MAT 121-G81 | Test 1 | Fall, 2011
100 Points

Name KEY

1. (7 pts) What is the domain of the function $f(x)=\sqrt{2x-7}$? Give your answer in set-builder notation (i.e., start with $\{x \mid \quad\quad\quad \}$), *and* interval notation.

Need $2x-7 \geq 0$

$2x \geq 7$

$\mathcal{D} = \{x \mid x \geq \frac{7}{2}\} = [\frac{7}{2}, \infty)$

2. Let $f(x)=\dfrac{x^2+13}{x^2-5}$. Find the following values:

a. (3 pts) $f(2) = \dfrac{4+13}{4-5} = \dfrac{17}{-1} = -17$

b. (3 pts) $f(-2) = -17$

3. (5 pts) What is the average rate of change of the function $r(x)=x^2+2x+7$, from $x=2$ to $x=3$?

$\dfrac{f(3)-f(2)}{3-2} = \dfrac{3^2+2(3)+7-(2^2+2(2)+7)}{1}$

$= 9+6+7-(4+4+7)$

$= 22-15$

$= \boxed{7 = m_{avg}}$

18

4. Determine whether each of the following relations represents a function. State the domain and range in each case. But if one is *not* a function, explain why.

a. (5 pts) $\{(2,-1),(3,2),(7,-1),(2,2)\}$

Domain: $\{2,3,7,\cancel{2}\}$ Repeats don't change the set, but they do affect the function designation. $\{1,2,3\}=\{1,2,2,3\}$ as sets.

Range: $\{-1,2\}$

Function? (If not, why not?) No. $(2,-1)$ & $(2,2)$ assign $x=2$ to two different y-values. is one way to word it. There's another. "$x=2$ corresponds to $y=-1$ and $y=2$, and $-1\neq 2$!"

b. (5 pts) $\{(2,-1),(3,2),(7,-1),(-1,2)\}$

Domain: $\{2,3,7,-1\}$

Range: $\{-1,2\}$

Function? (If not, why not?) Yes

5. (10 pts) Find the difference quotient of f, that is, find $\frac{f(x+h)-f(x)}{h}$, for $f(x)=2x^2-3x$. Simplify your answer. $(x+h)(x+h)=x^2+2xh+h^2$

$$\frac{f(x+h)-f(x)}{h} = \frac{2(x+h)^2-3(x+h)-(2x^2-3x)}{h}$$

$$= \frac{2(x^2+2xh+h^2)-3x-3h-2x^2+3x}{h}$$

$$= \frac{2x^2+4xh+2h^2-3h-2x^2}{h}$$

$$= \frac{4xh+2h^2-3h}{h}$$

$$= \frac{h(4x+2h-3)}{h} = 4x+2h-3$$

$f(x+h)=2(x+h)^2-3(x+h)$ is the hardest part. Expanding $2(x+h)^2 = 2(x+h)(x+h) = 2(x^2+2xh+h^2)$ is the next hardest part.

20

6. Let $f(x) = \sqrt{2x-6}$ and $g(x) = \dfrac{x+3}{x-1}$.

a. (5 pts) What is the domain of f ? (Set notation or interval notation)

$2x-6 \geq 0$

$2x \geq 6$

$x \geq 3$

$\{x \mid x \geq 3\} = \boxed{[3, \infty) = \mathcal{D}}$

b. (5 pts) What is the domain of g ? (Set notation or interval notation)

$x-1=0$

$x=1$ is bad

$\mathcal{D} = \{x \mid x \neq 1\}$

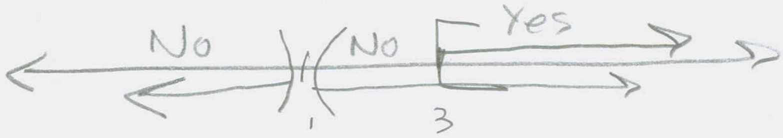
$= \boxed{(-\infty, 1) \cup (1, \infty)}$

c. Find the following functions and *find the domain of each one*. You do not need to simplify the functions.

i. (5 pts) $(f-g)(x)$

$\sqrt{2x-6} - \dfrac{x+3}{x-1}$

$[3, \infty) \cap \big((-\infty, 1) \cup (1, \infty)\big)$



$\mathcal{D} = [3, \infty)$

ii. (5 pts) $(g \circ f)(x)$ (The domain on this one is a little bit tricky.)

