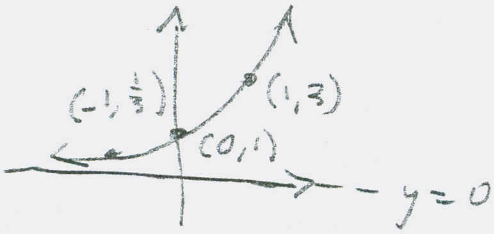
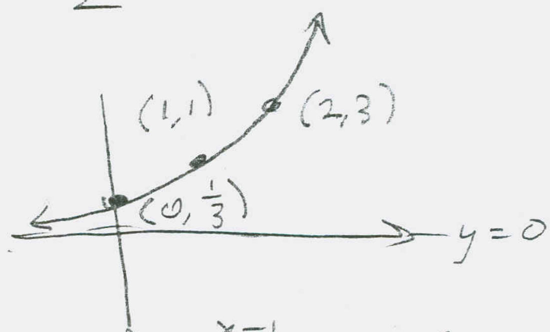
121-GS1 TEST 4

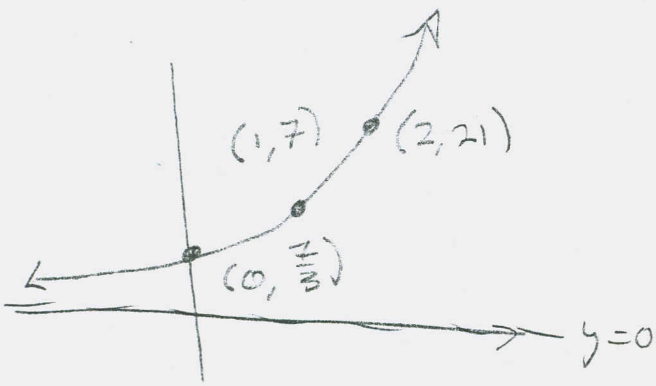
① Graph $7 \cdot 3^{x-1} - 2$



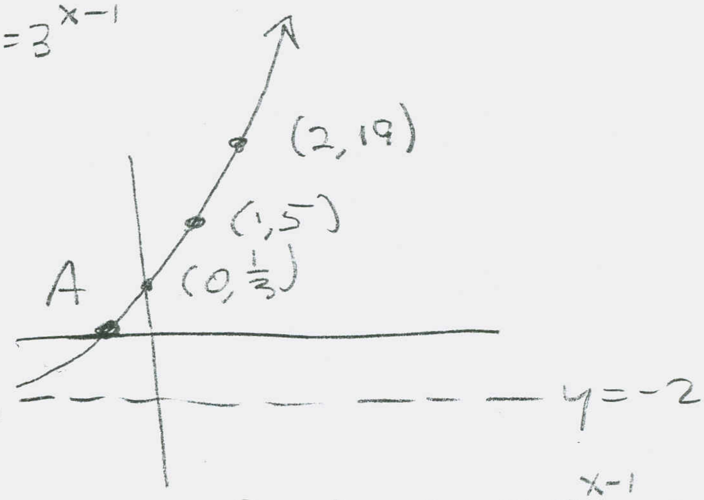
$f(x) = 3^x$



$f(x-1) = 3^{x-1}$



$7 \cdot f(x-1) = 7 \cdot 3^{x-1}$



$7f(x-1) - 2 = 7 \cdot 3^{x-1} - 2$

To find A: $7 \cdot 3^{x-1} - 2 \overset{\text{SET}}{=} 0$

$$7 \cdot 3^{x-1} = 2$$

$$3^{x-1} = \frac{2}{7}$$

$$x - 1 = \log_3\left(\frac{2}{7}\right)$$

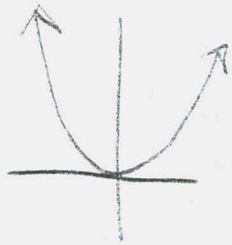
$$x = 1 + \log_3\left(\frac{2}{7}\right) =$$

$$= 1 + \frac{\ln(2/7)}{\ln(3)}$$

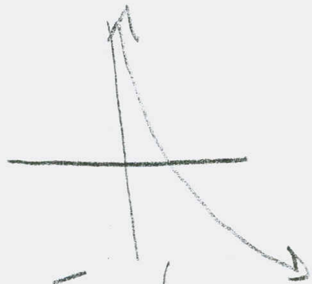
$$\approx (-.1403139956, 0)$$

$$\boxed{A = \left(1 + \log_3\left(\frac{2}{7}\right), 0\right) \approx (-.1403, 0)}$$

