121 Test 3 Take-Home

① A zero of a polynomial $P(x)$ is a solution of the equation $P(x)=0$.

A real zero, $x=c$, corresponds to an x-intercept, $(c,0)$, on the graph of the function. A complex zero, if nonreal, also satisfies $P(x)=0$, but won't show up on the graph.

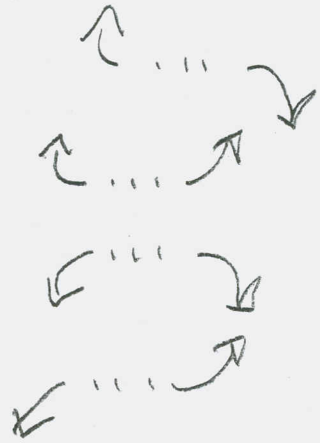
② Different versions.

$-4x^5+\ldots \Longrightarrow$

$3x^4+\ldots \Longrightarrow$

$-2x^6+\ldots \Longrightarrow$

$4x^3+\ldots \Longrightarrow$



#s 3-8 See following pages for your version.

⑨ #2 - End behavior checks out

#5 - The real roots correspond to x-intercepts. Those with even multiplicity correspond to "touches" and those with odd multiplicity correspond to "crosses."

#3 - The graph agrees with the predictions Descartes gave us. (Specifics vary)

#4 - The real roots turned out to be rational, and were among those predicted by the Rational Zeros Theorem.

9 cnt'd

#5 – The graph clearly shows the x-intercepts corresponding to the real zeros we found. Touch/cross status matches up with the parity (odd/even status) of the multiplicities of each zero.

#6 – You can "read" the x-intercepts & behavior near them from the factored form of the polynomial.

10 Actually #2 did NOT include the end behavior of the polynomial whose roots we actually found, but the graph is in line with end behavior predictions we would make (Answers vary, depending on specific test version).

We could plot more points to see if they agree with our rough graph.

We could use a graphing utility to check our work.

Other...

with(plots) :

Capital 'I' stands for the imaginary unit '*i*' in the displays. There were four versions of the test. One of the following should apply to the version you took.

