = \sqrt{14}$

$\Rightarrow x+3 = \sqrt{14}$ OR $x+3 = -\sqrt{14}$

$\Rightarrow x = -3+\sqrt{14}$ OR $x = -3-\sqrt{14}$ $\Rightarrow \boxed{x \in \{-3 \pm \sqrt{14}\}}$

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TEST 2

(4) $f(x) = x^2 - 10x + 21$

$= x^2 - 10x + 5^2 - 5^2 + 21$

$= (x-5)^2 - 4$

RIGHT 5 ↑ ↑ DOWN 4

Scratch: $-25 + 21 = -4$

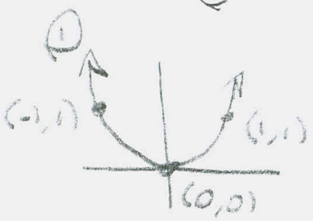
(3)

y-int: 4
x-int: 4
vertex: 4
AOS: 4

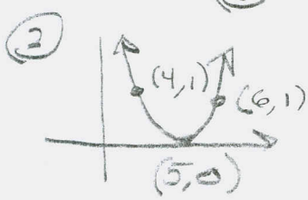
$(x-h)^2 + k$: 4
Range: 4

$x^2 \longrightarrow (x-5)^2 \longrightarrow (x-5)^2 - 4$

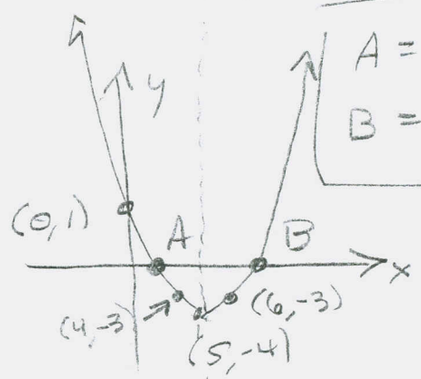
① ② ③



x^2



$(x-5)^2$



$(x-5)^2 - 4$

A = (3,0)
B = (7,0)

To find A & B, the x-intercepts, I use the completing-the-square work.

$f(x) = (x-5)^2 - 4 \overset{\text{SET}}{=} 0$

$\Rightarrow (x-5)^2 = 4$

$\Rightarrow \sqrt{(x-5)^2} = \sqrt{4}$

$\Rightarrow |x-5| = 2$

$\Rightarrow x - 5 = \pm 2$

$\Rightarrow x = 5 \pm 2$ → $x = 7$, $x = 3$

$x \in \{3, 7\}$

(5) $f(x) = 4x^2 - 12x + 10 \overset{\text{SET}}{=} 0$

① $\Rightarrow 2x^2 - 6x + 5 = 0$

② $\Rightarrow 2(x^2 - 3x \quad) + 5 = 0$

③ $\Rightarrow 2\left(x^2 - 3x + \left(\frac{3}{2}\right)^2\right) = -5 + \left(\frac{9}{4}\right)(2)$

④ $\Rightarrow 2\left(x - \frac{3}{2}\right)^2 = -\frac{1}{2}$

⑤ $\Rightarrow \left(x - \left(\frac{3}{2}\right)\right)^2 = -\frac{1}{4}$

Scratch: $-5 + \left(\frac{9}{4}\right)(2)$

$= -5 + \frac{9}{2}$

$= \frac{-10+9}{2}$

$= -\frac{1}{2}$

① Factor out '2'. Throw it away.

② Factor out 2 from x^2 & x terms.

③ Add $2\left(\frac{3}{2}\right)^2$ to both sides

ADDED $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$ inside. That's $\frac{9}{2}$ outside.

$x - \frac{3}{2} = \pm\sqrt{-\frac{1}{4}} = \pm\frac{1}{2}i$

$x = \frac{3}{2} \pm \frac{1}{2}i$ OR $\frac{3 \pm i}{2}$

$x \in \left\{\frac{3 \pm i}{2}\right\}$

(6) I misled you on this question, by getting the signs wrong on the "13x" and the "6" on the right-hand side. I MEANT to write $6x^2 - 13x \geq -6$, *sigh* I'll go ahead and work this one, for you, but I'm throwing out the prob.

(4)

$6x^2 + 13x \geq 6$

$6x^2 + 13x - 6 \geq 0$

$a = 6, b = 13, c = -6$

$b^2 - 4ac = 13^2 - 4(6)(-6)$

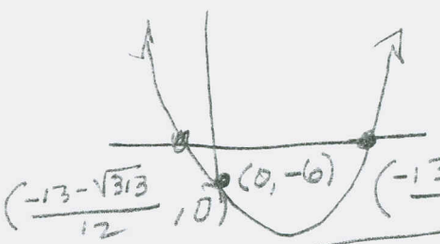
$= 169 + 144$

$= 313$

$$x = \frac{-13 \pm \sqrt{313}}{2(6)}$$

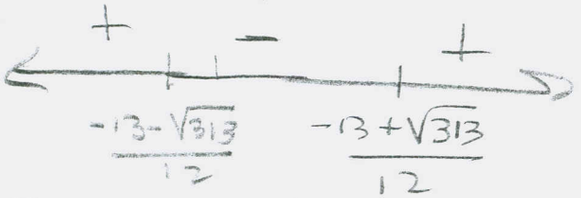
$$= \frac{-13 \pm \sqrt{313}}{12}$$

It's enough to know that $-13 + \sqrt{313} > 0$ to get an idea of how the graph looks: The y-int = (0, -6) confirms:



(-2.56, 0) (.39, 0) oops! Picture?

