MAT 121-G81 Test 1 Fall, 2010

100 Points

Name KEY

1. (8 pts) Determine whether the relation below represents a function. What is the domain and what is the range?

$$\{(-3,6), (7, 5), (4, 9), (-5, 6)\}$$

Yes

$D = \{-3, 7, 4, -5\}$

$R = \{6, 5, 9\}$

2. (5 pts) Determine whether the equation $x^2 - y = 9$ defines y as a function of x. If it does *not*, show/explain why not.

Yes, it does. $y = x^2 - 9$

3. Let $f(x) = \dfrac{4}{x-4}$. Determine the following, if possible. If not possible, state why:

a. (2 pts) $f(2) = \dfrac{4}{2-4} = \dfrac{4}{-2} = -2$

b. (2 pts) $f(3) = \dfrac{4}{3-4} = \dfrac{4}{-1} = -4$

c. (2 pts) $f(4) = \dfrac{4}{4-4} = \dfrac{4}{0}$ $\not\exists$. Division by zero

4. (7 pts) Find the domain of $g(x) = \dfrac{x^2 + 5x + 17}{x^2 - x - 12}$.

$D = \{x \mid x \neq -3 \ \& \ x \neq 4\}$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x = -3, 4$ make denominator zero.

5. (4 pts) Let $f(x) = 3x^2$. Find the average rate of change of f from $x = -1$ to $x = 1$.

$\dfrac{f(1) - f(-1)}{1 - (-1)} = \dfrac{3(1)^2 - 3(-1)^2}{2} = \dfrac{0}{2} = 0$

6. Let $f(x) = 2x - 6$ and $g(x) = \sqrt{x-3}$.
   a. (5 pts) Determine the domain of *f*.

$\boxed{(-\infty, \infty) = \mathcal{D}}$

   b. (5 pts) Determine the domain of *g*.

Need $x - 3 \geq 0 \Rightarrow x \geq 3 \Rightarrow \{x \mid x \geq 3\} = \boxed{\mathcal{D} = [3, \infty)}$

   c. Find the following functions and state the domain of each.
      i. (3 pts) $(f+g)(x)$

$= 2x - 6 + \sqrt{x-3}$ $\boxed{\mathcal{D} = [3, \infty)}$

      ii. (3 pts) $(f \cdot g)(x) = (2x-6)(\sqrt{x-3})$

$\mathcal{D} = [3, \infty)$

      iii. (3 pts) $\left(\frac{f}{g}\right)(x) = \dfrac{2x-6}{\sqrt{x-3}}$ $\mathcal{D} = (3, \infty)$

$= \{x \mid x > 3\}$

7. Determine algebraically whether each function is even, odd, or neither.

   a. (2 pts) $h(x) = 4x^2 + 6x^6 - 2$

$h(-x) = 4(-x)^2 + 6(-x)^6 - 2$

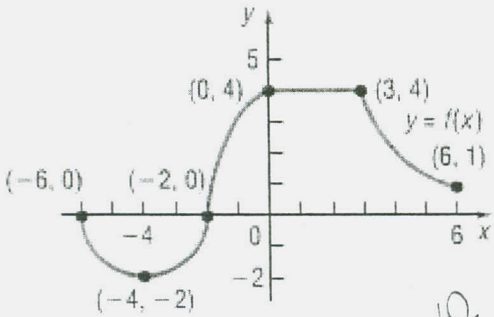
$= 4x^2 + 6x^6 - 2 = h(x) \Rightarrow$ Even

   b. (2 pts) $g(x) = 3x^3 - 5$

$g(-x) = 3(-x)^3 - 5 = -3x^3 - 5$ Neither.

$(-g(x) = -3x^3 + 5)$

8. Use the graph of the function f, below, to find:



a. (5 pts) The intercepts (Express answers in ordered pairs.

(-6,0), (-2,0), (0,4)

b. (5 pts) The domain and range.