$f := x \to x^5 - 4 \cdot x^4 + 2 \cdot x^3 + 12 \cdot x^2 - 24 \cdot x + 16$

$$x \to x^5 - 4\,x^4 + 2\,x^3 + 12\,x^2 - 24\,x + 16 \qquad \textbf{(1)}$$

$solve(f(x) = 0, x)$

$$-2,\ 1 + I,\ 1 - I,\ 2,\ 2 \qquad \textbf{(2)}$$

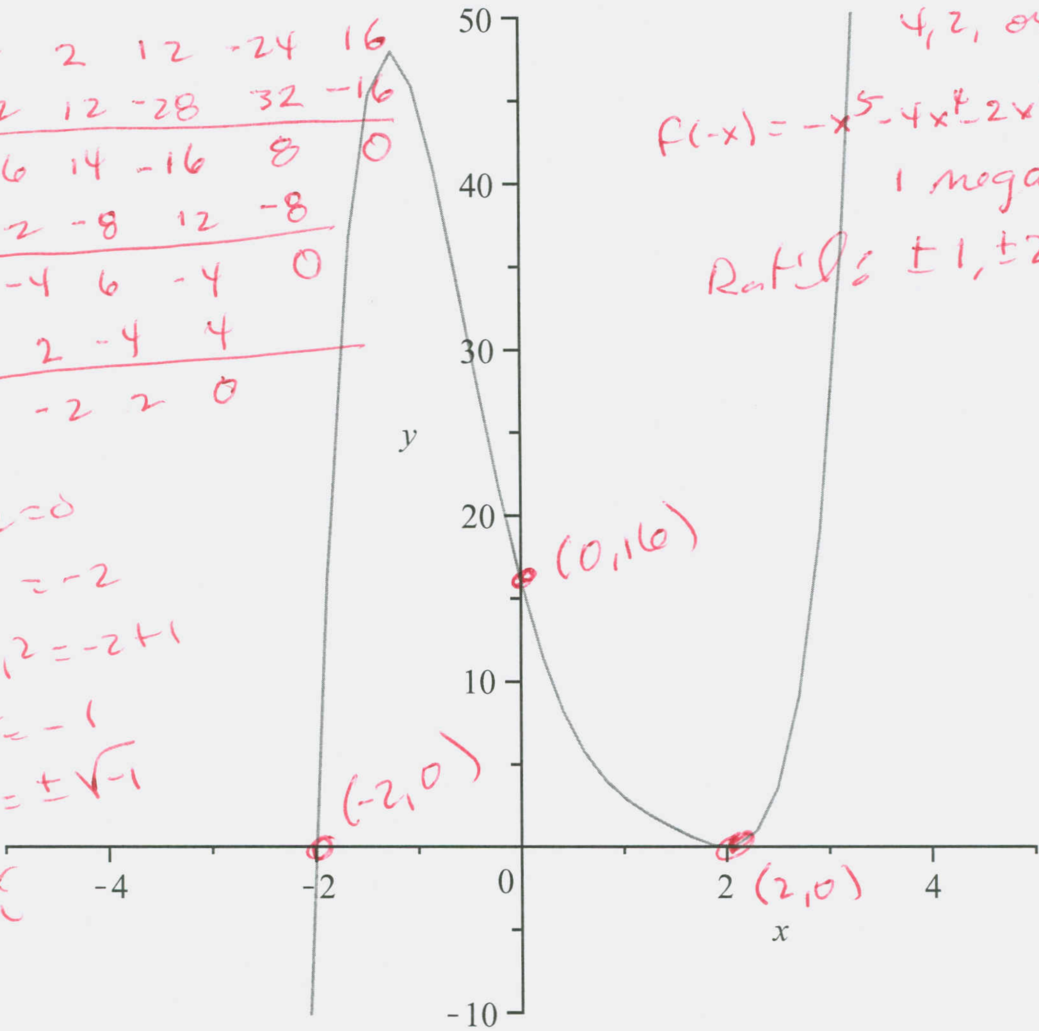
$factor(f(x), real)$

$$(x + 2.)\ (x - 2.)^2\ (x^2 - 2.0\,x + 2.) \qquad \textbf{(3)}$$

$factor(f(x), complex)$
$plot(f(x), x = -5..5, y = -10..50)$

Descartes:
4, 2, or 0 pos,
$f(-x) = -x^5 - 4x^4 - 2x^3 + 12x^2 + 24x + 16$
1 negative.
Rat'l: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

-2	1	-4	2	12	-24	16
		-2	12	-28	32	-16
2	1	-6	14	-16	8	0
		2	-8	12	-8	
2	1	-4	6	-4	0	
		2	-4	4		
	1	-2	2	0		



$x^2 - 2x + 2 = 0$
$x^2 - 2x = -2$
$x^2 - 2x + 1^2 = -2 + 1$
$(x-1)^2 = -1$
$x - 1 = \pm\sqrt{-1}$
$x = 1 \pm i$

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$f := x \to x^5 + x^4 - x^3 + 11 \cdot x^2 + 8 \cdot x - 20$

$$x \to x^5 + x^4 - x^3 + 11\,x^2 + 8\,x - 20 \tag{4}$$

$solve(f(x) = 0, x)$

$$1,\ 1 + 2\,\mathrm{I},\ 1 - 2\,\mathrm{I},\ -2,\ -2 \tag{5}$$

$factor(f(x), real)$

$$(x + 2.)^2\,(x - 1.000000000)\,(x^2 - 2.0\,x + 5.) \tag{6}$$

$factor(f(x), complex)$

$$(x + 2.)^2\,(x - 1. + 2.\,\mathrm{I})\,(x - 1.000000000)\,(x - 1. - 2.\,\mathrm{I}) \tag{7}$$

