MAT 121

TEST 1

(1) $\sqrt{6+2x} = f(x) \Rightarrow$ Domain

$= \{x \mid 6+2x \geq 0\}$

$= \boxed{\{x \mid x \geq -3\}}$ (a)

$= \boxed{[-3, \infty)}$ (b)

Scratch

$6+2x \geq 0$

$2x \geq -6$

$x \geq -\frac{6}{2} = -3$

(2) $f(x) = \frac{x-7}{x^2+2} \Rightarrow$ (a) $f(2) = \frac{2-7}{2^2+2} = \boxed{\frac{-5}{6} = f(2)}$

(b) $f(-2) = \frac{-2-7}{(-2)^2+2} = \frac{-9}{6} = \boxed{-\frac{3}{2} = f(-2)}$

(3) $f(x) = x^2 - 3x - 5 \Rightarrow$ Avg. rate of change from $x = 2$ to $x = 3$ is

$$\frac{f(3)-f(2)}{3-2} = \frac{3^2-3(3)-5-(2^2-3(2)-5)}{1}$$

Double-check #3

$$= \frac{9-9-5-(4-6-5)}{1} = -5-(-7) = -5+7 = \boxed{2}$$

(4a) $R = \{(2,-1), (3,2), (7,-1), (5,2)\} \Rightarrow$

$D = \{2, 3, 7, 5\}$,

$R = \{-1, 2\}$

R is a function. Yes

MAT 121 TEST 1

(4b) $\{(2,-1),(3,2),(7,-1),(3,-1)\} = R$ (2)

$\Rightarrow D = \{2,3,7\}$

$R = \{-1,2\}$

Not a function, b/c $x=3$ corresponds to $y=2$ and $y=-1$ and that's one too many y's for the one x.

(5) The difference quotient for $f(x) = 3x^2+2x$ is

$$\frac{f(x+h)-f(x)}{h} = \frac{3(x+h)^2+2(x+h)-(3x^2+2x)}{h}$$

$$= \frac{3(x^2+2xh+h^2)+2x+2h-3x^2-2x}{h}$$

$$= \frac{3x^2+6xh+3h^2+2x+2h-3x^2-2x}{h}$$

$$= \frac{6xh+3h^2+2h}{h} = \frac{h(6x+3h+2)}{h} = \boxed{6x+3h+2}$$

(6) $f(x)=\frac{x-2}{x-5}$ and $g(x)=\sqrt{2x-6}$ $\Rightarrow$

(a) $\mathcal{D}(f)=\{x \mid x-5\neq 0\}=\boxed{\{x \mid x\neq 5\}=\mathcal{D}(f)}$

(b) $\mathcal{D}(g)=\{x \mid 2x-6\geq 0\}=\boxed{\{x \mid x\geq 3\}=\mathcal{D}(g)}$

Scratch: $2x-6\geq 0$

$2x\geq 6$

$x\geq 3$

(c)(i) $\boxed{(f-g)(x)=\frac{x-2}{x-5}-\sqrt{2x-6}}$

$\mathcal{D}(f-g)=\{x \mid x\in\mathcal{D}(f) \text{ and } x\in\mathcal{D}(g)\}$

$=\boxed{\{x \mid x\neq 5 \text{ and } x\geq 3\}}$

$=$  AND

$=$ 

$=\boxed{[3,5)\cup(5,\infty)}$

(ii) $\boxed{(f\circ g)(x)=\frac{\sqrt{2x-6}-2}{\sqrt{2x-6}-5}}$

$\mathcal{D}(f\circ g)=\{x \mid x\in\mathcal{D}(g) \text{ AND } g(x)\in\mathcal{D}(f)\}$

$=\{x \mid x\geq 3 \text{ AND } \sqrt{2x-6}\neq 5\}$

$=\{x \mid x\geq 3 \text{ AND } x\neq \frac{31}{2}\}$

$(\sqrt{2x-6})^2\neq 5^2 \longrightarrow$ Scratch: $\sqrt{2x-6}\neq 5 \longleftarrow$