$\mathcal{D} = [-6,6]$, $\mathcal{R} = [-2,4]$

c. The local extreme points (Give actual points on the graph.)

i. (2 pts) Does f have any local maxima? Where? No

ii. (2 pts) Does f have any local minima? Where? Yes. @ (-4,-2)

d. The intervals on which f is increasing, decreasing, or constant.

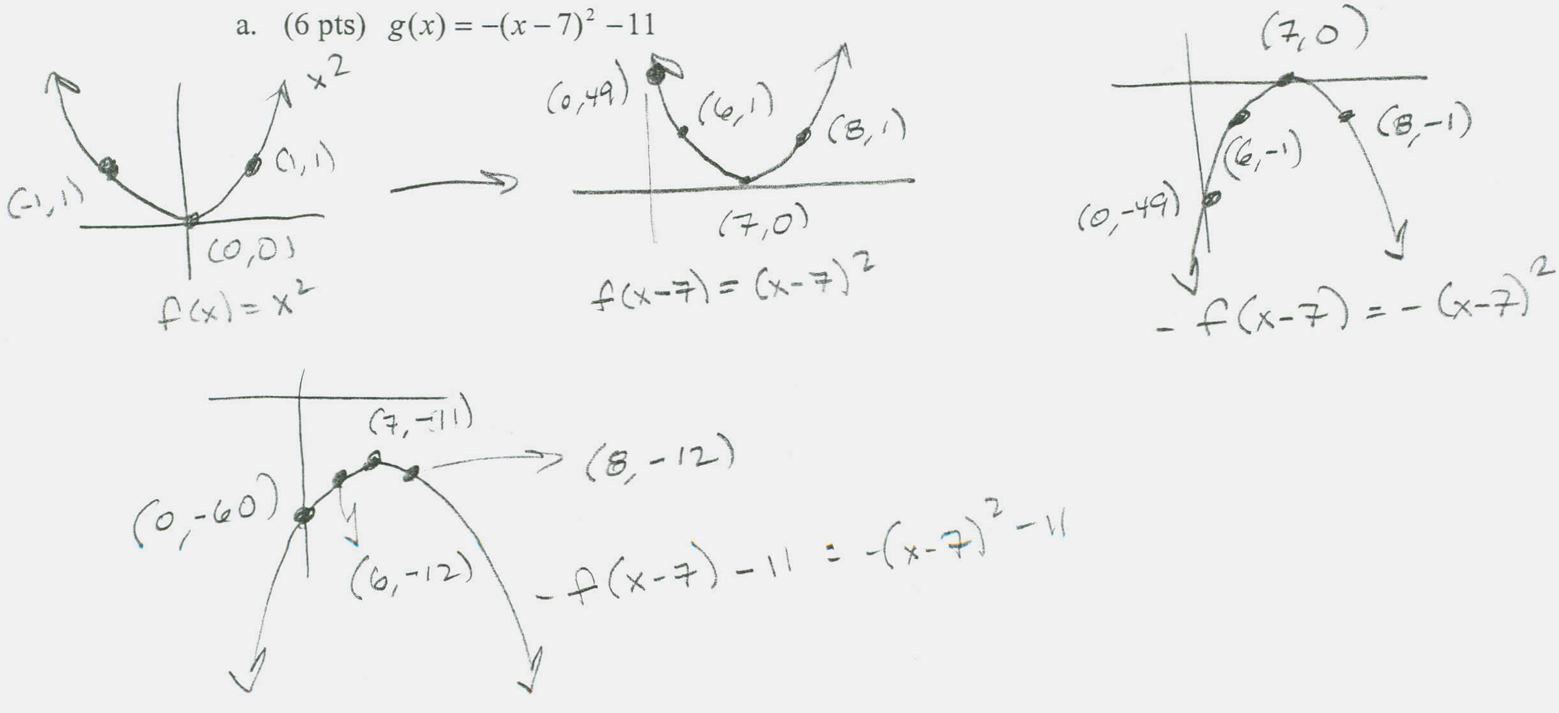
i. (2 pts) f is increasing on (-4, 0)

ii. (2 pts) f is decreasing on $(-6,-4) \cup (3,6)$

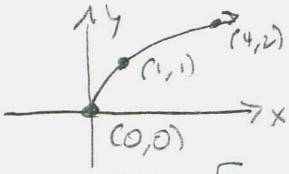
iii. (2 pts) f is constant on (0, 3)

9. Graph each of the following functions using the techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function and show all stages.