$\mathcal{D} = \{x \mid x \in \mathcal{D}(f) \text{ AND } f(x) \in \mathcal{D}(g)\}$

$\boxed{(g \circ f)(x) = \dfrac{\sqrt{2x-6} + 3}{\sqrt{2x-6} - 1}}$

Need $\sqrt{2x-6} \neq 1$ (See part (b))

$2x-6 \neq 1^2$

$2x \neq 7$

$x \neq \frac{7}{2}$

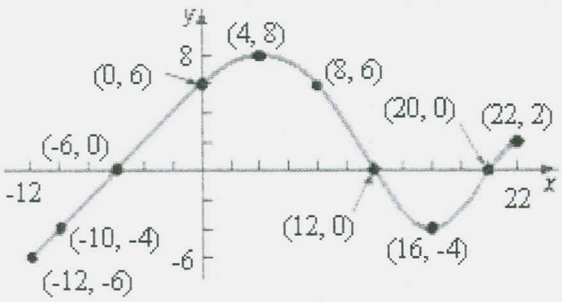
COMBINE:

$\{x \mid x \geq 3 \text{ and } x \neq \frac{7}{2}\}$

$= [3, \frac{7}{2}) \cup (\frac{7}{2}, \infty)$

20

7. Use the graph of the function *f*, below, to answer the following questions.



a. (2 pts) What is $f(-6)$? 0

b. (2 pts) Is $f(21)$ positive or negative? positive

c. (2 pts) How often does the line $y = 1$ intersect the graph of f?

3 times

d. (2 pts) What is the domain of f? [-12, 22]

Less

e. (2 pts) What is the range of f? [-6, 8]

Text defines these using open intervals

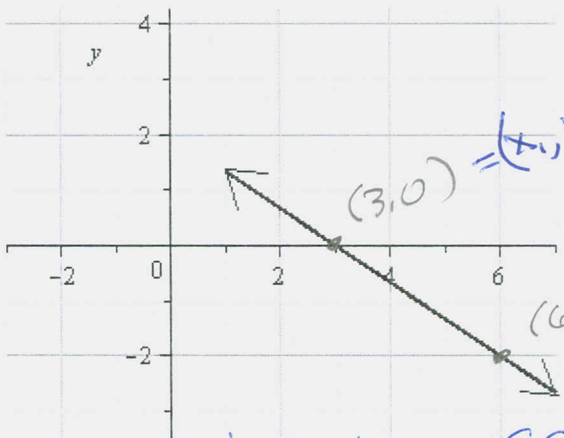
f. (2 pts) List the interval(s) on which f is increasing.

Some ambiguity, here: [-12, 4) OR $\boxed{(-12, 4)}$ → Like these better. Some authors use closed intervals (Yuck!)

Preference: OPEN INTERVALS OR $\boxed{(16, 22)}$ OR (16, 22]

8. (10 pts) Determine the equation of the line, below, from its graph. Give the equation in two forms:

a. point-slope
b. slope-intercept



(3, 0) = (x_1, y_1)

(6, -2) = (x_2, y_2)

$m = \frac{-2-0}{6-3} = \frac{-2}{3} = -\frac{2}{3}$

(a) $\boxed{y+2 = -\frac{2}{3}(x-6)}$

$y + 2 = -\frac{2}{3}x + 4$

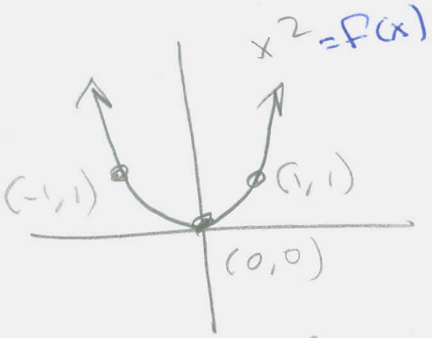
(b) $\boxed{y = -\frac{2}{3}x + 2}$

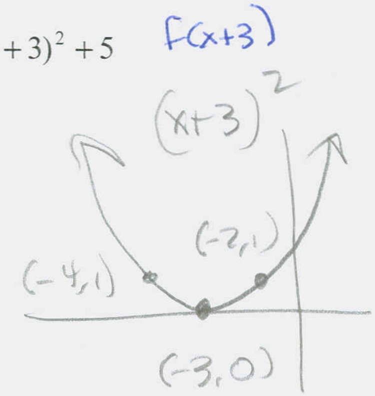
I expected this 099 question to be easy points.