$2x-6\neq 25$

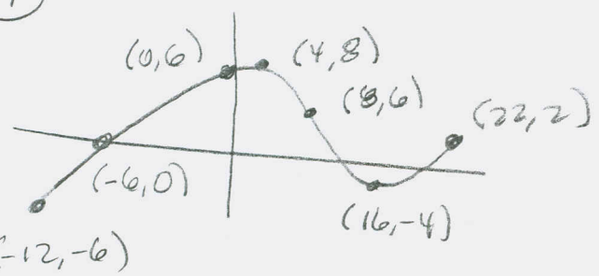
$2x\neq 31$

$x\neq \frac{31}{2}$

MAT 121

TEST 1

(7)



(4)

(a) $f(8) = 6$

(b) $f(-9)$ is negative

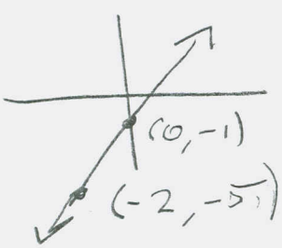
(c) $y=5$ intersects the graph once.

(d) $D(f) = [-12, 22]$ (e) $R(f) = [-6, 8]$

(f) f is increasing on $(-12, 4) \cup (16, 22)$

Including endpoints is OK, although it can depend on the textbook, which definition of increasing is used, so $[-12, 4] \cup [16, 22]$ is OK

(8)



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - (-1)}{-2 - 0} = \frac{-4}{-2} = 2$$

$$y = m(x - x_1) + y_1$$

$$y = 2(x - (-2)) + (-5)$$

(a) $\boxed{y = 2(x+2) - 5}$

$$y = 2x + 4 - 5$$

(b) $\boxed{y = 2x - 1}$

MAT 121 TEST 1

(5)

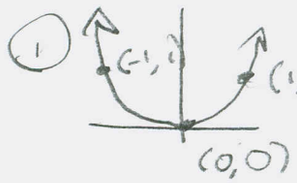
9a $g(x) = 2(x-5)^2 + 7$

① $f(x) = x^2$

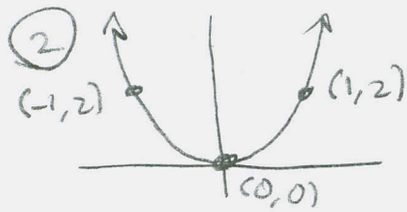
② $2f(x) = 2x^2$

③ $2f(x-5) = 2(x-5)^2$

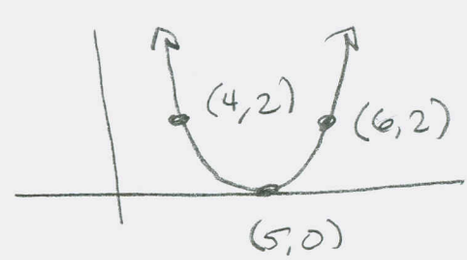
④ $2(f(x-5)) + 7 = g(x)$

① 

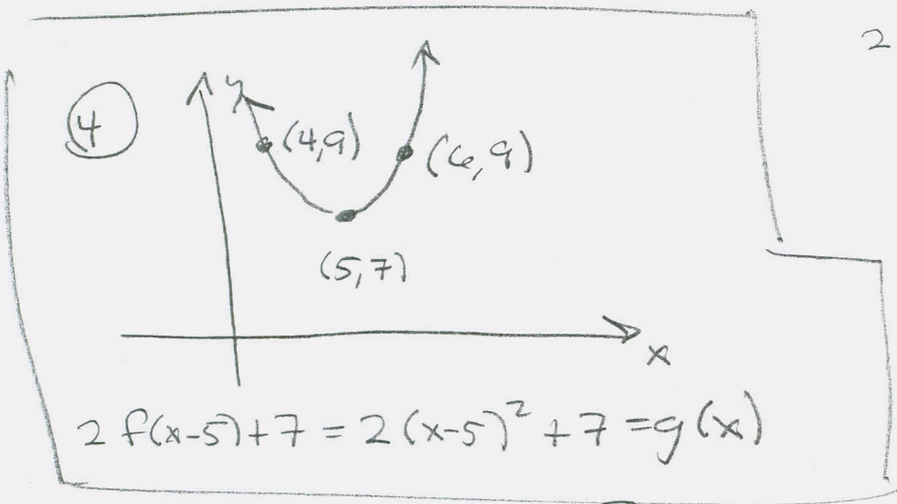
$f(x) = x^2$

② 

$2f(x) = 2x^2$

③ 

$2f(x-5) = 2(x-5)^2$

④ 

$2f(x-5)+7 = 2(x-5)^2 + 7 = g(x)$

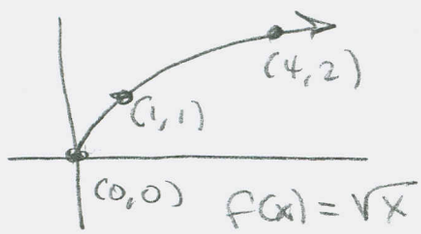
2 – Show stages
2 – Final Graph resulting from transformations
2 – Shape Basic Graph