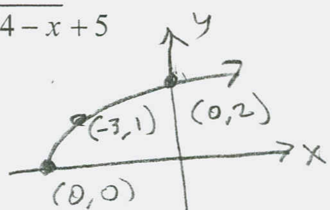
a. (6 pts) $g(x) = -(x-7)^2 - 11$



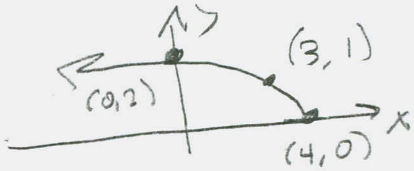
b. (7 pts) $g(x) = \sqrt{4-x} + 5$



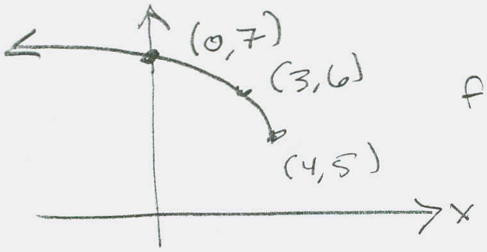
$f(x) = \sqrt{x}$



$f(x+4) = \sqrt{x+4}$



$f(-x+4) = f(4-x) = \sqrt{4-x}$

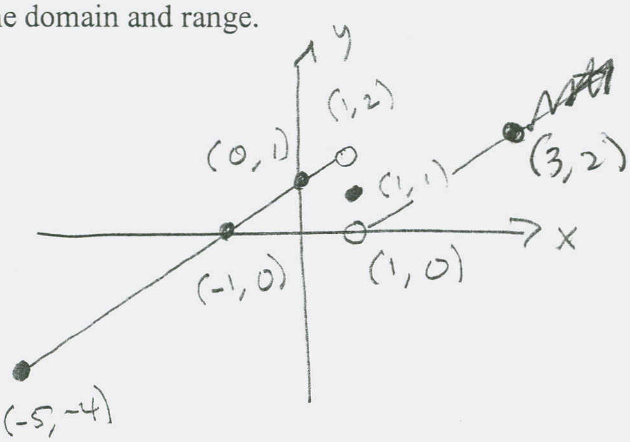


$f(4-x) + 5 = \sqrt{4-x} + 5$

$(-5,-4)$ $(1,2)$

10. (8 pts) Sketch the graph of $f(x) = \begin{cases} x+1 & \text{if } -5 \le x < 1 \\ 1 & \text{if } x = 1 \\ x-1 & \text{if } 1 < x \le 3 \end{cases}$. Include all intercepts.

State the domain and range.



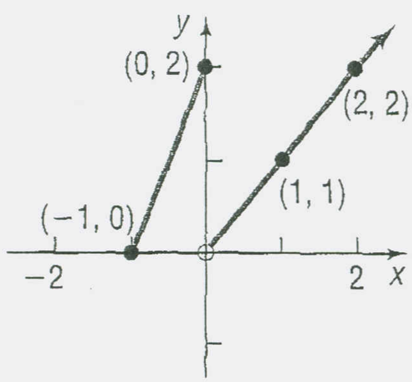
~~$\mathcal{D} = [-5, \infty)$~~

~~$\mathcal{R} = [-4, \infty)$~~

$\mathcal{D} = [-5, 3]$

$\mathcal{R} = [-4, 2]$

11. (6 pts) Determine the piecewise-defined function g from its graph, below.



$m = \frac{2-0}{0-(-1)} = \frac{2}{1} = 2$

$y = 2x + 2 \quad -1 \le x \le 0$

$m = \frac{2-1}{2-1} = 1$

$y = 1x + 0 \quad 0 < x$

$$\boxed{f(x) = \begin{cases} 2x+2 & \text{if } -1 \le x \le 0 \\ x & \text{if } x > 0 \end{cases}}$$