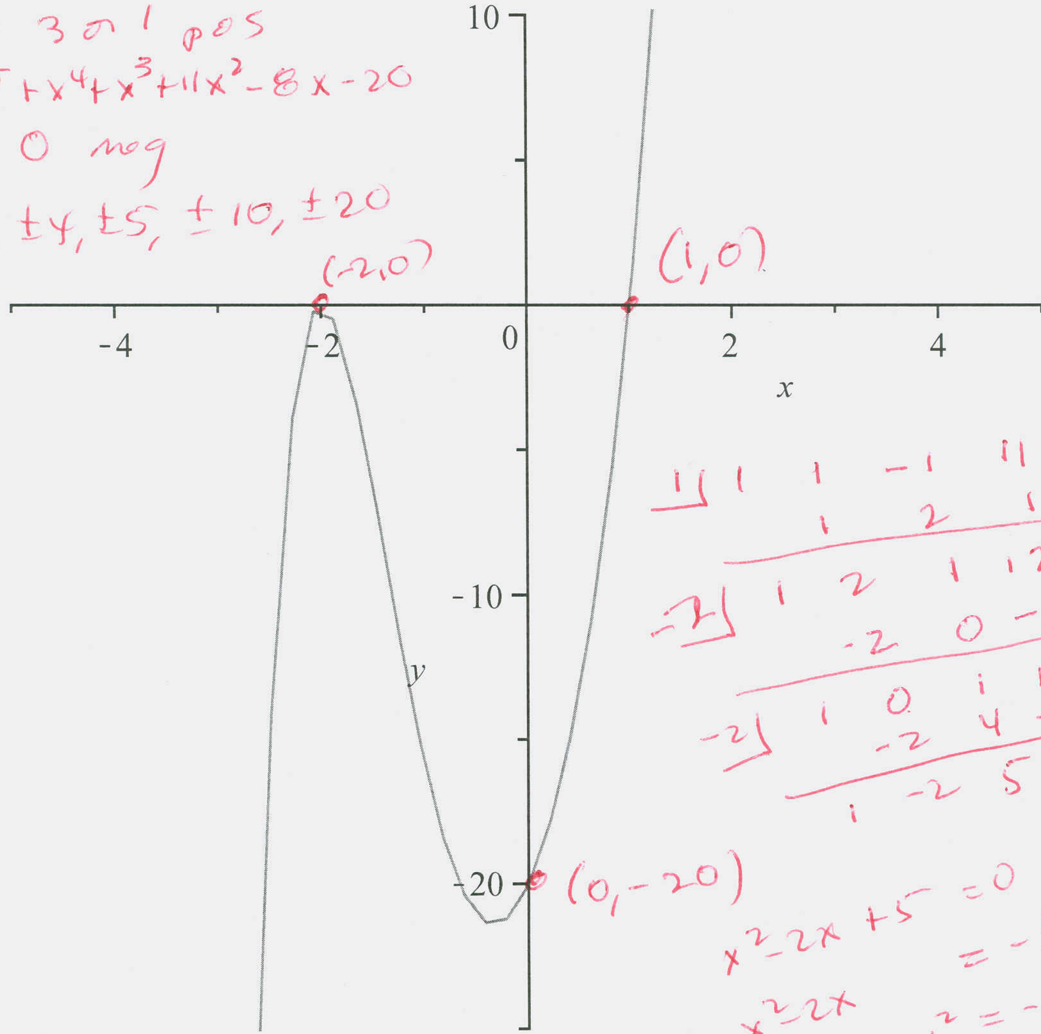
$plot(f(x), x = -5..5, y = -25..10)$

Descartes: 3 or 1 pos

$f(-x) = -x^5 + x^4 + x^3 + 11x^2 - 8x - 20$

2 or 0 neg

$\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$



$$\begin{array}{r|rrrrrr} 1 & 1 & 1 & -1 & 11 & 8 & -20 \\ & & 1 & 2 & 1 & 12 & 20 \\ \hline -2 & 1 & 2 & 1 & 12 & 20 & \\ & & -2 & 0 & -2 & -20 & \\ \hline -2 & 1 & 0 & 1 & 10 & & \\ & & -2 & 4 & -10 & & \\ \hline & 1 & -2 & 5 & & & \end{array}$$