9b $g(x) = \sqrt{x+2} - 3$

① $f(x) = \sqrt{x}$

② $f(x+2) = \sqrt{x+2}$

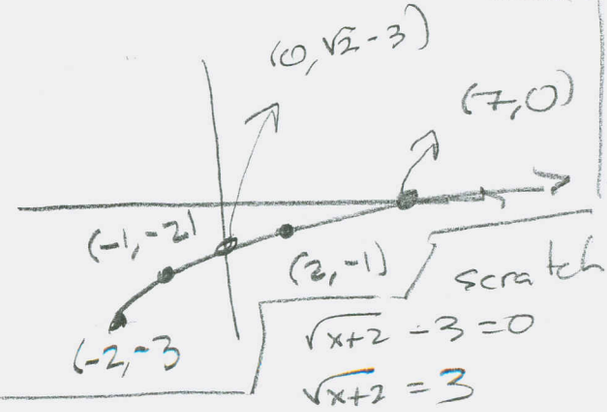
③ $f(x+2) - 3 = \sqrt{x+2} - 3 = g(x)$

① 

$f(x) = \sqrt{x}$

② 

$f(x+2) = \sqrt{x+2}$

③ 

scratch

$\sqrt{x+2} - 3 = 0$

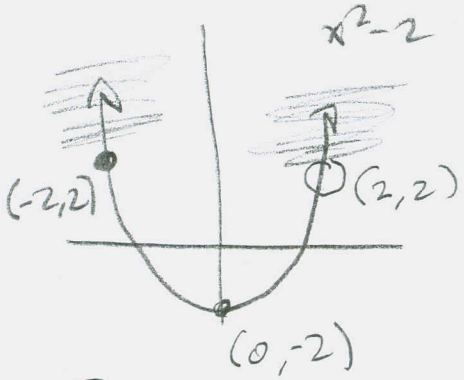
$\sqrt{x+2} = 3$

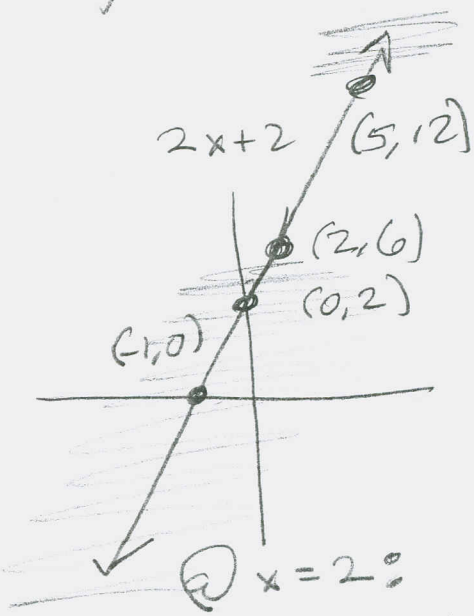
$x+2 = 9$

$x = 7$

(10) $f(x) = \begin{cases} x^2-2 & \text{if } -2 \le x < 2 \\ 2x+2 & \text{if } 2 \le x \le 5 \end{cases}$

Graph each piece, then find the suture points where the pieces fit together and the endpoints, if any.





$2x+2=0$
$2x=-2$
$x=-1$

① $x=-2$:
$(-2)^2-2=4-2=2$
$\rightsquigarrow (-2,2)$ ●

$x=2$:
$2^2-2=2$
$\rightsquigarrow (2,2)$ ○ ← open dot, from $x<2$

② $x=2$:
$2(2)+2=6$
$\rightsquigarrow (2,6)$ ●

② $x=5$:
$2(5)+2=12$
$\rightsquigarrow (5,12)$ ●