All we need for this problem is the position of the x-intercepts:



From our understanding of what the zeros (x-ints) say about the graph, we get the +, −, + pattern above. $\left(-\infty, \frac{-13-\sqrt{313}}{12}\right)$

$x = -3$: $6(-3)^2 + 13(-3) - 6 = 9$

$21 \geq 0$ ✓ + Yes

$\left(\frac{-13-\sqrt{313}}{12}, \frac{-13+\sqrt{313}}{12}\right)$

$x = 0$: $6(0)^2 + 13(0) - 6 = -6$

$-6 < 0$ − No

$\left(\frac{-13+\sqrt{313}}{12}, \infty\right)$

$x = 1$: $6(1)^2 + 13(1) - 6 = 13$

$13 \geq 0$ + Yes

$x \in \left(-\infty, \frac{-13-\sqrt{313}}{12}\right] \cup \left[\frac{-13+\sqrt{313}}{12}, \infty\right)$

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SET-BUILDER: $\{x \mid x \le \frac{-13-\sqrt{313}}{12}$ OR $x \ge \frac{-13+\sqrt{313}}{12}\}$ (5)

(6) Critical Regarding The "OOPS! Picture?" on the previous page:

When I punched out $\frac{13 \pm \sqrt{313}}{12}$ on my calculator, I FINALLY discovered that I got the vertex on the wrong side. 0.39 is closer to the y-axis than $x = -2.56$, so the vertex SHOULD be on the -2.56 side of the y-axis, since the vertex is halfway between the x-ints. This doesn't affect the problem we are solving. All we needed was

to solve the inequality!

The problem I wanted:

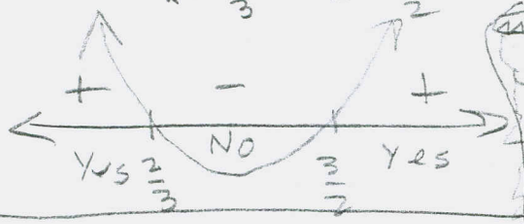
$6x^2 - 13x \ge -6$

$6x^2 - 13x + 6 \ge 0$

By Previous Work:

$(3x-2)(2x-3) \ge 0$

$x = \frac{2}{3}$ $x = \frac{3}{2}$



$x \in (-\infty, \frac{2}{3}] \cup [\frac{3}{2}, \infty)$

$= \{x \mid x \le \frac{2}{3}$ OR $x \ge \frac{3}{2}\}$

When in doubt, test:

$(-\infty, \frac{2}{3}]$ $x = 0$: $6(0)^2 - 13(0) + 6 = 6 > 0$ Yes

$[\frac{2}{3}, \frac{3}{2}]$ $x = 1$: $6(1)^2 - 13(1) + 6 = -1 < 0$ No

$[\frac{3}{2}, \infty)$ $x = 2$: $6(2)^2 - 13(2) + 6 = 24 - 26 + 6 = 4 > 0$ Yes

Want ≥ 0

Want +

Want the graph above the x-axis.

The thought process (I don't need to see it, but I like.)

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(6) See previous 2 pages.

(7) $|3x-5| = 2$ 4pts each

$3x-5 = 2$ OR $3x-5 = -2$

$3x = 7$ OR $3x = 3$

$\{x \mid x = \frac{7}{3}$ OR $x = 1\}$

$= \{1, \frac{7}{3}\}$

(8) $|3x-11| < 7$

$3x-11 < 7$ and $3x-11 > -7$

$3x < 18$ and $3x > 4$

$x < \frac{18}{3}$ and $x > \frac{4}{3}$

$\{x \mid x < 6$ and $x > \frac{4}{3}\}$

AND

No $\frac{4}{3}$ Yes 6 No

$x \in (\frac{4}{3}, 6)$

(9) $|3x-11| \geq 7$

$3x-11 \geq 7$ OR $3x-11 \leq -7$

$3x \geq 18$ OR $3x \leq 4$

$\{x \mid x \geq 6$ OR $x \leq \frac{4}{3}\}$

Yes $\frac{4}{3}$ NO 6 Yes

$x \in (-\infty, \frac{4}{3}] \cup [6, \infty)$

(10) $|3x-11| \geq -7$

$(-\infty, \infty)$

(11) $|3x-11| \leq -7$

$\emptyset$

(12) $|3x-11| = -7$

$\emptyset$

(6)