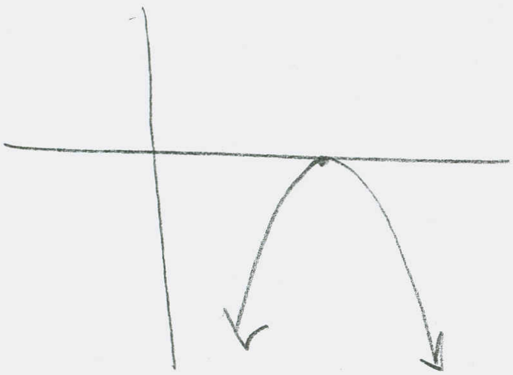
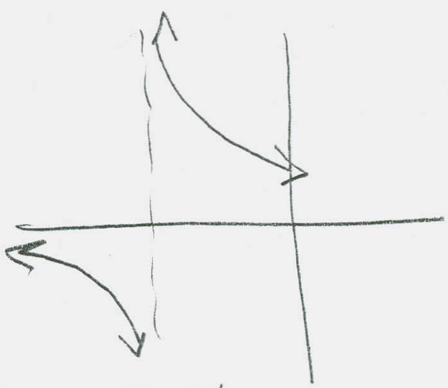
MAT 121 Online 80 Points   Fall, 2010
Test 3, Chapter 3   Name KEY

1. Give a (quick) rough sketch of the following:

a. (5 pts) $f(x) = -2(x-5)^4$



b. (5 pts) $f(x) = \dfrac{1}{(x+4)^3}$



$x = -4$

2. In each of the following, form a polynomial with real coefficients that has the given zeros and degree. Please do not expand the polynomial.

a. (5 pts) Zeros: 3, multiplicity 1; 5, multiplicity 3; - 2, multiplicity 1 Degree 5.

$(x-3)(x-5)^3(x+2)$

b. (5 pts) Zeros: 3, multiplicity 1; 5, multiplicity 2; $2+3i$, multiplicity 1. Degree 5.

$(x-3)(x-5)^2(x-(2+3i))(x-(2-3i))$

3. (5 pts) Expand $(x-(2-3i))(x-(2+3i))$

$= x^2 - (2+3i)x - (2-3i)x + (2-3i)(2+3i)$

$= x^2 - (2x+3ix) - (2x-3ix) + 2^2+3^2$

$= x^2 - 2x - 3ix - 2x + 3ix + 13$

$= x^2 - 4x + 13$

4. Let $f(x) = (x-1)(x+3)^2(x+1)^3$.

a. (5 pts) List each real zero and its multiplicity. Determine whether the graph of $f(x)$ touches or crosses the x-axis at each x-intercept.

$x=1$, $m=1$ cross

$x=-3$, $m=2$ touch

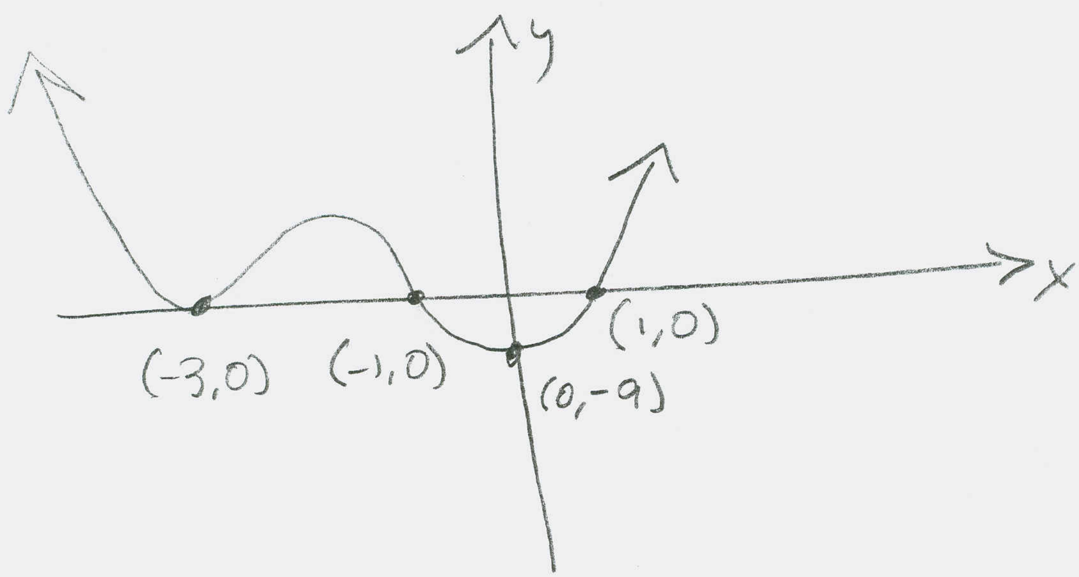
$x=-1$, $m=3$ cross

b. (5 pts) Determine the power function that $f(x)$ resembles for large $|x|$. This is the End Behavior part of the question. Show the end behavior with a simple figure or diagram.