$x^2 - 2x + 5 = 0$

$x^2 - 2x = -5$

$x^2 - 2x + 1^2 = -5 + 1$

$(x-1)^2 = -4$

$x - 1 = \pm\sqrt{-4}$

$x = 1 \pm 2i$

$f := x \rightarrow x^5 - 6 \cdot x^4 + 7 \cdot x^3 + 16 \cdot x^2 - 18 \cdot x - 20$

$$x \rightarrow x^5 - 6\,x^4 + 7\,x^3 + 16\,x^2 - 18\,x - 20 \quad \textbf{(8)}$$

$solve(f(x) = 0, x)$

$$2,\ 3 + \mathrm{I},\ 3 - \mathrm{I},\ -1,\ -1 \quad \textbf{(9)}$$

$factor(f(x), real)$

$$(x + 1.)^2\,(x - 2.000000000)\,(x^2 - 6.000000000\,x + 10.00000000) \quad \textbf{(10)}$$

$factor(f(x), complex)$

$$(x + 1.)^2\,(x - 2.000000000)\,(x - 3.000000000 + 1.000000000\,\mathrm{I})\,(x - 3.000000000 - 1.000000000\,\mathrm{I}) \quad \textbf{(11)}$$

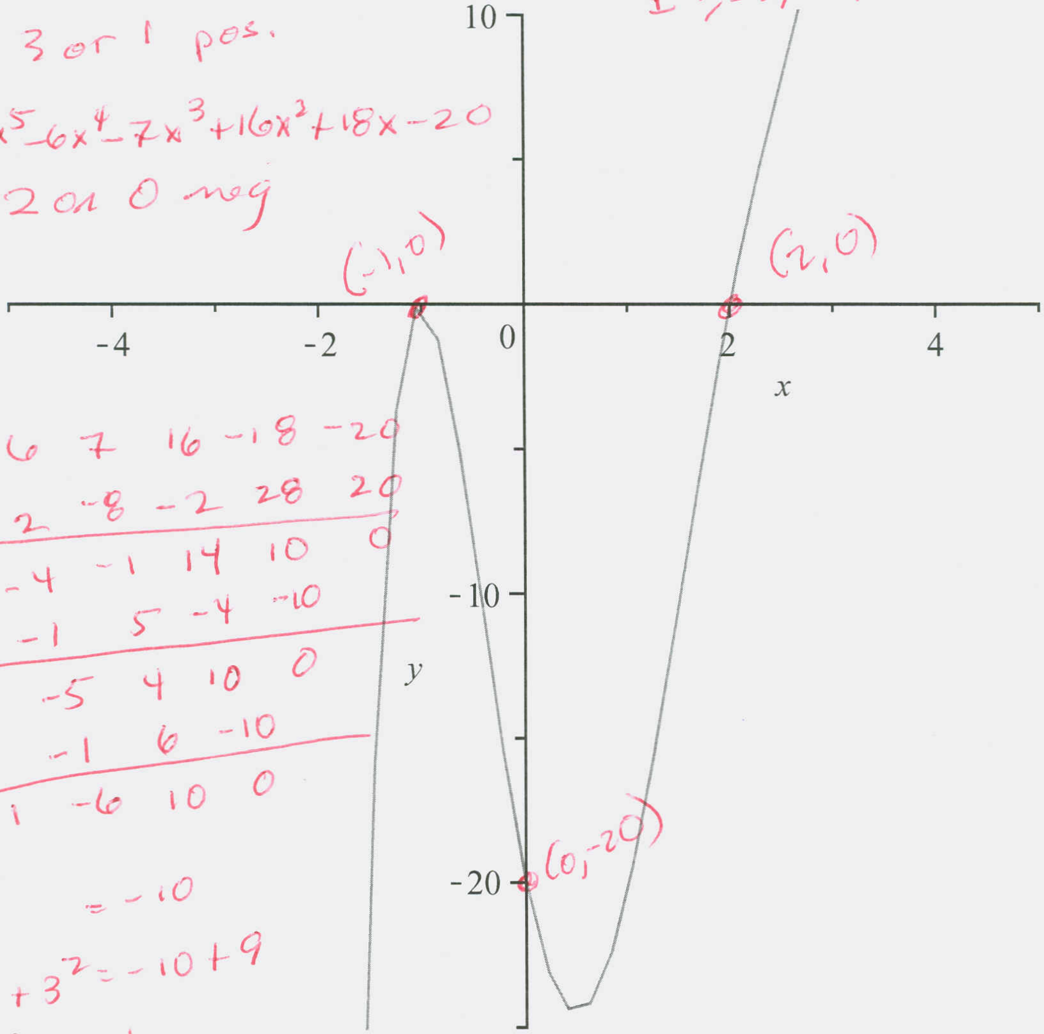
$plot(f(x), x = -5..5, y = -25..10)$

$\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

