MAT 121 Test 2

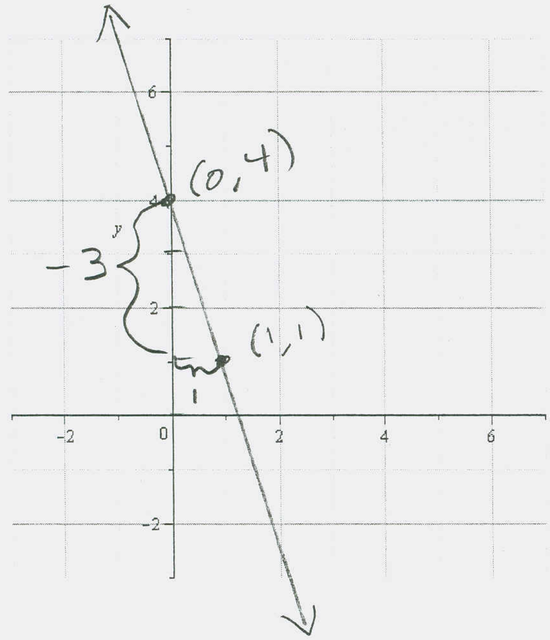
100 Points Name KEY

1. Let $f(x) = -3x + 4$ in the following:

a. (5 pts) Determine the slope and y-intercept of f.

$m = -3$, $y\text{-int} = (0,4)$

b. (5 pts) Use the slope and y-intercept to graph f here:



c. (5 pts) Determine the average rate of change of f.

$m = -3$

d. (5 pts) Is f increasing, decreasing or constant?

Decreasing

2. (5 pts) Suppose y varies jointly as m_1 and m_2 and inversely as the square of r. If $y = 2$ when $m_1 = 3$, $m_2 = 8$, and $r = 2$ what is y when $m_1 = 15$, $m_2 = 10$, and $r = 5$?

$$y = k\frac{m_1 m_2}{r}$$

$$2 = k\frac{(3)(8)}{2^2}$$

$$8 = 24k$$

$$\frac{1}{3} = k$$

$$y = \frac{1}{3}\frac{(15)(10)}{5^2}$$

$$= \frac{(5)(10)}{25}$$

$$= \boxed{2 = y}$$

3. Let $f(x) = x^2 - 8x - 33$.

a. (5 pts) Find the zeros of f by factoring.

$(x-11)(x+3)=0$

$\boxed{x \in \{-3, 11\}}$

b. (5 pts) Find the zeros of f by quadratic formula.

$a=1,\ b=-8,\ c=-33$

$b^2-4ac = (-8)^2-4(1)(-33)$

$= 64+132$

$=196$

$\sqrt{196} = 14$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{+8 \pm 14}{2}$

$\frac{22}{2} = 11$

$-\frac{6}{2} = -3$

$\boxed{x \in \{-3, 11\}}$

c. (5 pts) Find the zeros of f by completing the square.

$x^2-8x = 33$

$x^2-8x+4^2 = 33+16$

$(x-4)^2 = 49$

$x-4 = \pm 7$

$x = 4 \pm 7$

11

-3

$\boxed{x \in \{-3, 11\}}$

4. (20 pts) Complete the square for $f(x) = x^2 - 6x + 1$, and re-write it in the form $a(x-h)^2 + k$. Use your result to answer the questions, below. You don't *have* to graph the function, but you'll be answering questions related to its graph, so a rough sketch wouldn't hurt.