$(x)(x)^2(x)^3 = x^6$

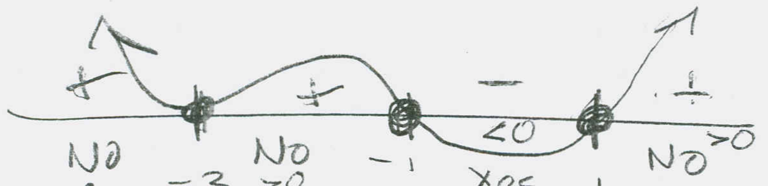
c. (5 pts) Use the information you reported to obtain a rough graph of $f(x)$. Show all intercepts, including the y-intercept.

$f(0) = (-1)(3)^2(1)^3 = -9 \rightarrow (0,-9)$



5. Solve the inequalities.

   a. (5 pts) $(x-1)(x+3)^2(x+1)^3 < 0$ (See previous work! If you know how to graph polynomials in factored form, this one is virtually a freebie!)



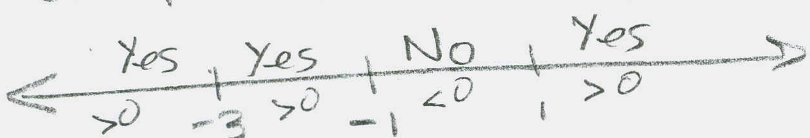
Want "< 0"

$(-\infty,-3)$ $(-3,-1)$ $(-1,1)$ $(1,\infty)$

$x \in (-1,1)$

   b. (5 pts) $\dfrac{(x-1)(x+3)^2}{(x+1)^3} \geq 0$ (See previous work!)

Not Defined at $x=-1$. Otherwise, include end points, because of the "$\geq$"



$$\boxed{(-\infty,-1) \cup [1,\infty)}$$

Bonus (5 pts) What is the domain of $\sqrt{\dfrac{(x-1)(x+3)^2}{(x+1)^3}}$?

$(-\infty,-1) \cup [1,\infty)$

6. (10 pts) Use Descarte's Rule of Signs and the Rational Zeros Theorem to find all the real zeros of $f(x) = 3x^5 - 17x^4 + 25t^3 + 65x^2 - 128x + 52$. Then use the *real* zeros to factor f over the real numbers. This is *likely* to involve an irreducible quadratic factor. I would advise using scratch paper to *find* the zeros and then do the work with *them* to break down f in the space below.

4, 2, or 0 positives

$f(-x) = -3x^5 - 17x^4 - 25x^3 + 65x^2 + 128x + 52$

$2 \lfloor 52$, $2 \lfloor 26$, 13

ONE Negative zero.

$p: 52$, $q: 3$ $\Big\}$ $\frac{p}{q} = \pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}, \pm 4, \pm \frac{4}{3}, \pm 13, \pm \frac{13}{3}, \pm 26, \pm \frac{26}{3}, \pm 52, \pm \frac{52}{3}$

Guess-and-check yields $\boxed{x = 1,\ x = \frac{2}{3},\ x = -2}$

$$\begin{array}{r|rrrrrr} 1 & 3 & -17 & 25 & 65 & -128 & 52 \\ & & 3 & -14 & 11 & 76 & -52 \\ \hline -2 & 3 & -14 & 11 & 76 & -52 & 0 \\ & & -6 & 40 & -102 & 52 & \\ \hline \frac{2}{3} & 3 & -20 & 51 & -26 & 0 & \\ & & 2 & -12 & 26 & & \\ \hline & 3 & -18 & 39 & 0 & & \end{array}$$

$(x-1)(3x^4 - 14x^3 + 11x^2 + 76x - 52)$

$(x-1)(x+2)(3x^3 - 20x^2 + 51x - 26)$

$\boxed{(x-1)(x+2)(x-\tfrac{2}{3})(3x^2 - 18x + 39)}$

Leaves $3x^2 - 18x + 39 \overset{SET}{=} 0 \Rightarrow 3(x^2 - 6x + 13) = 0$

$\Rightarrow b^2 - 4ac = (-6)^2 - 4(1)(13) = 36 - 52 = -16$

NO REAL ROOT

7. (5 pts) Based on your work in #6, above, find *all* the (real *and* nonreal) zeros of $f(x) = 3x^5 - 17x^4 + 25t^3 + 65x^2 - 128x + 52$. Use *all* the zeros to write $f(x)$ as the product of *linear* factors.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{-16}}{2(1)} = \frac{6 \pm 4i}{2} \Rightarrow \boxed{3 \pm 2i}$$

is the remaining pair of zeros

$f(x) = 3(x-1)(x+2)(x - \tfrac{2}{3})(x - (3+2i))(x - (3-2i))$

8. (5 pts) Divide $f(x)=2x^4-3x^3+x-3$ by $f(x)=x^2-2$. Write your final answer in the form $f(x)=(\text{Divisor})(\text{Quotient})+\text{Remainder}$