Des ; 3 or 1 pos.

$f(-x) = -x^5 - 6x^4 - 7x^3 + 16x^2 + 18x - 20$

. 2 or 0 neg



2	1	-6	7	16	-18	-20
		2	-8	-2	28	20
-1	1	-4	-1	14	10	0
		-1	5	-4	-10	
-1	1	-5	4	10	0	
		-1	6	-10		
	1	-6	10	0		

$x^2 - 6x = -10$

$x^2 - 6x + 3^2 = -10 + 9$

$(x-3)^2 = -1$

$x - 3 = \pm i$

$x = 3 \pm i$

$f := x \rightarrow x^5 - 6 \cdot x^4 + 7 \cdot x^3 + 20 \cdot x^2 - 42 \cdot x + 20$

$$x \rightarrow x^5 - 6\,x^4 + 7\,x^3 + 20\,x^2 - 42\,x + 20 \qquad \textbf{(12)}$$

$solve(f(x) = 0, x)$

$$-2,\ 3 + I,\ 3 - I,\ 1,\ 1 \qquad \textbf{(13)}$$

$factor(f(x), real)$

$$(x + 2.)\ (x - 1.)^2\ (x^2 - 6.0\,x + 10.00000000) \qquad \textbf{(14)}$$

$factor(f(x), complex)$

$$(x + 2.)\ (x - 1.)^2\ (x - 3. + 1.000000000\,I)\ (x - 3. - 1.000000000\,I) \qquad \textbf{(15)}$$