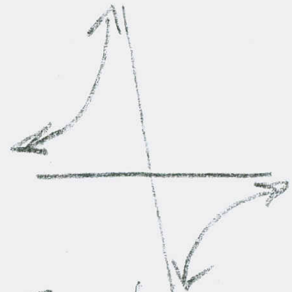
② Screwed <u>this</u> one up.



Function
Not 1-to-1



Function
1-to-1



Function
1-to-1

③ $f(x)=\sqrt{2x+1}$, $g(x)=\frac{1}{x}$

(a) $(f\circ g)(x) = \boxed{f(g(x)) = \sqrt{2\left(\frac{1}{x}\right)+1}} = \sqrt{\frac{2}{x}+1}$

$\mathcal{D}(f\circ g)$:

$\mathcal{D}(g) = \mathbb{R}\setminus\{0\} = (-\infty,0)\cup(0,\infty)$

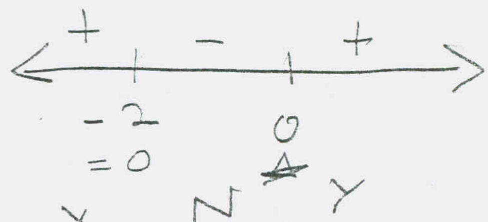
$\mathcal{D}(f) = \{x \mid 2x+1\geq 0\} = \{x \mid x \geq -\frac{1}{2}\} = [-\frac{1}{2},\infty)$

So $\mathcal{D}(f\circ g) = \{x \mid x\in\mathcal{D}(g) \text{ and } \frac{1}{x}\in\mathcal{D}(f)\}$

$\frac{1}{x}\in\mathcal{D}(f)$ piece: $\frac{1}{x}\geq -\frac{1}{2}$

$\frac{1}{x}+\frac{1}{2}\geq 0$

$\frac{2+x}{2x}\geq 0$



$\boxed{(-\infty,-2]\cup(0,\infty)}$

$= \mathcal{D}(f\circ g)$

(b) $(g\circ f)(x) = \frac{1}{\sqrt{2x+1}}$

Need $2x+1\geq 0$ and $\sqrt{2x+1}\neq 0 \Rightarrow$

$2x+1>0 \Rightarrow \mathcal{D} = \{x \mid x>-\frac{1}{2}\} = \boxed{(-\frac{1}{2},\infty) = \mathcal{D}(g\circ f)}$

