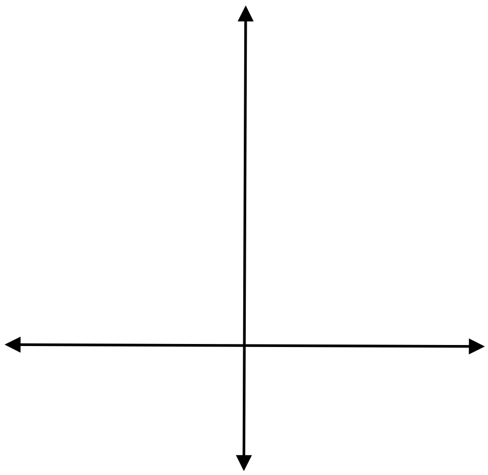
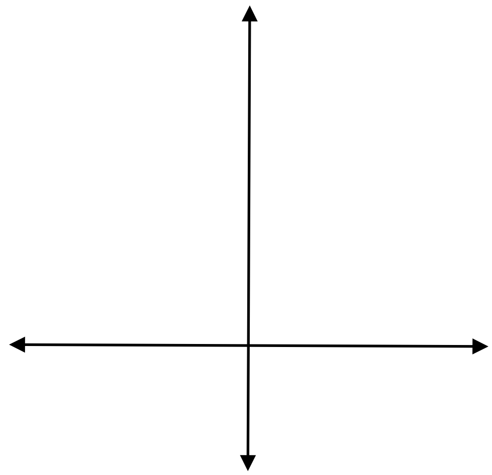
MAT 121-G81 | Test 4 – Spring, 2011 | Name________________________

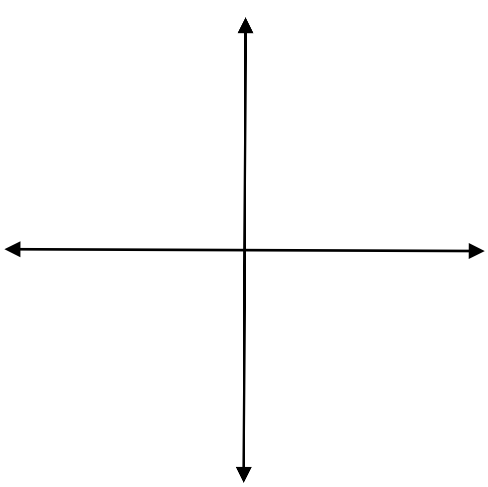
100 Points | Covers Chapter 4, plus 7.1, 7.3 | NO GRAPHING CALCULATOR!!!

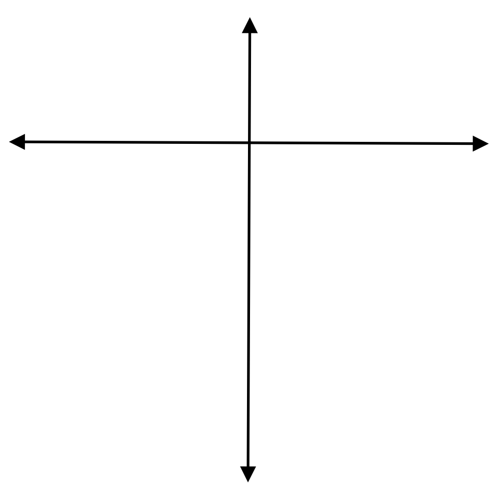
Graph:

1. (20 pts) Starting with $f(x) = 2^x$, sketch the graph of $g(x) = 4 \cdot 2^{x+1} - 5$ in 4 steps (counting $f(x) = 2^x$ as the first step). Use x = -1, x = 0, and x = 1 to find 3 points in the first graph, and show how these 3 points are moved around by each step in the transformation to $g(x)$. Your final graph should also show the y-intercept and, for 5 bonus points, the x-intercept.

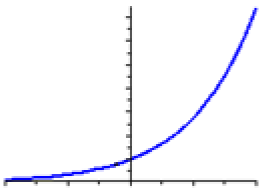
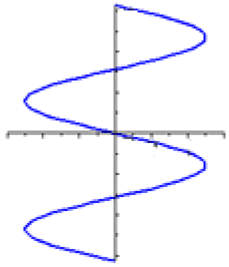
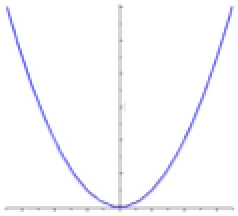
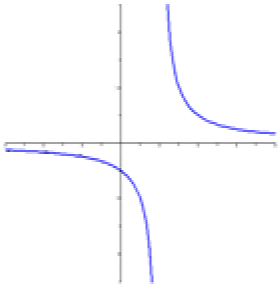








2. (10 pts) Determine which of the following are functions and whether they are one-to-one. So indicate by writing "Yes" or "No" in the appropriate spaces.

Function?	Function?	Function?	Function?
1-to-1?	1-to-1?	1-to-1?	1-to-1?

3. For $f(x)=\sqrt{x-1}$ and $g(x)=2x+1$, determine the following composite functions, *simplify them*, and state their domains:

a. (5 pts) $(f\circ g)(x)$

b. (5 pts) $(g\circ f)(x)$

4. (5 pts) What is the domain of $g(x) = \ln(-2x+3)$?

5. (5 pts) What is the domain of $\ln\left(\dfrac{(x-2)^2}{(x-3)(x+1)^3}\right)$? (This is like a Chapter 3 question!)

6. (5 pts) Let $f(x) = 5^{x+1} - 4$. Find $f^{-1}(x)$.

7. (5 pts) Find functions *f* and *g* so that $f \circ g = H$, given that $H(x) = \ln\left(x^2 - 1\right)$.

8. (5 pts) Evaluate $\log_2(96) - \log_2(3)$ *without a calculator* !!

9. (5 pts) Solve *without a calculator*: $5^{x-1} = 3^x$. All I want is a symbolic answer and the symbolic manipulations you perform to *get* there. For full credit, your answer should involve a logarithm or two in it.

10. (5 pts) Write the following as the logarithm of a single expression. Assume that variables represent positive numbers. $3\log_5(x+7) - 2\log_5(x-7) + \log_5 9$

11. (10 pts) Solve: $\ln(x-4)+\ln(x+1)=\ln(6)$ for x.

12. Find the geometric sums:

a. (5 pts) $2+\frac{2}{3}+\frac{2}{9}+\cdots+\frac{2}{2187}$ (Be careful finding your a, r, and n in $a\cdot r^{n-1}$)

b. (5 pts) $\sum_{k=1}^{\infty} 7\cdot\left(\frac{3}{5}\right)^{k-1}$

13. The half-life of carbon-14 is (approximately) 5500 years.

a. (5 pts) Derive the exponential decay model $A(t) = A_0 e^{-kt}$. The trick, here, is to find the decay rate, k, based on the half-life given.

b. (5 pts) Use your model from above to predict the age of an ancient fire pit, if a charcoal sample from the pit contains 20% of its original carbon-14. For ease of solving this problem, you may want to just use a symbolic k until the last step. Round your final answer to the nearest year.