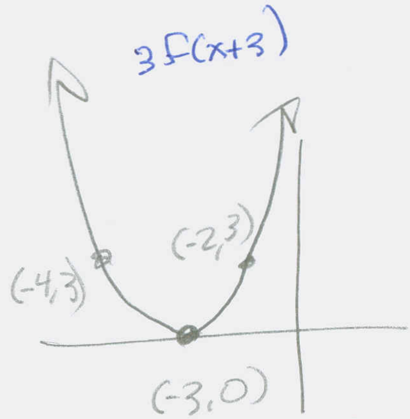
22

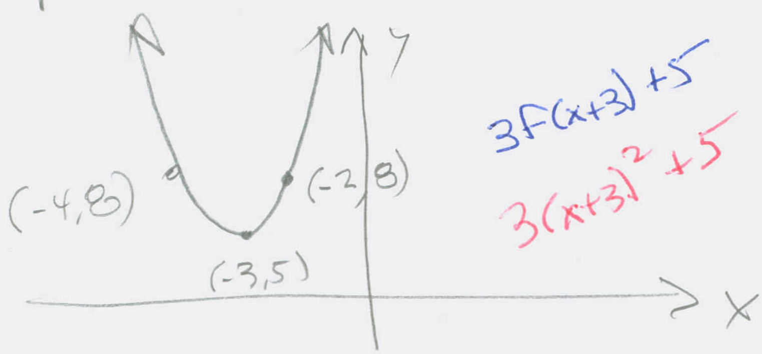
9. Graph each of the following functions using the techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function and show all stages.