(4) $\mathcal{D}(\ln(x+7))$

Need $x+7>0 \Rightarrow \boxed{\mathcal{D}=(-7,\infty)}$

(5) Domain of $\sqrt{\dfrac{(x-2)^2}{(x-3)(x+1)^3}}$

Need $\dfrac{(x-2)^2}{(x-3)(x+1)^3} \geq 0$

+	-1	−	2	−	3	+
	☆		$=0$		☆	

$x=2$, $m=2$ No change

$x=-1$, $m=3$ change

$x=3$, $m=1$ change

$\boxed{(-\infty,-1)\cup\{2\}\cup(3,\infty)}$

(6) $H(x)=e^{2x-5}$

Let $\boxed{f(x)=e^{x} \text{ and } g(x)=2x-5.}$

Then $H(x)=(f\circ g)(x)$

(7) $f(x) = \log_7(x+2)+3$

$\log_7(y+2)+3 = x$

$\log_7(y+2) = x-3$

$y+2 = 7^{x-3}$

$y = \boxed{7^{x-3}-2 = f^{-1}(x)}$

(8) $e^{x-7} = 4^{2x+1}$

$x-7 = \ln(4^{2x+1}) = (2x+1)\ln(4) = 2x\ln(4)+\ln(4)$

Let $a = \ln(4)$

Then $x-7 = 2ax+a \implies$

$x-2ax = a+7$

$x(1-2a) = a+7$

$x = \dfrac{a+7}{1-2a} = \boxed{\dfrac{\ln(4)+7}{1-2\ln(4)} = x}$

(9) (a) $4 - 2 + 1 - \frac{1}{2} + \cdots + \frac{1}{64}$

$a = 4$, $r = -\frac{1}{2}$, so we find n:

$$\frac{1}{64} = ar^{n-1} = 4\cdot\left(-\frac{1}{2}\right)^{n-1}$$

$$\frac{1}{64}\cdot\frac{1}{4} = \left(-\frac{1}{2}\right)^{n-1}$$

$$\frac{1}{2^6}\cdot\frac{1}{2^2} = \left(-\frac{1}{2}\right)^{n-1}$$

$$\frac{1}{2^8} = \left(-\frac{1}{2}\right)^{n-1} = (-1)^{n-1}\left(\frac{1}{2}\right)^{n-1}$$

$$\frac{1}{2^8} = (-1)^{n-1}\left(\frac{1}{2^{n-1}}\right)$$

$$(-1)^8\left(\frac{1}{2^8}\right) = (-1)^{n-1}\left(\frac{1}{2^{n-1}}\right)$$

$$\Rightarrow 8 = n-1 \Rightarrow 9 = n$$

$2 \,\overline{)\,64}$
$2 \,\overline{)\,32}$
$2 \,\overline{)\,16}$
$2 \,\overline{)\,8}$
$2 \,\overline{)\,4}$
2

$$S = \frac{a(1-r^n)}{1-r} = \frac{4\left(1-\left(-\frac{1}{2}\right)^9\right)}{1-\left(-\frac{1}{2}\right)}$$

$$= \frac{4\left(1+\frac{1}{2^9}\right)}{1+\frac{1}{2}} = \frac{4\left(1+\frac{1}{2^9}\right)}{\frac{3}{2}} = \frac{2}{3}\cdot 4\left(1+\frac{1}{2^9}\right)$$

$$= \frac{171}{64} = \boxed{2.671875}$$

(b) $$\sum_{k=1}^{\infty} 2\left(\frac{2}{5}\right)^{k-1} = \frac{a}{1-r} = \frac{2}{1-\frac{2}{5}} = \frac{2}{\frac{3}{5}} = \left(\frac{2}{1}\right)\left(\frac{5}{3}\right) = \boxed{\frac{10}{3}}$$

(10) $\ln(x-4) + \ln(x+1) = \ln(6)$

$\ln((x-4)(x+1)) = \ln(6)$

$x^2 - 3x - 4 = 6$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$\boxed{x = 5}$ OR $x = -2$

FINAL ANS.

Nope

(11) Doubling time is 10 yrs.

(a) $A(t) = A_0 e^{kt}$

$$A(10) = A_0 e^{10k} = 2A_0$$

$$e^{10k} = 2$$

$$10k = \ln(2)$$

$$\boxed{k = \frac{\ln(2)}{10}} \approx .069314718 1$$

(b) $A_0 = 500$ $A(t) = 6000$ in 1998.

Find when it started.

$$A_0 e^{kt} = 500 e^{kt} = 6000$$

$$e^{kt} = \frac{6000}{500} = 12$$

$$kt = \ln(12)$$

$$t = \frac{\ln(12)}{k} = \frac{\ln(12)}{\frac{\ln(2)}{10}} = \frac{10\ln(12)}{\ln(2)}$$

$\Rightarrow t \approx 35.8496250 1$

So, $1998 - 35.84962501 \approx 1962.150375$

That means the study started in $\boxed{1962}$