121 TEST 3
G81 KEY FALL 2011

(1) $\boxed{(x-1)^2(x+3)^2}$

(2) $(x+(2-7i))(x+(2+7i))$

$= x^2+2x+7ix+2x+4+14i-7ix-14i-49i^2$

$= x^2+4x+4+49 = \boxed{x^2+4x+53}$

(3)

$2\,|\,3 \quad 0 \quad -2 \quad 2 \quad -7 \quad 5$

$\qquad\quad 6 \quad 12 \quad 20 \quad 44 \quad 74$

$\qquad 3 \quad 6 \quad 10 \quad 22 \quad 37 \quad \boxed{79 = P(2)}$

(4)

$$x^2-2 \;\overline{)\; x^4-3x^3+2x^2+0x+5}$$

$$\underline{-(x^4 \qquad -2x^2)}$$

$$-3x^3+4x^2+0x+5$$

$$\underline{-(-3x^3 \qquad +6x)}$$

$$4x^2-6x+5$$

$$\underline{-(4x^2 \qquad -8)}$$

$$-6x+13$$

$$\boxed{f(x) = (x^2-2)(x^2-3x+4)-6x+13}$$

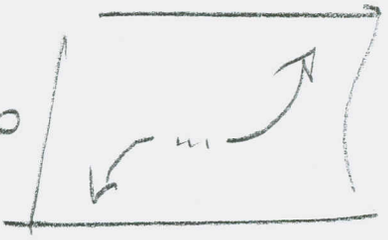
(5) $f(x) = 3x^5 - 15x^4 + 21x^3 + 3x^2 - 24x + 12$

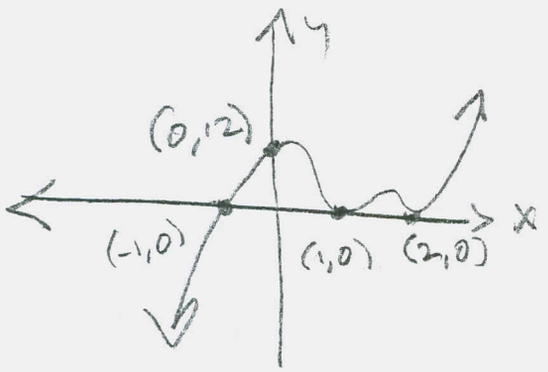
$= 3(x-2)^2(x+1)(x-1)^2$

(a) $x = -1$, $m = 1$ cross

$x = 1$, $m = 2$ touch

$x = 2$, $m = 2$ touch

(b) $\boxed{f(x) \approx 3x^5}$ as $x \to \pm\infty$ 

(c) 

(6) (a) $3(x-2)^2(x+1)(x-1)^2 \geq 0 \Rightarrow$

$\boxed{x \in [-1,1) \cup (1,2) \cup (2,\infty)}$

(b) $\dfrac{3(x-2)^2}{(x+1)(x-1)^2} \geq 0 \Rightarrow \boxed{x \in (-1,1) \cup (1,\infty)}$

(7)

5	1	-5	15	-5	-26
		5	0	75	350
	1	0	15	70	324

All nonnegative $\Rightarrow$ 5 is upper bound on real zeros.

(8) $f(x) = x^4 - 5x^3 + 15x^2 - 5x - 26$

$\pm 1, \pm 2, \pm 13, \pm 26$

-1 | 1 | -5 | 15 | -5 | -26
| | | -1 | 6 | -21 | 26

2 | 1 | -6 | 21 | -26
| | | 2 | -8 | 26
| | 1 | -4 | 13 | |

$x^2 - 4x + 13 = 0$

$x^2 - 4x = -13$

$x^2 - 4x + 2^2 = -13 + 4$

$(x-2)^2 = -9$

$x - 2 = \pm 3i$

$x = 2 \pm 3i$

$f(x) = (x+1)(x-2)(x^2-4x+13)$

(9) $f(x) = (x+1)(x-2)(x-(2+3i))(x-(2-3i))$

(10) $R(x) = \dfrac{x^3 - 8x^2 + x + 42}{x^3 - x^2 - 10x - 8} = \dfrac{(x-3)(x+2)(x-7)}{(x+2)(x-4)(x+1)} = \dfrac{(x-3)(x-7)}{(x-4)(x+1)}$ LOWEST TERMS

Domain: $\mathbb{R} \setminus \{-2, -1, 4\}$

HOLE: $x = -2$

Find Hole: $\dfrac{(-2-3)(-2-7)}{(-2-4)(-2+1)} = \dfrac{(-5)(-9)}{(-6)(-1)} = \dfrac{45}{6}$

$\left(-2, \dfrac{45}{6}\right)$

V.A.: $x = -1, x = 4$

H.A.: $y = \dfrac{x^3}{x^3} = 1 = y$

zeros: $x = 3, x = 7$

crosses @ both

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