a. (5 pts) $g(x) = 3(x+3)^2 + 5$









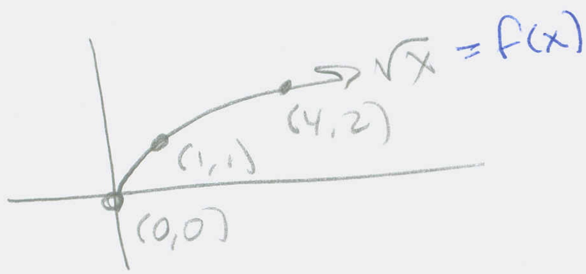
$3f(x+3)+5$

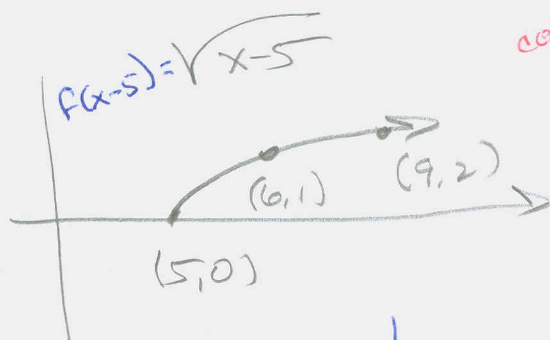
$3(x+3)^2+5$

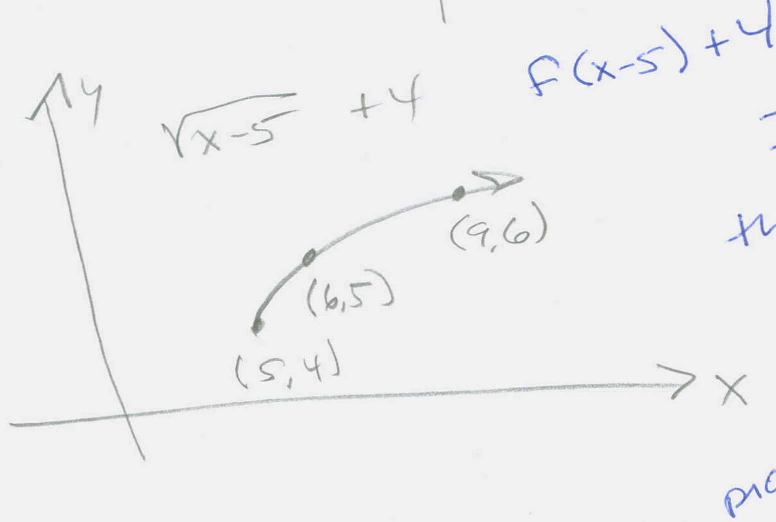
$3(x+3)^2$

This is wrong pg 5. Make copy of the correct pg 5.

b. (5 pts) $g(x) = \sqrt{x-5} + 4$







I'm giving the class 5 points for some of the problems we had with these. We will see them again.

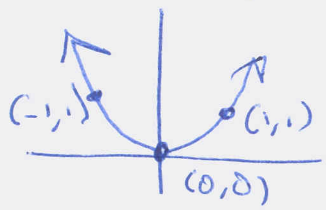
10

9a The ~~CORRECT~~ page 5:

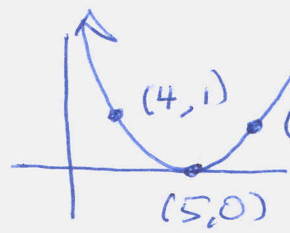
We'll see lots of these on Test 2, Chapter 2

$g(x) = 2(x-5)^2 + 7$

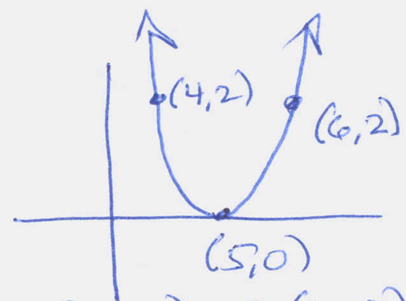
$f(x) = x^2$ is Basic Function



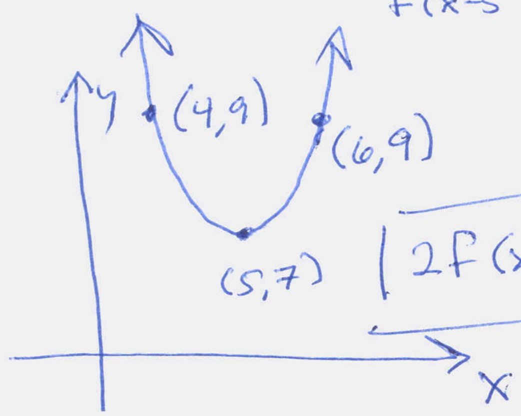
$f(x) = x^2$



$f(x-5) = (x-5)^2$



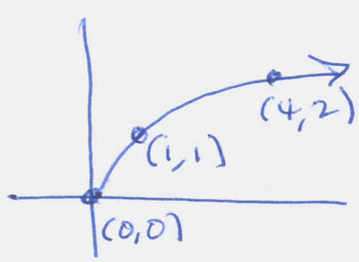
$2f(x-5) = 2(x-5)^2$



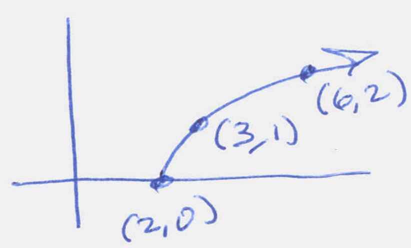
$$\boxed{2f(x-5) + 7 = 2(x-5)^2 + 7 = g(x)}$$

9b $g(x) = \sqrt{x-2} + 3$

$f(x) = \sqrt{x}$ is Basic Function

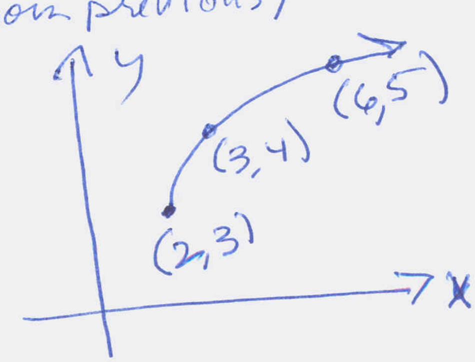


$f(x-2) = \sqrt{x-2}$ is Right 2 (from previous)