$f(x) = x^2 - 6x + 1$

$= x^2 - 6x + 3^2 - 9 + 1$

$= (x-3)^2 - 8 \overset{\text{SET}}{=} 0$

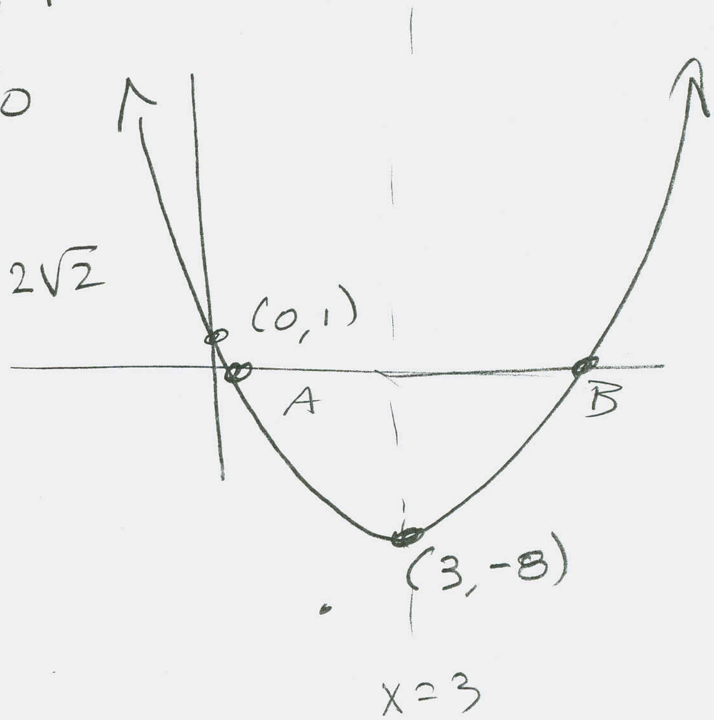
$\Rightarrow (x-3)^2 = 8$

$x - 3 = \pm\sqrt{8} = \pm 2\sqrt{2}$

$x = 3 \pm 2\sqrt{2}$

$A = (3 - 2\sqrt{2}, 0)$

$B = (3 + 2\sqrt{2}, 0)$



a. Give the location of the vertex. $(3, -8)$

b. State the equation of the axis of symmetry. $x = 3$

c. Give the location of the y-intercept. $(0, 1)$

d. Give the location of the x-intercept(s), if any. (Simplify any radicals as appropriate). $(3 - 2\sqrt{2}, 0)$, $(3 + 2\sqrt{2}, 0)$

e. State the domain in interval notation. $(-\infty, \infty)$

f. State the range in interval notation. $[-8, \infty)$

g. State the interval(s) of increase in interval notation. $(3, \infty)$

h. State the interval(s) of decrease in interval notation $(-\infty, 3)$

5. Consider the quadratic function $h(x) = 6x^2 - 5x + 3$.

a. (5 pts) Compute the discriminant for h.

$a=6, b=-5, c=3$

$b^2-4ac = (-5)^2-4(6)(3) = 25-72 = -47$

b. (5 pts) Based on your answer to part a., describe the nature of the zeros of h. In other words, state how many zeros h has, and whether they're real or nonreal. You do not need to solve the equation.

$\boxed{\text{2 nonreal zeros}}$

6. (10 pts) Find the complex zeros of $f(x) = 4x^2 - 8x + 13$

$a=4, b=-8, c=13$

$b^2-4ac = (-8)^2-4(4)(13)$

$= 64 - 16(13) =$

$= 16(4) - 16(13)$

$= -16(9)$

$= -144$

$\sqrt{-144} = 12i$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \dfrac{8 \pm 12i}{2(4)} = \dfrac{4(2 \pm 3i)}{2(4)}$

$= \boxed{\dfrac{2 \pm 3i}{2} = x}$

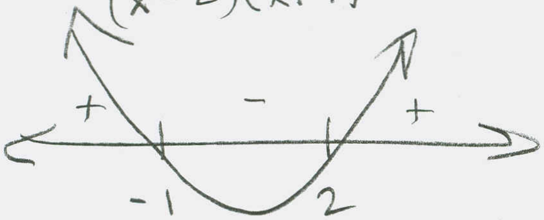
i

7. (10 pts) Solve $x^2 - x > 2$. Express your answer in both set-builder and interval notation. Visuals Two ways

$x^2 - x > 2$

$x^2 - x - 2 > 0$

$(x-2)(x+1) > 0$



">0" where it's above

$\boxed{x \in (-\infty, -1) \cup (2, \infty) = \{x \mid x < -1 \text{ OR } x > 2\}}$

TEST VALUES:

$(-\infty, -1)$ $x = -2 \Rightarrow 4 > 0$ Yes

$(-1, 2)$ $x = 0 \Rightarrow -2 < 0$ NO

$(2, \infty)$ $x = 3 \Rightarrow 4 > 0$ Yes

$(-\infty, -1) \cup (2, \infty)$ is where the Yes's live.

8. (5 pts) Solve $|2x + 3| = 3$

$2x + 3 = 3$ OR $2x + 3 = -3$

$2x = 0$ OR $2x = -6$

$x = 0$ OR $x = -3$

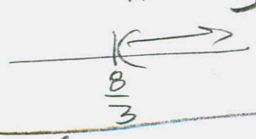
$x \in \{-3, 0\}$

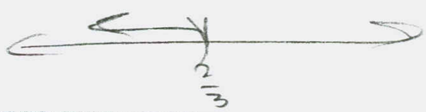
9. (5 pts) Solve $|3x - 5| > 3$

$3x - 5 > 3$ OR $3x - 5 < -3$

$3x > 8$ OR $3x < 2$

$x > \frac{8}{3}$ OR $x < \frac{2}{3}$





$\boxed{\left(\frac{8}{3}, \infty\right) \cup \left(-\infty, \frac{2}{3}\right)} = \{x \mid x < \frac{2}{3} \text{ OR } x > \frac{8}{3}\}$