$$
\begin{array}{r l}
 & 2x^2-3x+4 \quad r\ -5x+5 \\
x^2-2 \,\big) & 2x^4-3x^3+0x^2+x-3 \\
 & -(2x^4 \quad\quad -4x^2) \\
 & -3x^3+4x^2+x-3 \\
 & -(-3x^3+\quad 6x) \\
 & 4x^2-5x-3 \\
 & -(4x^2 \quad -8) \\
 & -5x+5
\end{array}
$$

$$\boxed{\therefore\ f(x)=(x^2-2)(2x^2-3x+4)+(-5x+5)}$$

x^3-2x^2-5x+6

9. (10 pts) Graph the function $R(x)=\dfrac{2x^3+10x^2+16x+8}{x^3-7x-6}=\dfrac{(2x^2+6x+4)(x+2)}{(x^2+x-2)(x-3)}$. Key features are asymptotes, holes (if any) and intercepts. I was kind enough to factor it for you.

x^3-2x^2-5x+6

DOMAIN: $(x+2)(x-1)(x-3)=0 \Longrightarrow \{x \mid x\neq -2, 1, 3\} = \mathcal{D}$

$2x^2+6x+4 = 2(x^2+3x+2) = 2(x+2)(x+1)$

$\Longrightarrow R(x)=\dfrac{2\cancel{(x+2)}(x+1)(x+2)}{\cancel{(x+2)}(x-1)(x-3)} = \dfrac{2(x+1)(x+2)}{(x-1)(x-3)} \quad (x\neq -2)$

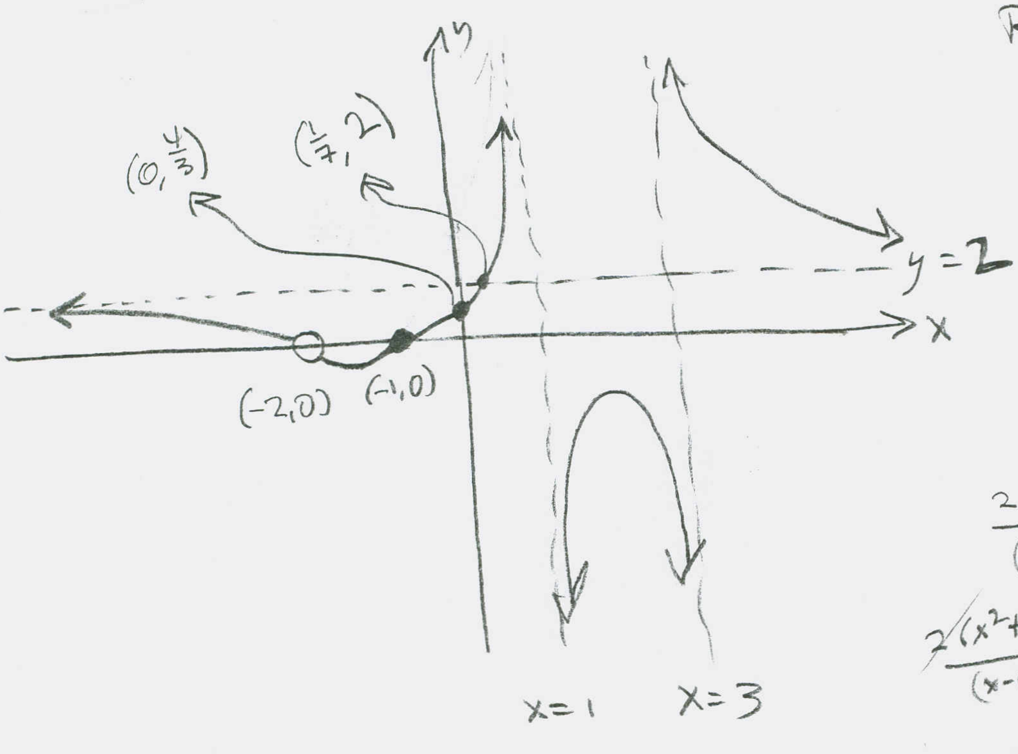
Hole @ $x=-2$: $\dfrac{2(-2+1)(-2+2)}{(-2-1)(-2-3)} = 0 \rightsquigarrow (-2,0)$

H.A.: $\dfrac{2x^3+\text{small}}{x^3+\text{small}} \longrightarrow \dfrac{2}{1}=2=y$ is Horiz. Asymp.

zeros: $x=-1$, $x=-2$ $\rightsquigarrow$ Hole

V.A.: $x=1$, $x=3$

$R(0)=\frac{8}{6}=\frac{4}{3}$



To get fancy, see if $R(x)$ crosses its H.A.:

$\dfrac{2(x+1)(x+2)}{(x-1)(x-3)} \overset{\text{SET}}{=} 2$

$\dfrac{\cancel{2}(x^2+3x+2)}{(x-1)(x-3)} = \dfrac{\cancel{2}(x^2-4x+3)}{(x-1)(x-3)} \Longrightarrow$

$3x+2=-4x+3 \Longrightarrow$

$7x=1 \Longrightarrow$

$x=\frac{1}{7} \rightsquigarrow (\frac{1}{7}$