$f(x-2)+3$ is up 3 (from previous)

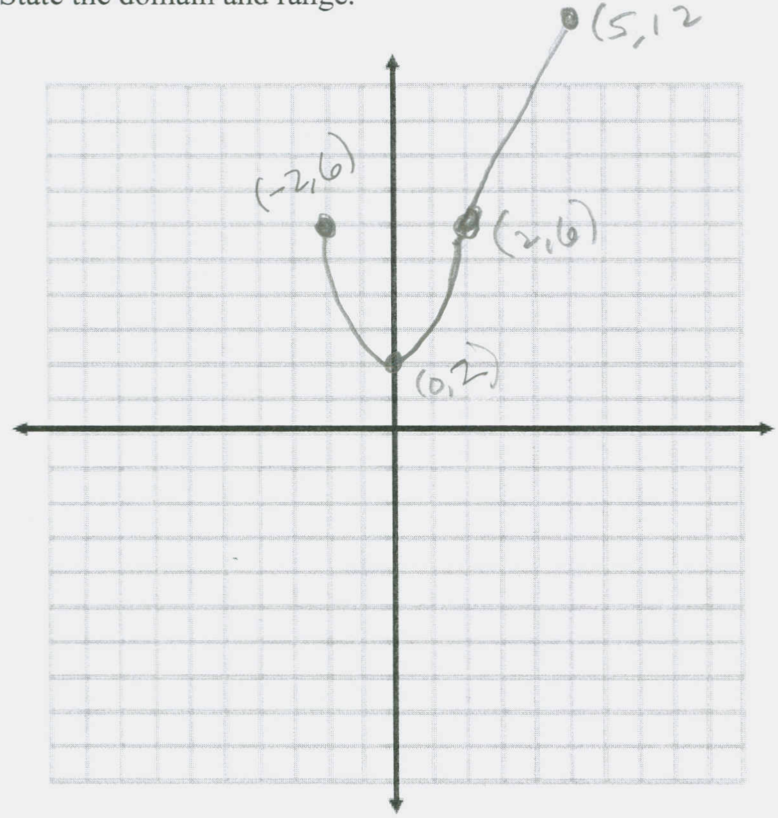
$= g(x) = \sqrt{x-2} + 3$

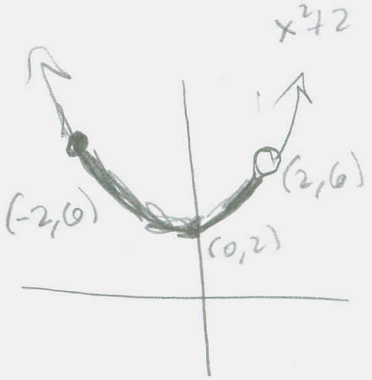


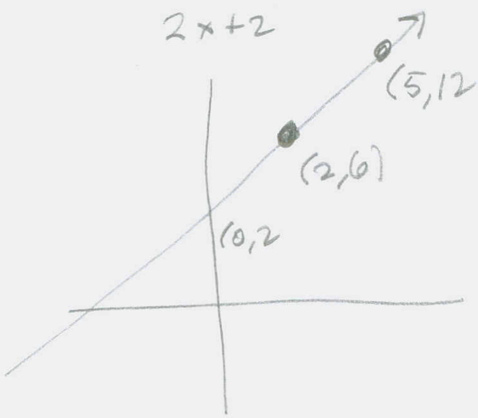
10. (10 pts) Sketch the graph of $f(x) = \begin{cases} x^2+2 & \text{if } -2 \le x < 2 \\ 2x+2 & \text{if } 2 \le x \le 5 \end{cases}$. Include all intercepts.

State the domain and range.

x=2 is suture point







$D = [-2, 5]$

$R = [2, 12]$

$-2 \le x < 2$

$x = 2 \Rightarrow$

$2^2 + 2 = 6 \rightarrow (2,6)$

HOLE

Left end:

$(-2)^2 + 2 = 6$

10