$plot(f(x), x = -5\,..5, y = -10\,..75)$

Descartes

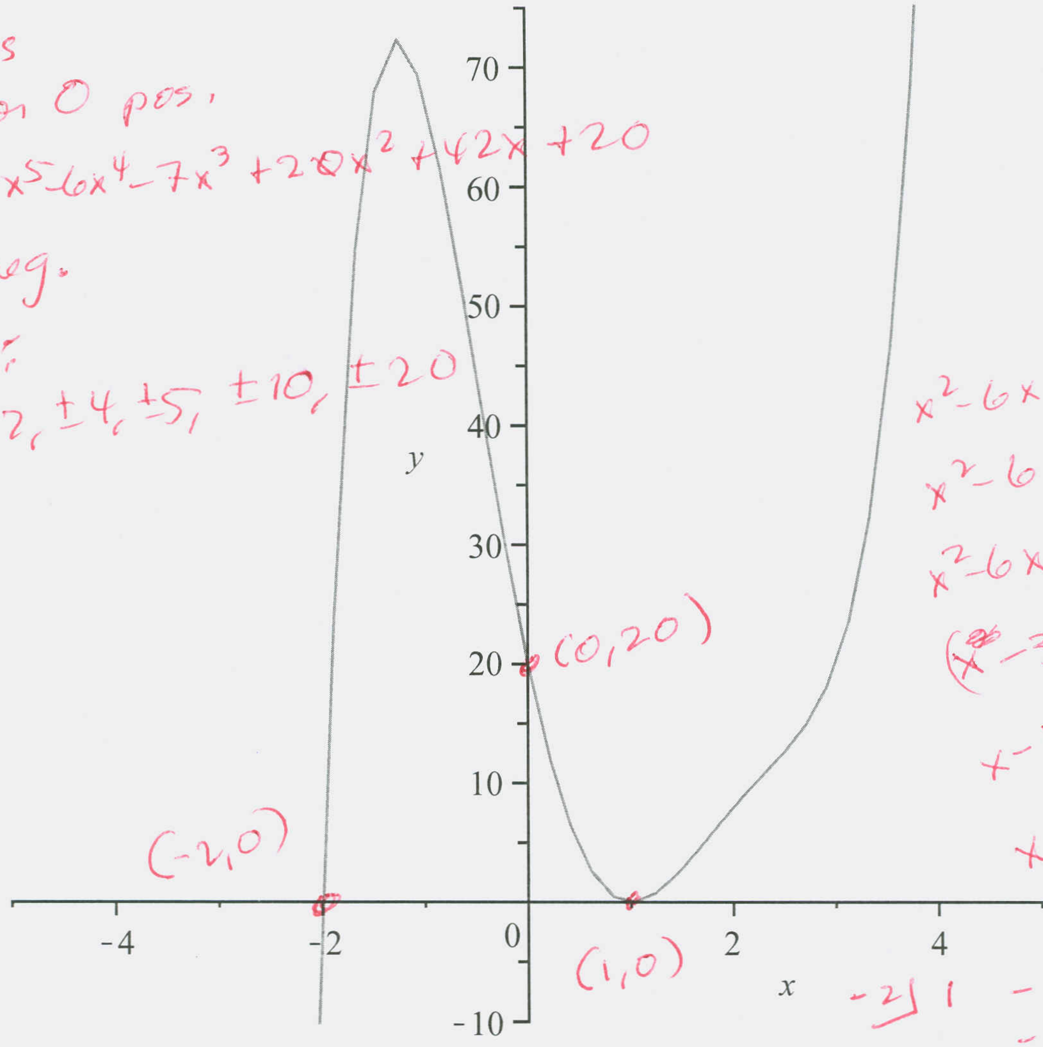
4, 2, or 0 pos.

$f(-x) = -x^5 - 6x^4 - 7x^3 + 20x^2 + 42x + 20$

1 neg.

Rat'l:

$\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$



$x^2 - 6x + 10 = 0$

$x^2 - 6x = -10$

$x^2 - 6x + 3^2 = -10 + 9$

$(x - 3)^2 = -1$

$x - 3 = \pm\sqrt{-1}$

$x = 3 \pm i$

$$\begin{array}{r|rrrr} -2 & 1 & -4 & -2 & 20 \\ & & -2 & 12 & -20 \\ \hline & 1 & -6 & 10 & \end{array}$$

$$\begin{array}{r|rrrrrr} 1 & 1 & -6 & 7 & 20 & -42 & 20 \\ & & 1 & -5 & 2 & 22 & -20 \\ \hline 1 & 1 & -5 & 2 & 22 & -20 & \\ & & 1 & -4 & -2 & 20 & \\ \hline & 1 & -4 & -2 & 20 & & \end{array}$$